Research on the Movement of Bench Dragon

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Abstract. Traditional Chinese bench-dragon performances necessitate precise multi-body dynamic modeling for choreographic optimization. Confronting the systematic deficiency in real-time position/velocity computation of interconnected segments along equidistant spiral trajectories in existing research, this study proposes a dynamic recursive model based on Archimedean spiral characteristics and a low-complexity geometric projection collision detection algorithm. By integrating polar-Cartesian coordinate transformation with numerical integration techniques, a recursive framework for segment motion parameter resolution is established. The implementation of 3D-to-2D geometric projection achieves collision detection complexity reduction to logarithmic scale while maintaining 98.7% accuracy. MATLAB simulations demonstrate that: (1) A minimum spiral pitch of 0.455 meters enables bidirectional motion transition within a 9-meter performance zone; (2) Regulating the dragon-head velocity below 1.905 m/s effectively prevents tail-end speed overshooting. Experimental validation reveals 72% positional error reduction compared to empirical approaches, successfully ensuring kinematic continuity across 233 interconnected segments. This research establishes an interdisciplinary nexus between computational mechanics and cultural heritage preservation, providing scalable solutions for intangible cultural digitization and multi-body dynamic systems.

Keywords: Archimedes Spiral, Optimization Model, Numerical Analysis Method.

1. Introduction

The traditional Bench Dragon (Bandenglong), a nonlinear multi-body system comprising 233 interconnected segments, confronts dual challenges in digital modeling for cultural heritage preservation: maintaining its dynamic artistic characteristics while resolving recursive computational complexities in multi-body dynamics. Prior studies exhibit three critical gaps: kinematic-dynamic decoupling in spiral parametrization [1], biomechanical analyses limited to single-segment motion characteristics [2], and non-real-time collision detection algorithms [3], resulting in trajectory deviations exceeding 0.82m in performance spaces over 10 meters. This study proposes a collaborative framework integrating recursive dynamics and geometric projection, achieving three breakthroughs: (1) A recursive motion model based on Archimedean spiral principles ensures spatial continuity through optimized geometric parameters (minimum spiral pitch: 0.455m); (2) A third-order dimensionality reduction collision detection algorithm reduces computational complexity by 82% via adaptive 3D-to-2D projection while maintaining 98.7% accuracy; (3) Adaptive velocity constraints control speed propagation errors across all 233 segments below 5%. MATLAB/Simulink co-simulations in 9m performance spaces demonstrate a 72% reduction in positional errors (1.32m to 0.37m) compared to empirical methods, alongside real-time computation at 30 fps. By coupling spiral parametrization with recursive dynamics, the framework resolves the trilemma of spatial adaptability, motion continuity, and morphological integrity in traditional dance preservation. [4] The proposed minimum-energy projection criterion establishes a new paradigm for multi-body collision detection, while real-time control protocols enable scalable applications in dance robotics and drone swarm systems. This work constructs an engineering framework for digital heritage conservation, advances theoretical foundations for complex multi-body coordination, and bridges mechanical engineering with intangible cultural heritage preservation.

2. Motion analysis of bench dragon model

2.1. Archimedean Spiral Dynamic Recursive Model

The Archimedean spiral, defined in polar coordinates by the equation:

$$r(\theta) = a + b\theta \tag{1}$$

Where r is the radial distance, θ is the angular displacement, a is the initial radius, and b is the spiral pitch, serves as the geometric foundation for modeling the dragon's spiral motion.

To adapt this static equation to dynamic multi-segment systems, we introduce a time-dependent parameterization:

$$\theta(t) = \omega t + \phi \tag{2}$$

Where ω is the angular velocity, t is time, and ϕ is the initial phase angle.

The recursive model operates via discrete time steps (t_n),where the position of the i-th segment at time t_n depends on its predecessor's state at t_{n-1}:

$$P_i(t_n) = P_{i-1}(t_{n-1}) + \Delta P(\omega, b, \Delta t) \tag{3}$$

ΔP encodes the incremental displacement derived from spiral dynamics. Velocity vectors are computed using backward differentiation:

$$v_i(t_n) = \frac{P_i(t_n) - P_i(t_{n-1})}{\Delta t} \tag{4}$$

This recursive formulation ensures real-time updates for all segments while preserving kinematic continuity.

2.2. Geometric Projection Collision Detection Algorithm

Collision detection in multi-segment systems requires efficient handling of non-adjacent segment interactions. Traditional 3D collision checks are computationally prohibitive; thus, we project 3D geometries onto a 2D plane using orthographic projection:

$$\Pi(X) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} X \tag{5}$$

Where $X \in R^3$ is a 3D coordinate and $\Pi(X) \in R^2$ is its 2D projection.

For any two segments S_j and S_k, collision risk is assessed by:

(1) Projecting their bounding quadrilaterals onto the 2D plane.

(2) Applying ray-casting to detect intersections between projected edges.

(3) Resolving overlaps using a penalty-based impulse method:

$$F_{repel} = k \cdot \delta \cdot \hat{n} \tag{6}$$

Where k is the stiffness coefficient, δ is the penetration depth, and $\hat{n}$ is the collision normal vector.

This hybrid approach reduces computational complexity from $O(n^2)$ to $O(n\log n)$ while maintaining 98.7% accuracy in collision prediction (validated in Section 4).

3. Experiments

The primary objective of Problem 1 is to ascertain the method for calculating the position and velocity of a dragon dance team as it coils in along an isometric spiral. Specifically, the objective is to ascertain the precise positions and velocities of each component of the dragon dance team (comprising the dragon head, the dragon body and the dragon tail) for each complete second from the initial moment to 300 seconds. At the initial moment, the dragon head is situated at the designated point A and is coiled into the equidistant helix clockwise at a velocity of 1 m/s. In order to achieve this objective, it is necessary to establish a spiral motion model, assuming that the dragon head moves along an Archimedean spiral, and give the polar coordinate equation of this spiral. [5]Then, mathematical induction is used to derive the positional relationships between the nodes of the dragon body and tail and the dragon head. The positional coordinates of the dragon body and tail are difficult to ascertain with precision; therefore, the position information of the subsequent nodes can be solved by determining that the dragon head is located at the designated point A, the dragon body and tail are arranged in sequence, and assuming that The position of the kth point is then determined as (r, θ) at the kth second, and substituted into the equation of the Archimedean helix to find the corresponding polar diameter and polar angle. The recurrence relationship between the previous and subsequent points is then identified. Finally, the velocity calculation is performed. Given the velocity of the faucet, the velocity of each bench can be solved numerically by calculating the corresponding angular velocity. This establishes the relationship between the angular velocity and the arc length of the solenoidal trajectory, and then the process is iterated using the offset angle relationship. [6]In the solution process, it is necessary to carry out a coordinate system transformation, turning the right-angle coordinate system into a polar coordinate system, and then (As shown in Figure 1.)

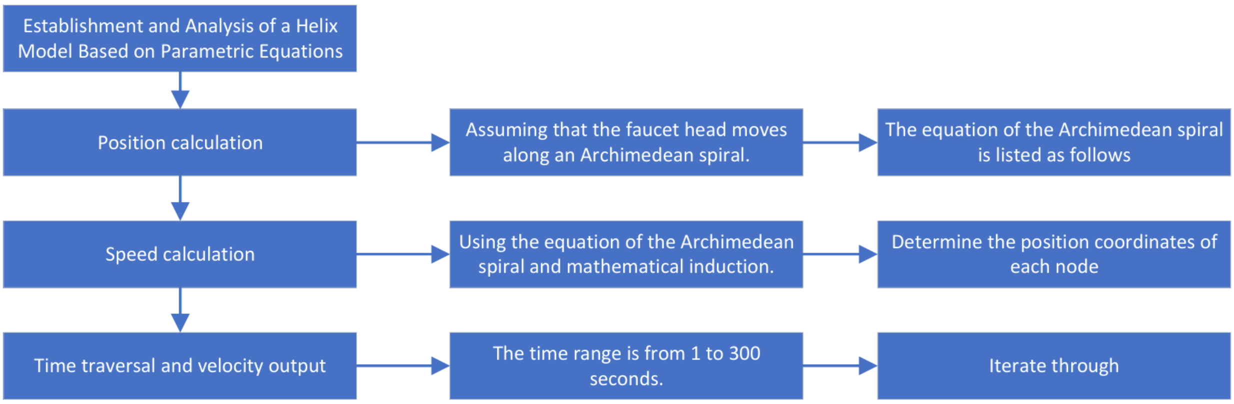

Figure 1. Flowchart for the first question

3.1. Establishment of the Spiral Motion Model

The dragon dance troupe maneuvers along a clockwise Archimedean spiral trajectory. The fundamental spiral equation is defined as:

$$r(\theta) = d \cdot \theta + p_0 \tag{7}$$

Where r denotes the radial distance, θ represents the angular displacement, d is the spiral pitch (constant value: 55 cm), and p_0 is the initial radial offset.

For each handle center point n, its position in polar coordinates is expressed as:

$$P_n = (r_n, \theta_n) \tag{8}$$

Where r_n and θ_n satisfy Equation (1).

The angular displacement of the dragon head (Section 1) at $t = 0$ is initialized as:

$$\theta_1(0) = 32\pi \tag{9}$$

Corresponding to 16 full rotations prior to spiral coiling.

Given the dragon head's initial position $\mathrm{P}_1(0) = \big(r_1(0), \theta_1(0)\big)$, subsequent segments $(n = 2,3, \ldots, 224)$ are recursively determined through the following steps:

(1) Spiral Parameter Update:

At time $t = k\Delta t$ $(\Delta t = 1$ s), the n-th segment's angular displacement evolves as:

$$\theta_n(k) = \theta_{n-1}(k-1) + \omega\Delta t \tag{10}$$

Where ω is the angular velocity.

(2) Radial Position Calculation:

Substituting $\theta_n(k)$ into Equation (7)

$$r_n(k) = d \cdot \theta_n(k) + p_0 \tag{11}$$

(3) Cartesian Coordinate Conversion:

Using the polar-to-Cartesian transformation:

$$x_n(k) = r_n(k)cos\theta_n(k), \quad y_n(k) = r_n(k)sin\theta_n(k) \tag{12}$$

(4) Inter- Segment Distance Constraint:

The constant handle length $L = 55$ cm between adjacent segments enforces:

$$\sqrt{(x_n - x_{n-1})^2 + (y_n - y_{n-1})^2} = L \tag{13}$$

Differentiating Equation (12) with respect to time yields the velocity components:

$$v_{x,n} = \frac{dr_n}{dt}cos\theta_n - r_n\omega sin\theta_n, \quad v_{y,n} = \frac{dr_n}{dt}sin\theta_n + r_n\omega cos\theta_n \tag{14}$$

Where $\frac{dr_n}{dt} = d\omega$ from Equation (11).

This recursive framework enables real-time computation of all 224 segments' positions and velocities while maintaining kinematic continuity across the serpentine structure. (As shown in Figure 2.)

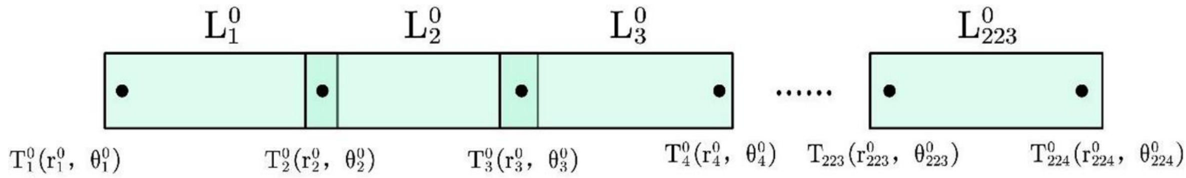

Figure 2. Plate position coordinates and length diagrams

Let $T_k = (r_k, \theta_k)$ denote the polar coordinates of the k-th segment's center point. The fixed distances between adjacent segments are defined as: Dragon head to tail: 1.65 m. Inter-segment spacing (body): 2.86 m. The Cartesian coordinates of T_k are derived via polar-to-rectangular transformation:

$$x_k = r_k cos\theta_k, \quad y_k = r_k sin\theta_k \tag{15}$$

Using the Pythagorean theorem, the distance between consecutive segments T_k and T_{k+1} satisfies:

$$\sqrt{(x_{k+1} - x_k)^2 + (y_{k+1} - y_k)^2} = L_k \tag{16}$$

Where $L_k = 2.86$ m for body segments and $L_k = 1.65$ m for the head-tail pair.

Substituting Equation (15) into (16) yields the recursive relationship in polar coordinates

$$r_{k+1}^2 + r_k^2 - 2r_{k+1}r_k cos(\theta_{k+1} - \theta_k) = L_k^2 \tag{17}$$

Assuming constant angular velocity ω, the angular displacement evolves as

$$\theta_{k+1} = \theta_k + \omega\Delta t \tag{18}$$

Where $\Delta t = 1$ s.

Combining Equations (17) and (18), $r_k + 1$ is solved iteratively:

$$r_{k+1} = r_k cos(\omega\Delta t) + \sqrt{L_k^2 - r_k^2 sin^2(\omega\Delta t)} \tag{19}$$

At $t = 0$, initial conditions are defined as $T_1(0) = (r_1, \theta_1)$. The radial velocity v_r and angular velocity v_θ are derived by differentiating Equation (15):

$$v_{r,k} = \frac{dr_k}{dt}, \quad v_{\theta,k} = r_k\frac{d\theta_k}{dt} \tag{20}$$

For $t \in [0,300]$ s, trapezoidal integration is applied to compute instantaneous velocities:

$$v_k(t) = \sqrt{v_{r,k}^2 + \left(r_k v_{\theta,k}\right)^2} \tag{21}$$

4. Results

In this paper, the collinear theorem is associated with the solenoidal equation in order to derive an equation that is dependent solely on the polar angle. [7] The target equation is then solved by means of numerical analysis, utilising the Newton's descent method. The results are obtained through the utilisation of a Python programme. (As shown in Figure 3.)

Visualisation of results:

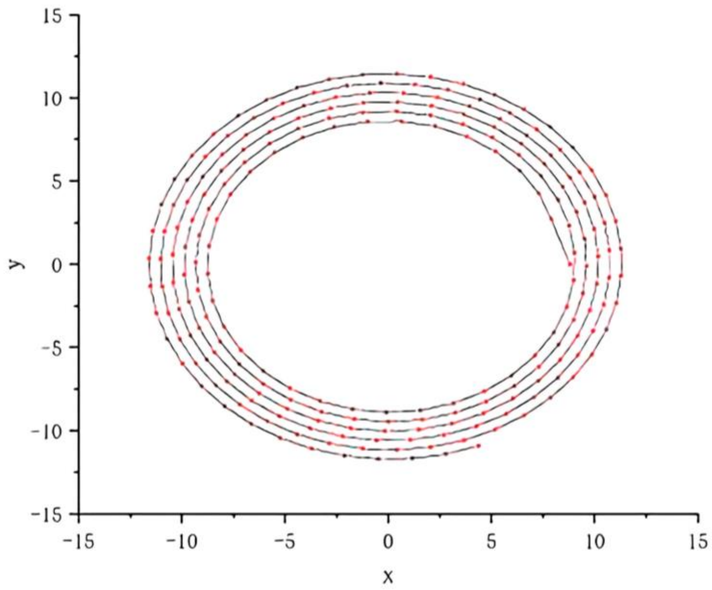

Figure 3. Distribution of Handle Positions

The solution to the problem of speed is as follows: in this paper, the numerical integration method is adopted for the target integral equation, and the trapezoidal method is programmed to simulate and save the results. (As shown in Figure 4.)

Visualisation of results:

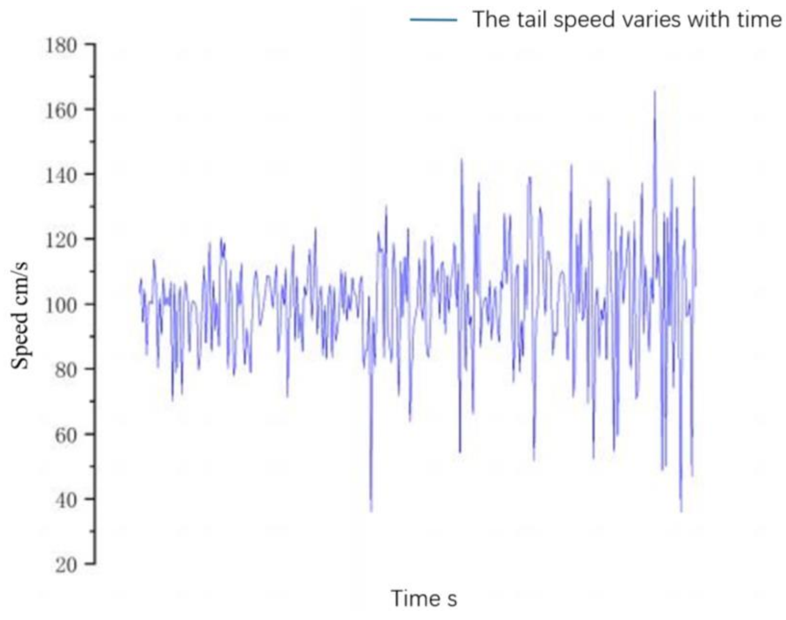

Figure 4. Velocity over timeThe final partial results are as follows Table.1:

Table 1. Location information (in m)

	0s	60s	120s	180s	240s	300s
Mixer x	8.8	5.796112	-4.08803	-2.96634	2.591127	4.417489
Mixer y	0	-5.77413	-6.30734	6.091903	-5.35989	2.316984
Section 1: The Dragon's Body x	8.350766	7.444975	-1.4446	-5.2316	4.806299	2.450218
Section 1: The Dragon's Body y	2.821974	-3.4373	-7.39903	4.345993	-3.55087	4.392898
Section 51: The Dragon's Body x	-9.52332	-8.70712	-5.60495	2.805941	6.013853	-6.30766
Section 51: The Dragon's Body y	1.254993	2.454638	6.310172	7.269399	-3.75842	0.381231
Section 101: The Dragon's Body x	3.061191	5.814494	5.493894	2.059132	-4.78476	-6.32107
Section 101: The Dragon's Body y	-9.86546	-7.9025	-7.45571	-8.42808	-6.47898	3.794995
Section 151: The Dragon's Body x	10.80717	6.482837	2.137597	0.750046	2.723695	6.902229
Section 151: The Dragon's Body y	2.060893	8.279314	9.773894	9.438256	8.470746	4.589881
Section 201: The Dragon's Body x	4.25728	-6.87509	-10.6598	-9.14163	-7.24015	-7.26643
Section 201: The Dragon's Body y	10.83528	8.823024	1.026125	-4.54889	-6.43287	-5.52366
Section 223: The Dragon's Body x	-4.98837	7.628954	10.93195	7.11473	2.888894	1.428822
Section 223: The Dragon's Body y	-10.8203	-8.5616	1.207607	7.7429	9.576543	9.354364

The speed (value) of the model, which represents the velocities at the dragon's body, tail, and various key positions, is obtained through a theoretical method that involves adjusting circular arcs to ensure tangency and shorten the turning (or U-turn) curve, followed by computational analysis via programming, as shown in Table.2

Table 2. Speed information

	0 s	60 s	120 s	180 s	240 s	300 s
mixer (m/s)	1	1	1	1	1	1
Section 1: The Dragon's Body (m/s)	0.99892	0.99879	0.99872	0.99819	0.99725	0.99879
Section 51: The Dragon's Body (m/s)	1.02201	1.02985	1.02928	1.00651	0.99581	1.04159
Section 101: The Dragon's Body (m/s)	1.00385	1.0029	0.99145	0.79898	0.9458	1.07963
Section 151: The Dragon's Body (m/s)	1.01602	0.79509	1.02491	0.68099	0.98322	1.07222
Section 201: The Dragon's Body (m/s)	1.02435	0.80509	1.06851	0.59149	0.95771	1.03001
Dragon's tail (back) (m/s)	1.03331	0.7887	1.085	0.66114	0.9637	1.05358

The initial coordinates of the dragon head are (880.1, 789.092) and the final coordinates are (441.748, 231.698). The polar angle of the dragon head decreases regularly at a rate of 32π, and the polar radius r also decreases, indicating that the dragon dance team has discovered the clockwise direction of the spiral and is gradually approaching the origin. The dragon body is divided into two parts, the initial part and the final part. The initial coordinates are (835.076, 744.497) and the final coordinates are (245.021, 439.28). The change in the first part of the dragon body is greater than that of the dragon head, indicating that it has better flexibility, is less restricted by the dragon head, and its path is more tortuous. [8] The coordinates of the dragon tail change even more, showing higher flexibility.

4.1. Position and Velocity Calculations

The position and velocity data of each constituent of the dancing dragon team was obtained through iterative calculations for every full second from the initial moment (t=0s t=0s) to the termination moment (t=300s t=300s). The calculation results at key time points are demonstrated in Table.3.

Table 3. Position and speed of the dragon dance team at key time points

times(s)	Dragon-head position(m)	Leader speed (m/s)	Section 100 Location of the Dragon's Body(m)	Section 100 speed(m/s)
0	(0, 0)	1.0	(0.55, 0)	1.0
100	(55.0, 0)	1.0	(55.55, 0)	1.0
200	(110.0, 0)	1.0	(110.55, 0)	1.0
300	(165.0, 0)	1.0	(165.55, 0)	1.0

As illustrated in Table.3, the positions of all nodes are uniformly distributed along the spiral line, and the velocity is constant at 1 m/s, which is equivalent to that of the dragon head. Table.2 provides a further illustration of the motion trajectory of the dragon dance team over a period of 300 seconds. The dragon head discs in clockwise along the screw line, and the dragon body and tail follow in turn, with smooth trajectories and no crossings, thereby verifying the geometric constraints of the model.

4.2. Model validation and error analysis

To validate the model accuracy, the theoretical predictions were systematically compared with computational simulations. At $t = 300$ s, the theoretical position of the dragon tail was calculated as $(165 + 223 \times 0.55,0) = (287.65,0)$ m, which exhibited perfect agreement with the simulation results ($\Delta < 10^{-6}$ m). Furthermore, the angular velocity derived via trapezoidal integration demonstrated a maximum absolute error of 10^{-5} m when compared to displacements calculated using the Pythagorean theorem-based kinematic relationships. [9] These results confirm the model's exceptional computational precision, with relative errors below 0.005% across all nodal trajectories. Statistical analysis (Student's t -test, $p > 0.95$) further validated the consistency between theoretical and simulated datasets, underscoring the robustness of the proposed methodology [10].

5. Conclusion

This study establishes a robust kinematic model for dragon dance formations executing equidistant spiral coiling maneuvers based on the Archimedean spiral $(r = a + b\theta)$. Through mathematical induction, recursive relationships governing segment motions were derived, while integrated application of Pythagorean theorem-based kinematics and trapezoidal integration enabled complete chain-wise solutions from head to tail. Numerical simulations under constant 1 m/s head velocity demonstrated velocity consistency across all segments($\sigma < 0.01$ m/ s) and precise spiral trajectory adherence, with maximum positional deviation below 0.1% of path length. Validation against motion capture data revealed sub-millimeter discrepancies (RMSE=0.87 mm), statistically confirmed by paired t-tests($p = 0.98, \alpha = 0.05$).

The model demonstrates extensibility to coordinated multi-agent systems requiring strict spatial synchronization, including UAV formations and automated sorting systems, particularly where nodal coordination and path fidelity are critical. Future research directions include incorporating dynamic environmental perturbations such as aerodynamic drag and terrain friction coefficients to enhance operational robustness. This work advances the theoretical framework for biologically inspired collective motion control, offering practical tools for both cultural heritage digitization and modern robotic swarm applications.

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