Analysis of Vibration and Mechanical Properties of Soundboards Based on Thin Plate Theory

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Abstract. In order to optimize the material selection and structure in the design of musical instruments, the vibration characteristics of a rectangular soundboard are modeled and simulated based on the thin plate vibration theory. Firstly, the geometrical dimensions (length, width and thickness) of the soundboard are set as the basic parameters, and the vibration model is constructed to derive the partial differential equations. After simplifying the equations by the method of separating variables, the finite difference method was utilized to solve the equations in Matlab to obtain the first few orders of the modal vibration pattern images, and the first three orders of the intrinsic frequency data were extracted. In order to study the material effects, four typical materials were selected, and polynomials were fitted to the frequency data with the help of numpy, matplotlib and scipy libraries in Python to obtain the expressions of vibration equations. The results show that the material properties (e.g., modulus of elasticity, density and Poisson's ratio) have a significant effect on the vibration frequencies and modes, which provides theoretical support for the design of musical instruments.

Keywords: Thin Plate Theory, Separated Variables Method, Finite Difference Method, Polynomial Fitting.

1. Introduction

By the end of the 20th century, the world's musical instrument industry has formed a perfect production system, capable of processing the world's major types of musical instruments, covering a wide range of categories and varieties. In the field of stringed instruments (e.g., piano, violin, guitar and erhu), the soundboard, as a core component, plays a decisive role in the quality of sound. Due to the low efficiency of sound energy radiation from string vibration, soundboard vibration needs to be stimulated to enhance the sound energy radiation. The soundboard, as a continuous thin elastic plate, generates various vibration modes under the excitation of the strings, thus generating rich and pleasant harmonics.

With the development of numerical computation techniques such as the finite difference method (FDM) and the separated variables method (SVM), they have been widely used in the fields of mechanical engineering and aerospace to solve complex partial differential equations (PDEs) and engineering problems. The finite difference method can efficiently simulate phenomena such as fluid dynamics, heat transfer and structural mechanics by discretizing continuous problems, for example, simulating the aerodynamic performance of an aircraft wing or analyzing the thermal stress distribution of an engine component [1][2]. Separation of variables methods are often used to solve fluctuation and heat transfer equations, e.g., to study the intrinsic frequencies and vibration shapes of structures or to analyze the dynamic response of aircraft under aerodynamic loading. These methods provide powerful tools for design and performance evaluation and have significantly contributed to technological progress and innovation.

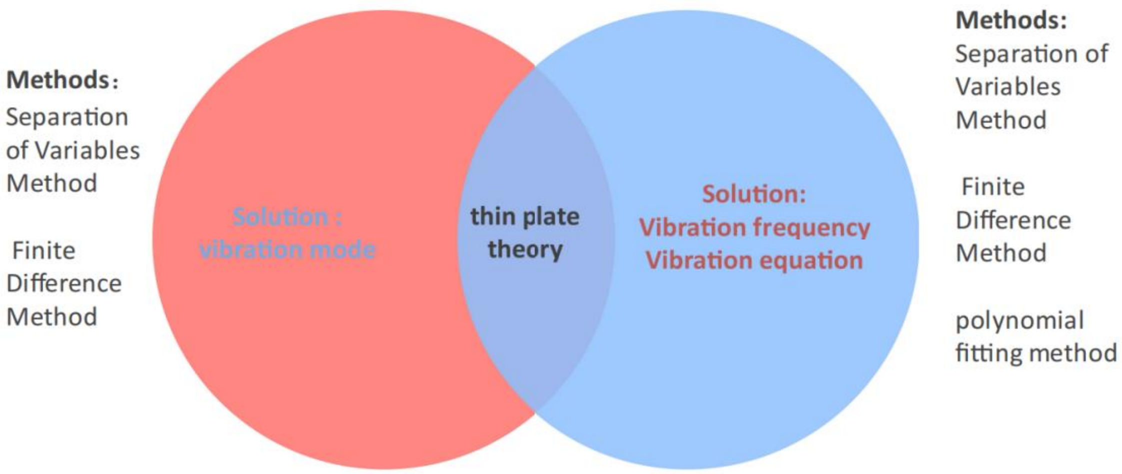

Figure 1. Pipeline of our work

Based on our completed work, we have prepared a job summary as shown in the Figure 1.The vibration modes of the sound board are closely related to its geometry, thickness and material properties (e.g. density, Young's modulus, shear modulus and Poisson's ratio). In order to study its vibration characteristics, based on the thin-plate theory, combined with the assumed rectangular plate parameters and spruce material data, the vibration equations of the soundboard are established, and the equations are simplified by the separation of variables method, and finally solved by the finite-difference method to obtain the first five-order free vibration modal images. Moreover, with the progress of musical instrument manufacturing technology, designers and manufacturers pay more attention to the scientific selection and optimization of materials. By studying the vibration characteristics of materials, we can provide a theoretical basis for the design of musical instruments and help optimize the material selection, structural design and manufacturing process. This study focuses on the vibration characterization of thin sound boards with uniform orientation.

We conduct extensive experiments on the finite element simulation to evaluate our approach. Our contributions can be summarized as follows:

(1) Vibration equations and intrinsic frequencies under different materials, we selected four materials, namely, spruce wood, carbon fiber, copper and PLA, and obtained their vibration images, and then extracted the image data, and obtained the vibration equations of the first few orders of the different materials and the corresponding intrinsic frequencies by using polynomial fitting method.

(2) Using the finite difference method in matlab, we solved the first five orders of the vibration modal diagrams of a rectangular thin plate made of cloudy wood, and then by extracting the data from the vibration modal diagrams, we obtained the vibration equations of the four materials by using numpy, matplotlib and scipy libraries in python.

(3) We conducted extensive experiments on the vibration modes of thin plates and ultimately obtained the first five orders of vibration mode diagrams for spruce wood thin plates.

2. Related work

Based on the thin plate theory, we establish the vibration equation of a rectangular thin plate and simplify it by using the separated variable method. The thin plate theory is widely used in aerospace, machinery and material fields to study the bending deformation and internal force of thin plates under load. Combining the composite beam mechanical model and the thin plate elasticity theory, the spatial and temporal evolution characteristics of the overburden separation zone of weakly consolidated rock formation under the influence of mining are systematically analyzed by numerical simulation, which provides theoretical basis for the prevention and control of water damage on the roof plate of the mine [3]. The separation of variables method is used to solve the problem by decomposing the partial differential equation into several univariate ordinary differential equations, and it is used to analyze the vibration effect of the blast seismicwave on the surrounding rock and the supporting structure in the explosion vibration science [4]. Finally, we obtained the vibration equations of the first few orders for rectangular thin plates of different materials.

The finite difference method is a numerical solution method for solving partial differential (or ordinary differential) equations and systems of equations by replacing the differential quotient by the difference quotient and approximating differential equations by difference equations. We use this method to solve the first five orders of vibration modal maps of a rectangular sound board. In the field of wind power generation, the finite difference method is used to quickly design the full-size static loading scheme for wind turbine blades [5]; in the field of robotics, the method combines the fuzzy regression neural network and the sliding mode control algorithm [6-8], which significantly improves the control accuracy and vibration suppression effect of the flexible joint system [9-10], avoids the complex derivation and the measurement of the diagonal acceleration, and applies to the arbitrary joint flexible stiffness, and the performance is better than that of the traditional method. Finally, we solved the first five orders of vibration modal cloud diagrams of spruce wood rectangular thin plate by finite difference method.

3. Methods

When formulating the thin plate theory, we denote “w” as “Deflection”,“x” as “the transverse coordinates of the thin plate”,“y” as “longitudinal coordinates of thin plates”,“h” as “longitudinal coordinates of thin plates”,“t” as “Vibration time”,“f” as “Frequency”,“z” as “Amplitude”,“D” as “bending stiffness”,“q” as “unit load”,“Mind” as “deflection component”,and“φ(x, y)” as “the vibrational shape function”.

To develop our framework , we assume that the length of the sheet is 100mm and the width is 100mm .The original line segment perpendicular to the midplane of the plate remains perpendicular to the deformed midplane , the positive stress perpendicular to the center plane (see stress) is much smaller than the stress component parallel to the center plane, so it can be ignored and when bending under the load perpendicular to the center surface of the plate, the center surface of the plate is not stretched.

The following formula is shown:

A. Bending equation:

$$D\nabla^4 w = q(x,y) \tag{1}$$

B. Bending stiffness:

$$D = \frac{Eh^3}{12(1-v^2)} \tag{2}$$

C. Fluctuation equations

$$D\nabla^4 w = \rho h \frac{\partial w^4}{\partial t^4} \tag{3}$$

When formulating separate variables approach, we denote “k” as “the number of waves in the x-direction” and “l” as “the number of waves in the y-direction”.

To develop our framework , we assume that the original linear equation is chi-square and the boundary conditions are chi-square.

The following formula is shown:

$$w(x,y) = \sum_{k=1}^{\infty}\sum_{l=1}^{\infty} A_{at}\sin(\frac{k\pi x}{L})\sin(\frac{l\pi y}{L}) \tag{4}$$

4. Experiments

4.1. Preprocessing

Following prior studies, we found physical information data of related materials sourced from the MatWeb.

Table 1. The physical information data of different materials

Material	Density (kg/m^3)	Young's Modulus (GPa)	Poisson's ratio
Spruce wood	350	10	0.35
copper	8960	110	0.3
Carbon fiber	1500	150	0.2
PLA	1000	1	0.3

4.2. Experimental procedure and findings

Using the known numerical parameters of the thin plate above and using the data in Table 1 ,we solved the vibration images of the thin plate of different materials are basically similar, but their amplitude values and frequency sizes are different, taking the two-dimensional and three-dimensional views of the first five modes of spruce wood as an example, the vibration images of the thin plate of different materials are basically similar, but their amplitude values and frequency sizes are different.

The vibration images are shown below, where the z-axis represents the amplitude (red indicates higher amplitudes, blue indicates lower amplitudes). Figure 2 displays the vibration pattern of spruce wood in the first mode; Figure 3 corresponds to the second mode; Figure 4 illustrates the third mode; Figure 5 demonstrates the fourth mode; and Figure 6 demonstrates the fifth mode.

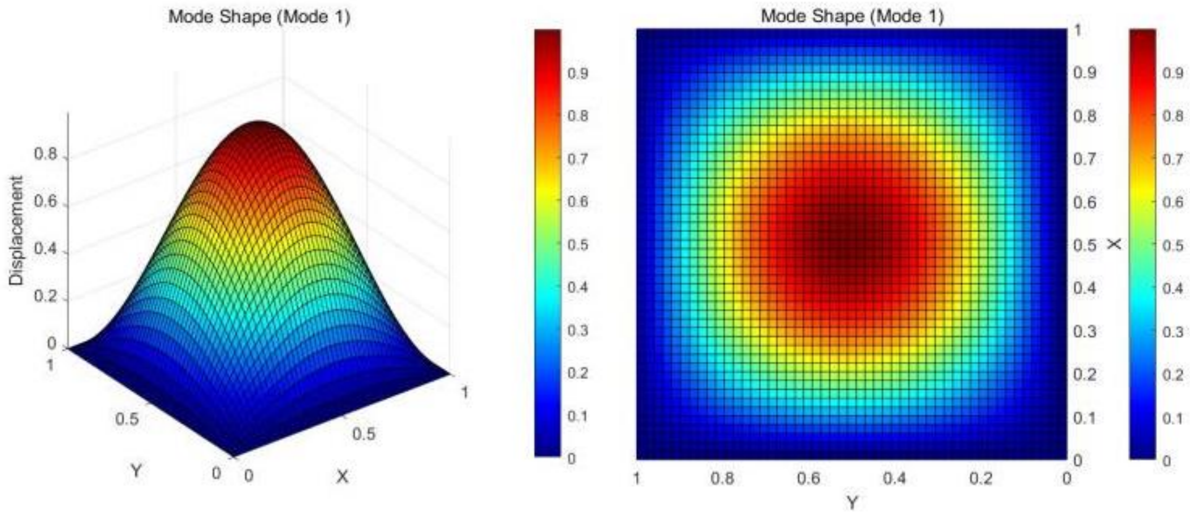

Figure 2. Model I

The first - order vibration diagram shows a single - peak shape. The displacement reaches its maximum at the center of the plane and gradually decreases towards the edges, reflecting the most fundamental vibration mode of the structure. In the cloud diagram, the color of the central area is the darkest (indicating large displacement), and the color becomes lighter towards the periphery (indicating small displacement), visually demonstrating the displacement distribution, with the most intense vibration at the center.

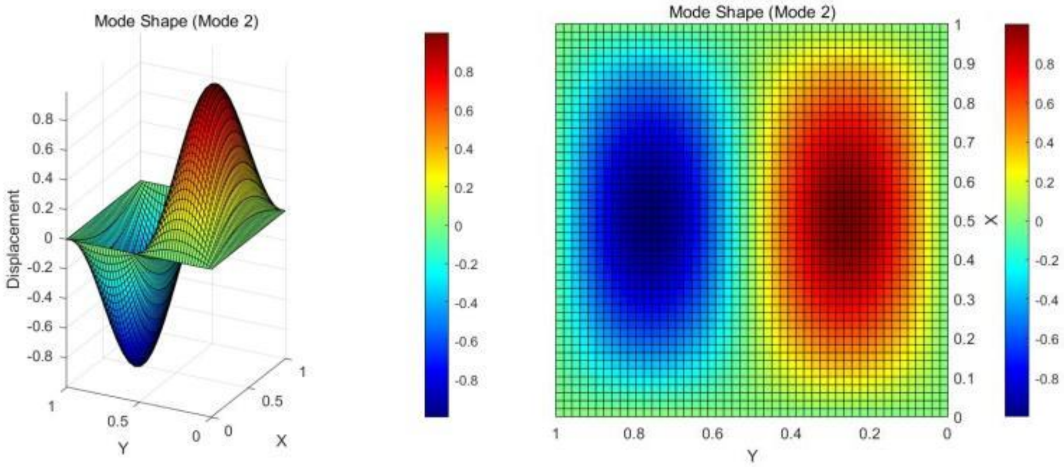

Figure 3. Model II

In the second - order vibration diagram, one peak and one trough appear. When the structure vibrates, positive and negative displacement regions are presented, showing obvious characteristics of out - of - phase vibration. In the cloud diagram, there are two distinct regions colored red and blue, representing different displacement directions of positive and negative. This indicates that the structure has two opposite vibration trends in this mode.

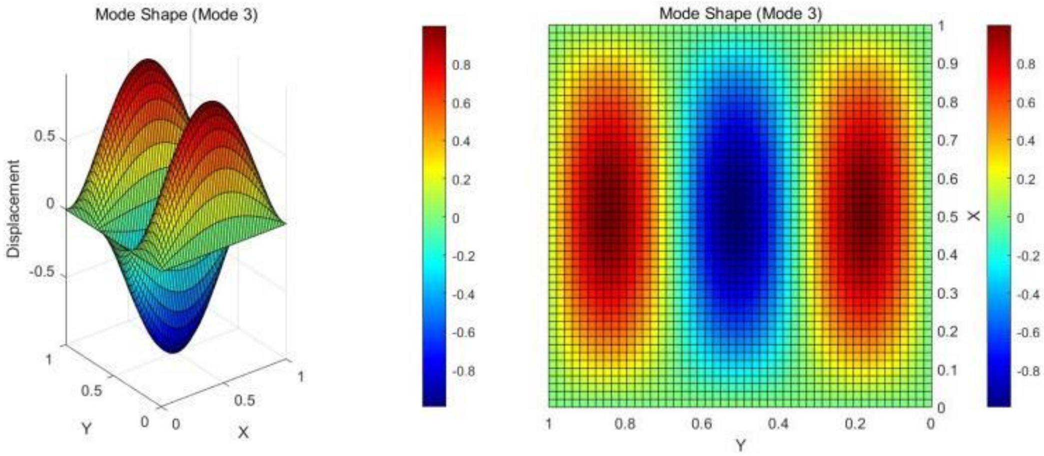

Figure 4. Modal III

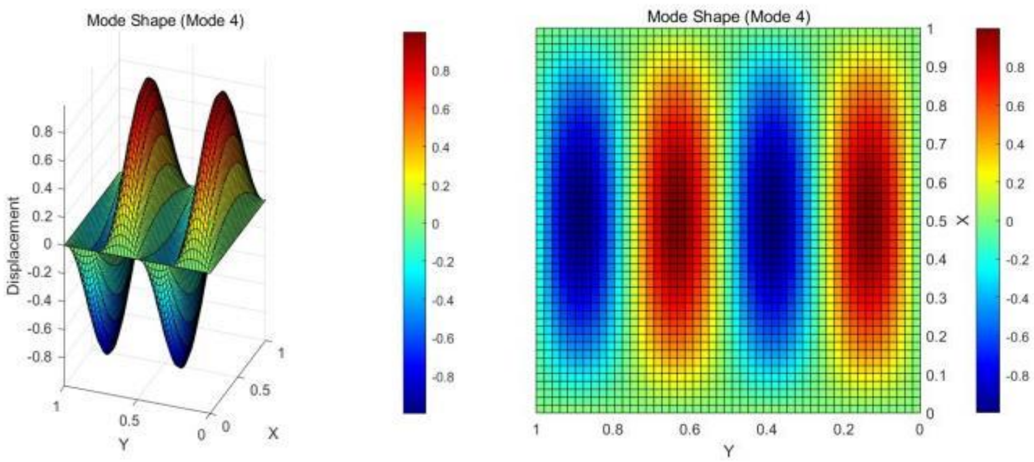

Figure 5. Modal IV

The 3D vibration plot features two peaks and two troughs. This results in a more intricate vibration pattern, with a greater differentiation in the vibration behavior across various parts of the structure.In the contour plot, alternating blue and red regions are visible. This clearly shows a more refined partition of displacements, highlighting the distinct vibration characteristics of the structure in this mode.

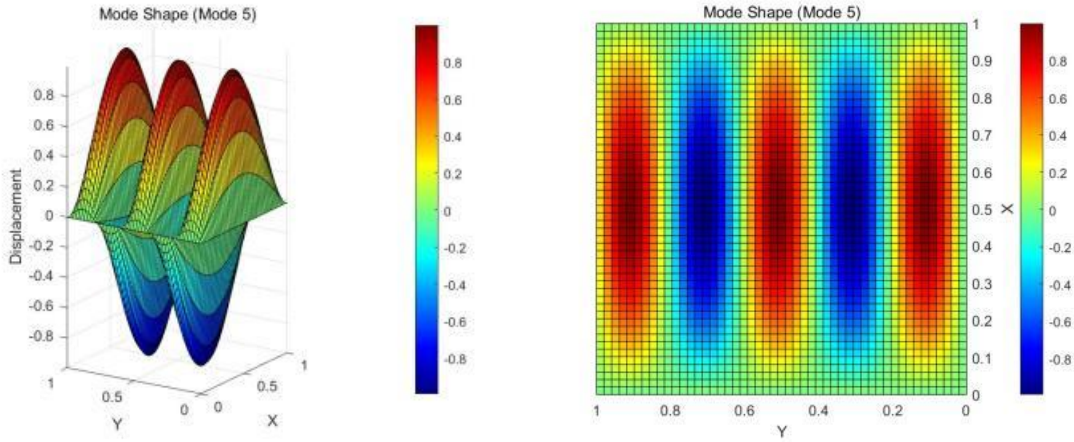

Figure 6. Modal V

The vibration equations corresponding to the following four materials and their frequencies in the first three modes were obtained by fitting the vibration patterns of spruce wood, copper, carbon fiber, and PLA (w denotes the amplitude of the fn denotes the frequency, n is the number of corresponding modes).

Figure 7 displays the first-mode vibration pattern of spruce wood, Figure 8 corresponds to copper, Figure 9 to carbon fiber, and Figure 10 to PLA. A comparative analysis reveals significant amplitude differences among these materials under their respective first vibration modes.

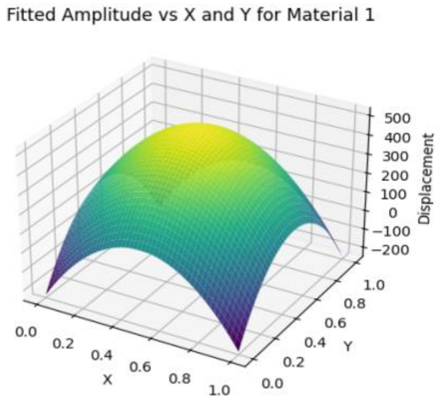

Figure 7. Spruce wood

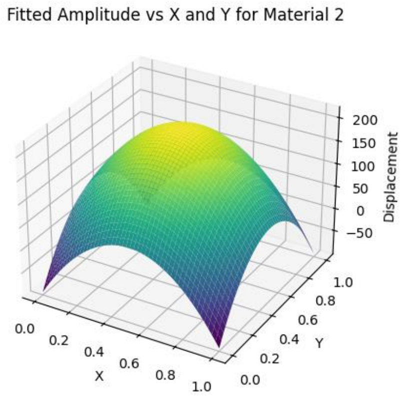

Figure 8. Copper

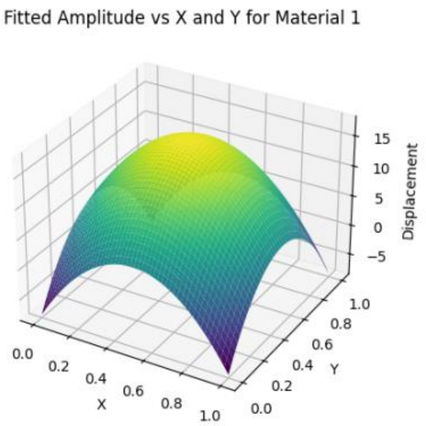

Figure 9. Carbon Fiber

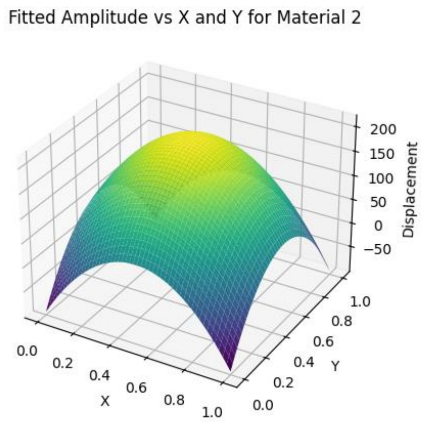

Figure 10. PLA

By applying polynomial fitting methods, we can derive the vibration equations and determine the first three natural frequencies of vibration for different materials in Table 2.

Table 2. the first three natural frequencies of different materials

Material	f1/Hz	f2/Hz	f3/Hz
Sprucewood.	51.74896	129.37241	206.99586
Copper	105.33699	263.34247	421.34795
CarbonFiber.	92.56006	231.40015	370.24024
PLA	9.50689	23.76723	38.02757

5. Conclusion

Firstly, the length, width and thickness of the original thin plate are assumed, and then the vibration model of the sound board and the vibration equations of the sound board are established by using the thin plate theory, and then the simplification of the equations is carried out by using the method of separating the variables, and the simplified equations are simplified and differentiated; then the simplified equations are solved by using the finite difference method to obtain the vibration images of the first few orders of the modes of vibration of the rectangular sound board in the Matlab software, and the vibration equations of four different materials of rectangular sound boards are obtained by extracting the first three order of the frequency data of the image in Then, the vibration equations of the rectangular sound boards of four different materials were obtained by extracting the first three orders of frequency data from the images and fitting the polynomials in python using numpy, Matplotlib, and scipy libraries.

Fitting higher-order polynomials faces challenges like increased computational complexity, overfitting risks, and numerical instability. For diverse sound panel materials and shapes, parameters can be adjusted in code/text for tailored recommendations. Vibration mode simulations can incorporate variables like temperature to improve accuracy by refining frequency and shape predictions.

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