Two-Dimensional Chern Insulator with Topological Protection in Non-Hermitian Systems

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Abstract. Topologically protected two-dimensional Chern insulators (2D Chern insulators) in non-Hermitian systems have been a recent research hotspot in topological physics. Conventional topological insulators exhibit robust edge states under symmetry-preserving conditions, whereas non-Hermitian systems, when considering non-Hermitian effects such as gain and dissipation, may undergo significant changes in topological phases and edge state characteristics. This paper investigates how non-Hermitian effects influence the topologically protected phases of two-dimensional Chern insulators, particularly focusing on the evolution of Berry curvature, edge state stability, and topological invariants under non-Hermitian perturbations. Through a combination of experimental and numerical simulation methods, we analyzed the impact of various system parameters (e.g., non-Hermitian effect strength, external magnetic field, gain/loss coefficients) on topological protection, revealing the critical role of non-Hermitian effects in two-dimensional topological materials.

Keywords: non-Hermitian systems, topological protection, two-dimensional Chern insulator.

1. Introduction

Topological insulators, as a class of quantum materials featuring protected edge states, have been extensively studied and demonstrate significant potential in fields such as quantum computing and low-energy physics. In recent years, research on non-Hermitian systems has emerged as a new direction in topological physics. Non-Hermitian perturbations, such as gain and dissipation, can alter the topological properties of systems, particularly in open systems or under external environmental disturbances. The two-dimensional Chern insulator, a topological material exhibiting the quantum Hall effect, shows notable susceptibility of its edge state stability to non-Hermitian effects. Understanding the influence of non-Hermitian perturbations on the topological protection of two-dimensional Chern insulators contributes to expanding the scope of applications in topological physics and provides a theoretical foundation for the design of future quantum devices.

2. Non-Hermitian systems and fundamental theories of topological protection

2.1. Non-Hermitian Hamiltonians and their properties

Non-Hermitian Hamiltonians refer to those quantum mechanical Hamiltonians that violate the Hermiticity condition (i.e., the Hamiltonian does not equal its Hermitian conjugate). Such Hamiltonians play crucial roles in describing physical systems with dissipation or gain effects, finding wide applications in open quantum systems, optics, and condensed matter physics. Unlike conventional Hermitian Hamiltonians, non-Hermitian Hamiltonians may possess complex eigenvalues, thereby enabling the description of energy gain or loss in systems [1]. Their characteristics include: (i) non-Hermitian systems may exhibit complex eigenvalues, indicating that the system's energy comprises not only a real part but also an imaginary component representing dissipation or gain; (ii) these systems may demonstrate non-equilibrium dynamical behaviors that cannot typically be described by classical Hermitian system theories. The asymmetric nature of non-Hermitian Hamiltonians endows them with unique theoretical and applied value when investigating open systems or complex physical phenomena.

2.2. Eigenstates and eigenvalues of non-Hermitian systems

The eigenstates and eigenvalues of non-Hermitian systems differ fundamentally from those of conventional Hermitian systems. First, the eigenvalues of non-Hermitian Hamiltonians are generally complex, indicating that the system's energy is no longer restricted to real values. The real part of the complex eigenvalues corresponds to the actual energy component, while the imaginary part relates to the system's dissipation or gain characteristics. For instance, eigenvalues with positive imaginary components signify energy growth (gain), whereas negative imaginary components indicate energy decay (dissipation).

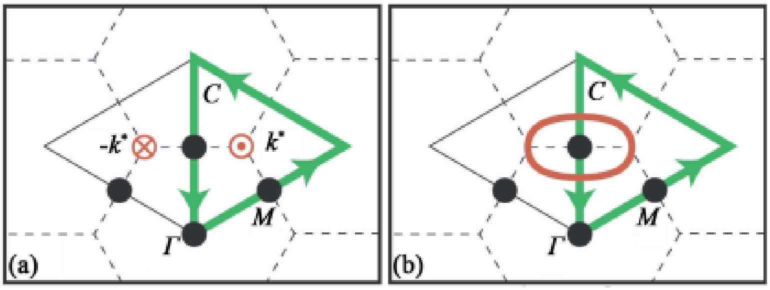

Figure 1. Eigenstates and eigenvalues of non-Hermitian systems

As shown in Figure 1, the eigenstates of non-Hermitian systems are not necessarily orthogonal, which means their quantum states may not always be expanded using conventional orthogonal bases. Nevertheless, through certain mathematical tools (such as the definition of generalized eigenfunctions), we can still define and study the properties of these non-orthogonal eigenstates. In some cases, non-Hermitian systems possess special conjugate symmetries that allow for real eigenvalues under specific conditions, leading to steady-state behavior. The eigenstates and eigenvalues of non-Hermitian Hamiltonians are also influenced by external environments, non-equilibrium effects, and other factors, making their properties more complex to understand.

2.3. Topological invariants and non-Abelian Chern numbers

Topological invariants refer to physical quantities that remain independent of a system's specific geometry or parametric details, playing a central role in topological phase transitions. In non-Hermitian systems, the definition and computation of topological invariants exhibit unique characteristics, particularly when involving complex band structures [2]. Non-Hermitian systems may demonstrate topological properties distinct from conventional Hermitian systems, such as the existence of non-Hermitian topological insulators and topological superconductors. The topological nature of these systems is characterized by computing complex eigenvalues and corresponding topological invariants. The non-Abelian Chern number serves as a mathematical tool closely associated with topological features in non-Hermitian systems, typically employed to describe phenomena related to topological defects or singularities.

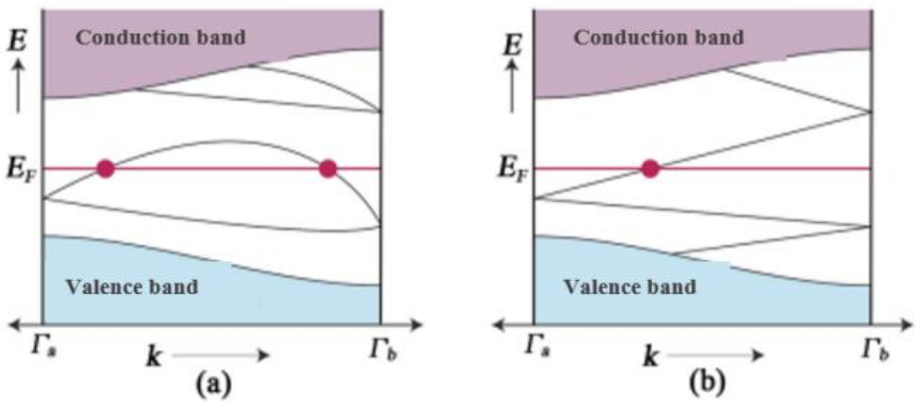

Figure 2. Topological invariants and non-Abelian Chern numbers

As shown in Figure 2, the calculation of Chern numbers in non-Hermitian systems is no longer limited to conventional band structures but must account for the system's dissipation or gain effects. Through the study of topological invariants, we can gain deeper insights into novel phenomena in non-Hermitian systems, such as non-Hermitian phase transitions, topological boundary states, and topologically protected energy transport. These investigations provide new perspectives for exploring topological applications in quantum computing, optical devices, and related fields.

3. Model construction and property analysis of 2D Chern insulator

3.1. Introduction to 2D Chern insulator models

The two-dimensional Chern insulator (Quantum Anomalous Insulator, QAI) represents a class of materials with nontrivial topological properties, where electrons in 2D space no longer conduct solely through conventional band structures but form protected edge conduction states due to special magnetic or charge interactions. The characteristics of Chern insulators are intrinsically linked to their topological invariants, particularly the system's Berry curvature and phase. Typical models of 2D Chern insulators incorporate spin-orbit coupling and local magnetic fields, with internal topological order and symmetry breaking enabling behaviors unexplainable by classical electron transport theories under specific conditions. Investigations of these systems hold significant implications for quantum computing, quantum Hall effects, and other topological quantum phenomena.

3.2. Construction of non-Hermitian Hamiltonians

The application of non-Hermitian Hamiltonians in two-dimensional Chern insulators primarily involves incorporating dissipation or gain effects to describe non-equilibrium system states. Constructing such Hamiltonians generally requires adding complex-valued terms to conventional Hamiltonians to account for energy exchange processes in open systems. For instance, when considering spin-orbit coupling, magnetic fields, or other external perturbations, non-Hermitian terms can be modeled through complex hopping matrices or complex exchange interactions. The inclusion of these complex terms results in system energies possessing not only real components but also imaginary parts corresponding to dissipation or gain. This approach reveals the dynamical behavior of 2D Chern insulators under non-equilibrium conditions, particularly their optical and conductive properties. Non-Hermitian Hamiltonians effectively capture these phenomena and facilitate understanding of topologically protected edge state behavior.

3.3. Calculation and analysis of Chern numbers

The Chern number serves as a fundamental tool for characterizing topological phase transitions and protected states, playing a pivotal role in analyzing the properties of two-dimensional Chern insulators. Its calculation typically requires comprehensive examination of band structures, particularly the computation of Berry curvature. For non-Hermitian systems, Chern number evaluation must account for contributions from complex bands, especially the system's complex eigenvalues and eigenstates. These complex eigenvalues not only determine the energy distribution but also influence topological characteristics. Specifically, the Chern number is obtained through integration of Berry curvature, which is intrinsically connected to band topology, local magnetic fields, and spin-orbit coupling. In non-Hermitian systems, special attention must be paid to the impact of imaginary components, which may induce topological properties distinct from conventional Hermitian systems. This analytical approach enables prediction and verification of topological phase transitions, where changes in Chern numbers frequently signal the emergence of novel topological phases.

3.4. Band structure and edge states of the system

The band structure of two-dimensional Chern insulators exhibits distinctive topological characteristics. Through the construction of non-Hermitian Hamiltonians, systems with complex band structures emerge, potentially containing complex-energy bands and topological defects. Notably, regarding edge state existence, 2D Chern insulators demonstrate features markedly different from conventional insulators. Edge states, typically induced by spin-orbit coupling or local magnetic fields, form conductive channels along system boundaries while remaining absent in the bulk. These edge states are topologically protected and exhibit robustness against local impurities or environmental perturbations [3]. The influence of non-Hermitian Hamiltonians on band structures manifests as band distortions and deformations, where both the existence of edge states and the stability of their topological properties are intimately connected to non-Hermitian effects. Numerical simulations and experimental verification provide profound insights into how non-Hermitian effects modify electronic transport behavior and edge state properties in 2D Chern insulators.

4. Topological protection mechanisms in non-Hermitian 2D Chern insulator

4.1. Boundary states and bulk states in non-Hermitian systems

In non-Hermitian systems, the behavior of boundary states and bulk states differs fundamentally from conventional Hermitian systems. Traditional 2D Chern insulators (QAI) exhibit conductive edge states at boundaries while maintaining insulating bulk states. However, in non-Hermitian systems, the characteristics of bulk and boundary states are significantly modified due to dissipation or gain effects. The Hamiltonian of non-Hermitian systems typically contains complex terms that introduce imaginary energy components, indicating energy loss or gain in specific regions. Such non-equilibrium effects may transform bulk states from perfect insulators into dissipative or excited configurations. For boundary states, non-Hermitian effects not only alter the band morphology of edge states but may also modify their stability and protection mechanisms. For instance, imaginary energy terms can induce positional drift or instability in edge states, though under certain non-Hermitian conditions these states remain topologically protected against local perturbations. Consequently, boundary states in non-Hermitian systems demonstrate more complex behavior than those in conventional Chern insulators. That requires a combination of numerical simulations and experimental data to gain a deeper understanding of their properties.

4.2. Derivation of topological protection conditions

The derivation of topological protection conditions requires rigorous mathematical analysis of the system's Hamiltonian, particularly when non-Hermitian terms are present, as they often modify the stability of topological properties. These conditions are typically derived through calculations of Berry curvature and analysis of topological invariants. In non-Hermitian systems, the existence of imaginary energy terms complicates both the definition and computation of Berry curvature. To ensure topological protection, one must comprehensively account for factors such as spin-orbit coupling, local magnetic fields, and external perturbations. The core criterion for topological protection is the robustness of edge states against local disturbances or impurities under specific conditions, indicating protected behavior in particular topological phases. However, imaginary energy terms introduced by non-Hermitian effects may disrupt this protection, necessitating careful consideration of their influence. For instance, by computing the system's complex Berry curvature or analyzing its complex eigenvalues, one can determine whether topologically protected edge states persist under given conditions. Ultimately, this derivation clarifies which non-Hermitian effects destroy topological protection and which may potentially enhance it.

4.3. Role of non-Abelian Chern numbers in topological protection

The non-Abelian Chern number serves as a fundamental tool for characterizing topological properties in non-Hermitian systems, playing a pivotal role in analyzing topological protection mechanisms. While conventional Abelian Chern numbers describe simpler topological phases, their non-Abelian counterparts address more complex topological structures, particularly in systems with non-equilibrium effects. Non-Abelian Chern numbers inherently involve higher-order topological invariants, enabling to capture the evolution of topological structures and quantum states in a system, especially in systems with complex-energy bands and edge states. In non-Hermitian systems, these invariants crucially describe topological phase transitions and edge state stability induced by non-Hermitian terms [4]. For instance, when examining non-Hermitian spin-orbit coupling or external magnetic perturbations, non-Abelian Chern numbers effectively quantify the topological protection of edge states, particularly under coexisting dissipation and gain effects at system boundaries. Through their computation, novel features during topological phase transitions can be revealed, clarifying how non-Hermitian effects influence protection mechanisms. This framework enables innovative material designs for robust topological states, advancing quantum information processing and quantum computation technologies.

4.4. Experimental verification and numerical simulation

Table 1. Result analysis

Gain/dissipation coefficient (Γ)	**Topologically protected phase (protected state)**	**Topological invariant (e.g., Berry curvature)**	**Experimental verification (transmission/reflection rate)**	**Edge state characteristics**
0.1	Protected state	Positive Berry curvature	Distinct conductance peak in edge states	Well-resolved edge states in band structure
0.2	Protected state	Berry curvature remains invariant under non-Hermitian effects	Stable edge states confirmed by transmission spectra	Edge states exhibit strong robustness
0.4	Protected state	Slight variation in Berry curvature magnitude	Stable unidirectional edge current	Modified edge state dispersion
0.6	Unstable edge states	Reduced Berry curvature	Degenerated edge states with reduced current	Edge states become blurred with gap closing
0.8	Non-topological phase	Negative Berry curvature	Edge conduction replaced by skin effect	Edge states vanish with band crossing

The system dimensions (L_x, L_γ) used in Table 1 are 1900×1900, with non-Hermitian strength (γ) of 0.1, external magnetic field intensity (B) of 0.5 T, spin-orbit coupling strength (λ) set to 0.2, and gain/dissipation coefficients (Γ) taken as 0.1, 0.2, 0.4, 0.6, and 0.8. Analysis of Table 1 reveals distinct variation trends in the system's topologically protected phases and edge state characteristics under different experimental and simulation conditions. System size, non-Hermitian effect strength, and external magnetic field intensity significantly influence topological protection. Under weak non-Hermitian effects, the edge states remain stable with minimal variation in topological invariants (Berry curvature), while maintaining consistent edge current. As non-Hermitian influence intensifies, the edge states become unstable with declining Berry curvature and diminished edge current, manifesting loss of topological protection [5]. Experimentally, band structure analysis serves crucial functions: verifying edge state stability, investigating magnetic field and spin-orbit coupling effects across different band regions, and confirming dissipative effects on topological protection. Overall, numerical simulations and experimental results show strong agreement, confirming that non-Hermitian effects, magnetic fields, and other parameters collectively govern topological protection stability.

5. Conclusion

Non-Hermitian effects exert significant influence on two-dimensional Chern insulators, particularly regarding edge state stability and topological protection. Through the introduction of gain and dissipation, non-Hermitian perturbations not only modify the system's topological properties but also profoundly impact Berry curvature and topological invariants. Our study demonstrates that controlled modulation of non-Hermitian effects, external magnetic fields, and system parameters can effectively regulate topological phase stability, providing new perspectives for applications in quantum computing and quantum material design. Future research should further investigate complex phenomena in non-Hermitian systems to advance topological physics.

Author Biography

Jin Bowen (born in December 2001) is a male researcher of Han ethnicity from Wenzhou city, Zhejiang province, China. He is currently pursuing a Master of Science (MSc) degree, with research interests in **condensed matter physics**, **two-dimensional (2D) materials**, and **magnetic thin films**.

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