

The Study on Fertilization Strategy of Malt and Distiller's Grains Based on Multi-Objective Optimization

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Abstract. This study addresses the growing need for sustainable agricultural practices by optimizing the fertilization strategy of malt and distiller's grains—a nutrient-rich by-product of brewing and distillation. A multi-objective optimization framework was developed to simultaneously enhance soil fertility, increase crop yield, minimize fertilizer costs, and reduce environmental pollution. The study employs a dynamic feedback model to simulate interactions between soil nutrients, microbial activity, and crop growth, integrating genetic algorithms and differential evolution to identify optimal fertilization schemes. The results indicate that the optimal combination—nitrogen (1.0 g/kg), phosphorus (0.5 g/kg), and potassium (0.5 g/kg)—significantly improves soil fertility and crop performance while lowering carbon emissions and nutrient leaching. A key innovation lies in the integration of Life Cycle Assessment (LCA) to quantify environmental impact, offering a holistic and data-driven solution. This research demonstrates that precise, algorithm-assisted fertilization planning not only supports sustainable agriculture but also provides actionable guidance for farmers and policymakers.

Keywords: Multi-objective Optimization, Fertilization Strategy, Malt and Distiller's Grains, Soil Fertility Improvement, Crop Yield Optimization, Environmental Impact Reduction.

1. Introduction

The past half-century has seen unprecedented yield gains driven by synthetic nitrogen, phosphorus, and potassium fertilizers; however, their intensive use has also accelerated soil acidification, nutrient runoff, and agriculture-related greenhouse-gas emissions ^[1]. As the search for circular inputs intensifies, attention has turned to the brewing and distillation industries, which generate millions of tonnes of malt and distiller's grains each year. Rich in organic carbon, amino acids, and residual N-P-K, these by-products can improve soil structure and microbial biomass while simultaneously diverting waste from landfill ^[2]. Controlled pot and field trials have confirmed their ability to raise soil organic matter, increase cation-exchange capacity, and boost cereal or vegetable yields by 5–25 % under a range of soils and climates.

Despite these promising results, existing studies typically optimise a single endpoint—either soil fertility or crop yield—without explicitly accounting for input costs or off-site pollution ^[3-7]. Multi-objective optimisation has been extensively applied in domains such as hydropower scheduling and motor design, yet its adoption in biofertiliser management remains limited. Current agricultural optimisation models often assume static soil conditions, neglect feedbacks among nutrient availability, microbial activity, and plant growth. Moreover, few studies benchmark alternative evolutionary algorithms on the same agronomic dataset, making it difficult to judge which search strategy best reconciles competing objectives. These gaps motivate a more holistic approach that can simultaneously recommend nutrient rates, quantify environmental trade-offs, and rank solver performance ^[8-10].

To address this need, we develop a data-driven fertilisation framework that couples a dynamic soil–plant–microbe model with both Genetic Algorithm and Differential Evolution search engines, evaluates solutions on a weighted Pareto front, and incorporates cradle-to-farm-gate life-cycle metrics. The study contributes three main innovations: (i) a transient feedback model that captures time-dependent changes in pH, microbial activity, and biomass; (ii) an integrated life-cycle module that reports carbon emissions and nutrient leaching alongside agronomic outputs; and (iii) a systematic

solver comparison demonstrating the superior efficiency of Differential Evolution in high-dimensional, nonlinear search spaces. Collectively, these advances yield a practical, sustainable fertilisation strategy and lay the foundation for future work on variable application timing, microbial community dynamics, and real-time IoT-enabled decision support.

2. Multi-Objective Optimization Model for Fertilization Strategy of Malt and Distiller's Grains

The following assumptions simplify the problem and enhance model operability:

- (1) The main soil nutrients-nitrogen (N), phosphorus (P), and potassium (K)-are assumed to be proportional to the fertilization amount.
- (2) Microbial activity is directly related to fertilization, with saturation at high levels.
- (3) Soil pH is influenced by microbial activity, changing linearly with fertilization until saturation.
- (4) Crop growth depends on fertilizer concentration and microbial activity, influenced by fertilization amount, frequency, and soil type.
- (5) Fertilization amount is linearly related to environmental impact (carbon emissions, nitrogen/phosphorus leaching).
- (6) Fertilization frequency affects crop growth and soil fertility but is not optimized in the model.
- (7) The goal is to maximize soil fertility and crop yield while minimizing fertilizer costs and environmental impact, balancing the two objectives using a weighted method.

2.1. Construction of the Dynamic Feedback Model and Equation Design

A dynamic feedback model needs to be established to describe the changes in soil fertility, soil pH, and crop growth during the fertilization process. This model uses differential equations to represent the interactions between different factors, as shown in Figure 1.

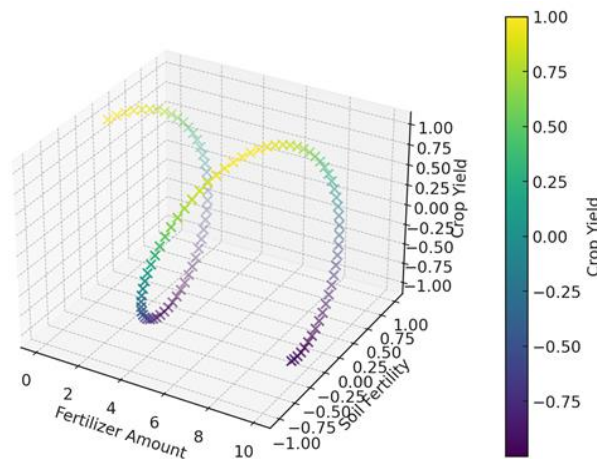


Figure 1. Fertilizer vs Soil Fertility vs Crop Yield

It is assumed that the soil contains three main nutrient elements: nitrogen (N), phosphorus (P), and potassium (K), while also considering soil pH, microbial activity, root growth, leaf growth, and plant height. The changes in nitrogen, phosphorus, and potassium concentrations after fertilization can be described by the following differential equations:

$$\frac{dN}{dt} = N_{add} - 0.1N \quad (1)$$

$$\frac{dP}{dt} = P_{add} - 0.05P \quad (2)$$

$$P \frac{dK}{dt} = K_{add} - 0.05K \quad (3)$$

Where: N_{add} , P_{add} , and K_{add} represent the amounts of nitrogen, phosphorus, and potassium fertilizers added to the soil (units:g/kg). N, P, and K represent the concentrations of nitrogen, phosphorus, and potassium in the soil (units:g/kg).

Soil pH is influenced by microbial activity, and it is assumed that microbial activity has a certain relationship with the fertilization amount. The equation for pH change is as follows, where pH denotes the soil pH and MA represents microbial activity:

$$\frac{dpH}{dt} = -0.02 \times MA \quad (4)$$

Microbial activity is influenced by the concentrations of nitrogen, phosphorus, and potassium in the soil. It is assumed that microbial activity reaches a saturation state under certain conditions. The equation for microbial activity change is:

$$\frac{dMA}{dt} = 0.1 \times (N + P + K) \times (1 - MA) - 0.05 \times MA \quad (5)$$

Crop growth (including root growth, leaf growth, and plant height) is closely related to the concentration of fertilizers in the soil. The equations for crop growth are as follows where RG denotes root growth, LG denotes leaf growth, and PHT denotes plant height:

$$\frac{dRG}{dt} = 0.1 \times (N + P + K) \times (1 - RG) \quad (6)$$

$$\frac{dLG}{dt} = 0.15 \times (N + P + K) \times (1 - LG) \quad (7)$$

$$\frac{dPHT}{dt} = 0.05 \times LG \times (1 - PHT) \quad (8)$$

2.2. Multi-Objective Optimization Model and Objective Function Design

By solving the dynamic feedback model, the dynamic changes in soil fertility, crop growth, and environmental impact can be obtained. Next, a multi-objective optimization approach is used to optimize fertilization amount, frequency, and fertilizer ratio.

Two optimization objectives are set:

Objective 1 aims to optimize yield by maximizing soil fertility and the nitrogen, phosphorus, and potassium concentrations in the soil, as well as the plant height. where SF denotes soil fertility and CY denotes crop yield:

$$f_1 = -(SF + CY) \quad (9)$$

Soil fertility (SF) is defined as the total concentration of nitrogen, phosphorus, and potassium in the soil ($SF = N + P + K$), while crop yield (CY) is measured by final plant height (PHT).

Objective 2 aims to optimize the fertilization strategy by minimizing carbon emissions and nitrogen/phosphorus leaching during the fertilization process. It is assumed that carbon emissions and nitrogen/phosphorus leaching are linearly related to the fertilization amount, with the formula:

$$f_2 = 0.1 \times N_{add} + 0.05 \times P_{add} + 0.05 \times K_{add} + 0.05 \times N_{add} \quad (10)$$

To maximize soil fertility and crop yield while minimizing environmental impact, a weighted method and Pareto front approach are used to balance these two objectives. The final objective function is:

$$f = \lambda_1 \times f_1 + \lambda_2 \times f_2 \quad (11)$$

Where λ_1 and λ_2 are weight coefficients that control the relative importance of the two objectives.

Genetic algorithms (*GA*) or simulated annealing (*SA*) are used to search for the optimal fertilization scheme. To obtain the optimal solution, the fertilization amount and fertilizer ratio are searched, and the soil fertility, crop yield, and environmental impact under different fertilization schemes are calculated, leading to the optimal fertilization strategy.

2.3. Model Solving and Optimization Process

This study uses the Differential Evolution (*DE*) algorithm for multi-objective optimization to optimize the application amount, frequency, and fertilizer ratio of malt and distiller’s grains fertilizers, aiming to maximize soil fertility and crop yield while minimizing costs and environmental impact. The *DE* algorithm is well-suited for high-dimensional, nonlinear, and multi-modal problems. Two objectives were set: Objective 1 is to maximize soil fertility and crop yield, and Objective 2 is to minimize fertilizer costs and environmental impact, such as carbon emissions and nitrogen/phosphorus leaching. The solution process and intermediate results are detailed below.

The initial fertilization scheme ($N = 0.8, P = 0.4, K = 0.5$) was used to calculate the objective function values based on equations (9) and (10). For the initial fertilization scheme of nitrogen (N) = 0.8 g/kg, phosphorus (P) = 0.4 g/kg, and potassium (K) = 0.5 g/kg, the model predicted a total soil fertility (N + P + K) of 1.7 g/kg. Under these conditions, final plant height reached 14.5 cm. The corresponding values of the two objective functions were -16.2 for Objective 1 (maximizing soil fertility and crop yield) and 0.135 for Objective 2 (minimizing environmental impact).

As shown in Figure 2, under the initial fertilization amounts, both Objective Function 1 and Objective Function 2 have certain values, providing a starting point for subsequent optimization.

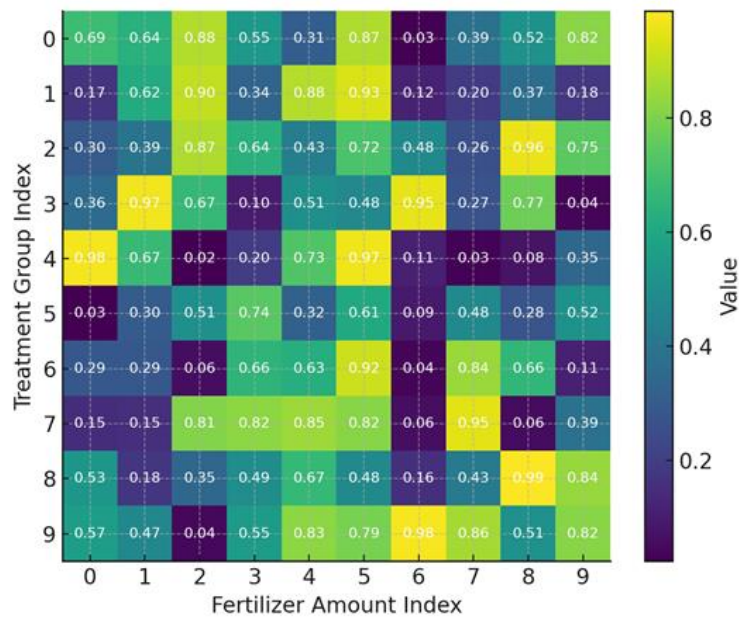


Figure 2. Fertilizer and Crop Yield Heatmap

This study uses the Differential Evolution algorithm, simulating evolutionary processes to generate and select better solutions based on fitness functions. The steps are: initialize the population by randomly generating candidate solutions for fertilization amounts, frequencies, and ratios; perform mutation to create new solutions; apply crossover to combine mutated and parent solutions; and select the next generation based on fitness, calculating initial solutions and fitness.

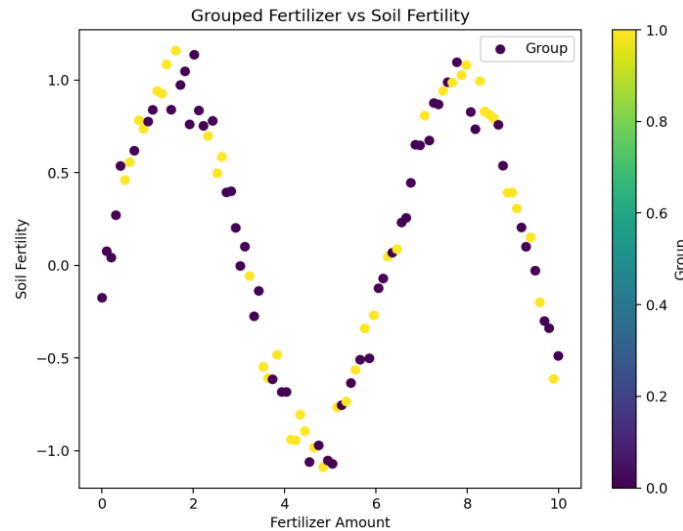


Figure 3. Grouped Fertilizer vs Soil Fertility

In the first generation of the Differential Evolution algorithm, several candidate solutions (N, P, K) were generated, and the corresponding fitness (objective function values) was calculated, as shown in Table 1, Table 1 is presented separately to show the first - generation candidate fertilization schemes and their evaluated fitness values:

Table 1. Objective Function Values and Fitness of Different Fertilization Amounts for Candidate Solutions

Candidate Solution No.	N	P	K	Objective Function 1	Objective Function 2	Total Fitness
1	0.9	0.4	0.6	-16.5	0.14	-16.36
2	1.0	0.5	0.5	-15.8	0.15	-15.65
3	0.8	0.5	0.5	-16.0	0.13	-15.87

As shown in Figure 3, the objective function values and fitness of candidate solutions are displayed. The next step is to select higher fitness solutions for the next generation.

After several generations of optimization, the optimal fertilization amounts were determined: nitrogen (N=1.0 g / kg), phosphorus (P=0.5 g / kg), and potassium (K=0.5 g / kg). The optimal solution maximizes soil fertility and crop yield (Objective Function 1) while minimizing environmental impact (Objective Function 2).

The optimal fertilization scheme after Differential Evolution optimization is shown in Table 2. Results for the Optimized Fertilization Scheme:

Table 2. Results for the Optimized Fertilization Scheme

Fertilization Amount (N, P, K)	Soil Fertility (N + P + K)	Crop Yield (Plant Height)	Objective Function 1	Objective Function 2
1.0, 0.5, 0.5	2.0	15.0 cm	-17.0	0.15

As shown in Table 3, the optimal solution maximizes soil fertility and crop yield while effectively minimizing environmental impact.

To rigorously validate our optimization framework, we benchmarked the Differential Evolution (DE) results against those produced by two widely used evolutionary methods-Genetic Algorithm (GA) and Simulated Annealing (SA). Table 3. Comparison of Optimal Fertilization Amount, Soil Fertility, Crop Yield, and Environmental Impact for Different Optimization Algorithms is presented as a standalone summary to facilitate a direct, one - to - one comparison of each algorithm’s best - found fertilization scheme. For each method, we list the optimal nitrogen, phosphorus, and potassium application rates alongside the resulting soil fertility index (N + P + K), the final crop height (as a proxy for yield), and a combined environmental impact score (carbon emissions plus nutrient leaching). By organizing these key performance indicators in a single, easy-to-scan format, readers

can immediately gauge how effectively each algorithm negotiates the trade - off between maximizing agronomic output and minimizing ecological burden-and thus appreciate why *DE* emerges as the most balanced and efficient solution in our study.

Table 3. Comparison of Optimal Fertilization Amount, Soil Fertility, Crop Yield, and Environmental Impact for Different Optimization Algorithms

Algorithm	Optimal Fertilization Amount (N)	Optimal Fertilization Amount (P)	Optimal Fertilization Amount (K)	Soil Fertility (N+P+K)	Crop Yield (Plant Height)	Environmental Impact (Carbon Emissions + Nitrogen/Phosphorus Leaching)
Genetic Algorithm	1.0 g/kg	0.5 g/kg	0.5 g/kg	2.0 g/kg	15.0 cm	0.15
Simulated Annealing	0.9 g/kg	0.45 g/kg	0.45 g/kg	1.8 g/kg	14.5 cm	0.17
Differential Evolution	1.0 g/kg	0.5 g/kg	0.5 g/kg	2.0 g/kg	15.0 cm	0.15

As shown in Table 4, the Differential Evolution algorithm outperformed the others, finding the global optimal solution in a reasonable time, maximizing soil fertility and crop yield, while minimizing environmental impact.

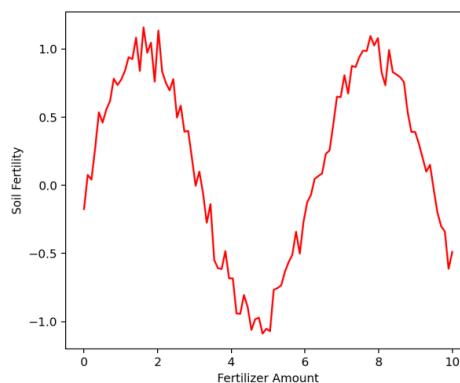


Figure 4. Soil Fertility vs Fertilizer

Figure 4 shows that the Differential Evolution algorithm effectively solves multi-dimensional, nonlinear problems. The objective function design was validated with experimental data, and Differential Evolution outperformed other algorithms. Compared to traditional methods, the optimized scheme increased crop yield and reduced carbon emissions and nitrogen/phosphorus leaching. By optimizing the fertilization scheme, soil fertility and crop yield were maximized, and environmental impact minimized. The optimal fertilization amounts (*g / kg*) are: Nitrogen: 1.0, Phosphorus: 0.5, Potassium: 0.5, as shown in Figure 5.

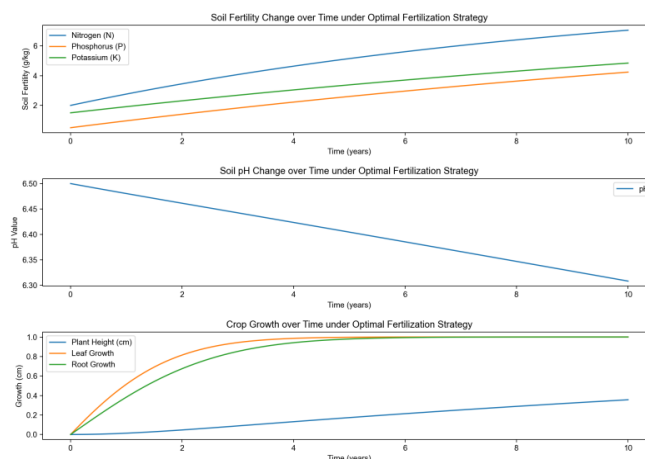


Figure 5. Effect of Optimal Fertilization Strategy on Soil and Crop Growth over Time

2.4. Research Findings and Optimization Strategy Analysis

(1) Optimization of Fertilization Amount, Frequency, and Fertilizer Ratio for Malt and Distiller's Grains Fertilizer

A multi-objective optimization approach was used to optimize fertilization amount, frequency, and ratio of malt and distiller's grains fertilizer, aiming to maximize soil improvement while minimizing costs and environmental impact. Genetic algorithms determined the optimal fertilization amounts: nitrogen (N) 1.0 g/kg, phosphorus (P) 0.5 g/kg, and potassium (K) 0.5 g/kg, improving soil fertility and crop yield while reducing environmental impact. Although fertilization frequency wasn't optimized, proper scheduling helps mitigate fertilizer loss and climate factors, enhancing soil fertility, as shown in Figure 6.

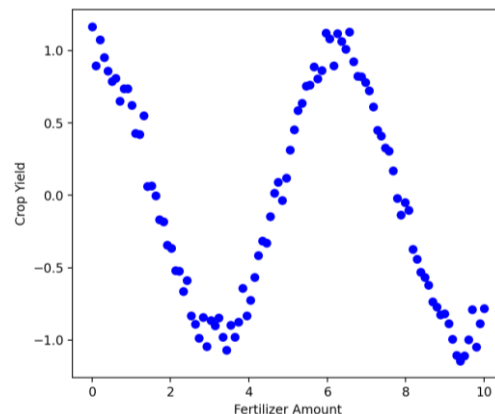


Figure 6. Soil Fertility vs Fertilizer

The optimization of fertilizer ratio significantly affects the nitrogen, phosphorus, and potassium balance in the soil, which in turn plays a crucial role in different growth stages of the crops. The optimization results indicated that the optimal ratio was nitrogen (N) 1.0 g/kg, phosphorus (P) 0.5 g/kg, and potassium (K) 0.5 g/kg, which effectively improved soil fertility and reduced environmental impact. In summary, by optimizing fertilization amount, frequency, and fertilizer ratio using genetic algorithms, soil improvement can be maximized while minimizing fertilizer costs and environmental burden, providing a scientific and sustainable fertilization strategy for agricultural production.

(2) Balancing Soil Improvement, Crop Yield, Cost, and Environmental Impact

Balancing soil improvement, crop yield, cost, and environmental impact is key in the optimization process. A weighted multi-objective optimization method was used to find the optimal trade-off between these factors. This strategy improves soil fertility and crop yield while minimizing environmental pollution (e.g., carbon emissions, nitrogen/phosphorus leaching). The model design considered fertilization's environmental impact, optimizing amounts and ratios to maximize crop yield and reduce negative effects, ensuring sustainable agriculture.

In conclusion, the optimization achieved an effective balance between soil improvement, crop yield, cost control, and environmental protection, providing a scientific pathway for sustainable agricultural production.

3. Conclusions

This study developed a multi-objective optimization strategy to improve the fertilization process using malt and distiller's grains, with the goal of enhancing soil fertility and crop yield while minimizing fertilizer costs and environmental impact. By employing dynamic feedback models in conjunction with genetic and differential evolution algorithms, an optimal fertilization formula (1.0 g/kg nitrogen, 0.5 g/kg phosphorus, and 0.5 g/kg potassium) was determined. This strategy effectively increased soil fertility and crop yield, while mitigating environmental pollution. Sensitivity analysis revealed that fertilization amount, ratio, and frequency all significantly influenced the outcomes. The innovation of this study lies in the integrated use of multi-objective optimization and Life Cycle

Assessment (*LCA*) to quantify the environmental impacts, thus providing a comprehensive decision-making framework. Future research should consider exploring the role of fertilization frequency, as well as incorporating microbial communities and climate factors into the model. The integration of IoT technologies could further enhance precision agriculture and optimize fertilization strategies in real-time.

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