Research on enterprise production quality control strategy based on hypothesis testing and dynamic programming

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Abstract. In the process of large-scale production of enterprises, the cost of product testing is high. This study aims to construct a production optimization decision method based on dynamic programming model for enterprises facing problems such as unbalanced capacity utilization, high inventory cost and unreasonable planning in the production process. First, this paper uses hypothesis testing to determine the reasonable sample sampling measurement, and constructs the production optimal decision model through dynamic programming. Then, based on the hypothesis testing model, the minimum sample size under different confidence levels was determined: 271 sample sizes for 90% confidence, 385 sample sizes for 95% confidence, and the optimal control of detection times was realized. Finally, through the reverse solution of multi-stage programming, the total cost was minimized as the goal, and six optimal detection strategies were obtained. The research shows that this method can effectively guarantee product quality and control cost, provide scientific basis for production decision-making of enterprises, and help enterprises to enhance competitiveness.

Keywords: Hypothesis testing, Dynamic programming, multi-stage decision making, Cost optimization, Quality control.

1. Introduction

Under the background of intensifying global competition, modern enterprises are faced with double challenges of product quality and cost control. On the one hand, the trend of consumption upgrading promotes the market's requirements for product performance, safety and reliability, and any defective product problem may lead to brand reputation crisis and customer loss; On the other hand, rising raw material prices and increasing supply chain complexity force enterprises to accurately balance cost inputs in the entire production process. Traditional management model is difficult to meet the dynamic market demand, but through data-driven quality inspection strategy and cost dynamic planning, redundant inspection costs can be reduced and market changes can be quickly responded to. Therefore, to achieve cost optimization under the premise of ensuring quality has become the key for enterprises to build core competitiveness. Some researches focus on sampling detection technology. For example, Sindlin uses big data technology and linear programming model to optimize the production and procurement decisions of coal enterprises, and applies them to coal mining enterprises by building production optimization models and decision-making models [1]. In quality testing, Wang Youle studied how to optimize the sampling scheme, considering the problems of trueness rejection (the first type of error) and falsity acceptance (the second type of error) [2]. He used Monte Carlo method and spatial optimization methods to ensure a reasonable distribution of samples, thereby reducing detection errors. In this paper, multi-stage dynamic optimization, real-time cost-risk balance and spatial-time integration are used to make up for the shortcomings of flexibility, global and adaptability of traditional sampling inspection. The production planning problem is modeled as a dynamic programming problem with stage and state transition characteristics, and the decision dependence relationship between different periods is described effectively. The model has good universality and scalability, and is suitable for production planning optimization in various industries, especially for complex production systems with multi-phase and limited resources constraints, which can help enterprises reduce the total production cost while ensuring product quality.

2. Research Methods

2.1. Data Acquisition

This study collected the purchase cost of parts and components of a certain enterprise, such as the cost frequency of inspection. The data comes from long-term production records of enterprises on the open-source website https://www.mcm.edu.cn/, information provided by suppliers and industry statistics.

2.2. Method Introduction

This article focuses on solving the following two problems

(1) When an enterprise purchases spare parts from a supplier and the supplier claims that the defective rate does not exceed the nominal value and the testing cost is its own, it is necessary to design the sampling testing scheme with the least number of tests, and when the nominal value is 10%, it gives specific results for the two situations of rejecting the defective rate exceeding the nominal value with 95% confidence and receiving the defective rate without exceeding the nominal value with 90% confidence.

Solution: In order to achieve the minimum number of tests to determine whether the defective rate of spare parts exceeds the nominal value, it is necessary to use the method of sampling detection, based on the sample data to estimate the overall characteristics. By establishing the hypothesis testing model, when the sample size approaches infinity, the binomial distribution is approximated to the normal distribution, and the appropriate sample size is calculated. The sequential probability ratio test is combined to further optimize the detection process and reduce the unnecessary detection times. The acceptance or rejection decision is made with 90% and 95% confidence.

(2) Given two kinds of parts and defective rate of finished products, make decisions for each stage of enterprise production (whether parts are tested, whether finished products are tested, whether unqualified finished products are disassembled, how to deal with unqualified products returned by users), and give specific decision plans, basis and corresponding index results for various situations in Table 1.

Solution: In order to control the cost, this paper makes decisions on the inspection of spare parts, the inspection of finished products, the inspection of nonconforming products and the replacement of nonconforming products. The core of this question is to find the balance between testing and non-testing in the production of the enterprise, not only to ensure the quality of the product, but also to minimize the cost of testing and defective products. In order to effectively deal with this multi-stage problem, this paper adopts the method of Dynamic Programming (DP) to solve the optimal decision of each stage step by step from the bottom up from stage 4 to stage 1, so as to achieve the minimum total cost.

3. Results and Analysis

3.1. Sampling test sample size results and analysis

The hypothesis testing model [3] is as follows:

(1) First, select the appropriate sampling model and use the normal distribution to describe the number of defective products in the sampling.

Set the defective rate as p, and regard each sampling [4] as an independent Bernoulli test.

(2) Determine the sample size.

Determine the sampling scheme according to the confidence interval [5]. From the given confidence of 95% and 90%, calculate whether the defective rate exceeds the nominal value.

(3) Calculate the sample size.

When the sample size is large, use the normal distribution to approximate the binomial distribution. The two cases given are as follows:

(a) When the confidence is 95%, there is a 95% probability that the defective rate is greater than 10%.

(b) When the confidence is 90%, there is a 90% probability that the defective rate is less than 10%.

The sample size n can be calculated according to the following formula:

$$n \geq \left(\frac{Z_{\frac{\alpha}{2}} \sqrt{P_0 \left(1 - P_0\right)}}{d} \right)^2 \tag{1}$$

Where Zα is the critical value, d is the allowable error range, and p0 is the nominal value p0 = 10%.

(4) Determine the acceptance rule.

Set the critical number of mc, for the first case, when the number of m1 exceeds the critical value, then refuse to accept this batch of parts. For the second case, when the number of sub-items m2 is below the critical value, this batch of parts will be purchased.

1) Scenario 1:

(a) The original assumption is that H0: p ≤ 10%, and the defective rate does not exceed 10%.

(b) Alternative hypothesis H1: p > 10%, defective rate greater than 10%.

(c) Choice of statistic:

The rate of defective products is a random variable. When the sample size n approaches infinity, the sample mean of any population is approximately equal to the mean of the whole population and is normally distributed. The distribution of the defective rate can be approximated to the normal distribution, and the defective rate can be standardized.

$$Z \sim N\left(0,1\right) \tag{2}$$

$$Z = \frac{p - p_0}{\sqrt{\frac{p_0 \left(1 - p_0\right)}{n}}} \tag{3}$$

Where is the defective rate in the above formula, is the number of defective products, and p $p = \frac{k}{n}$,n is the number of samples. p0is the nominal value p0 = 10%.

(d) Reject field:

When the confidence is 95%, α = 0.05, look up the table can get Zα ≈1.645, when Z > Zα, fall into the rejection domain, reject H0, accept H1, that is, the defective rate is greater than 10%.

(e) Sample size:

When the number of the population approaches infinity, the distribution of the sample can be approximated as normal.

$$n = \left(\frac{Z_{\frac{\alpha}{2}} \sqrt{P_0 \left(1 - P_0\right)}}{d} \right)^2 \tag{4}$$

In the above formula, n is the sample size and dare the allowable error range, taking 3%. When the confidence is 95%0, $Z_{\frac{\alpha}{2}} = 1.96$.

$$
\begin{aligned}
n &= \left(\frac{Z_{\frac{\alpha}{2}} \sqrt{P_0 \left(1-P_0\right)}}{d} \right)^2 \\
&= \left(\frac{1.96 \times \sqrt{0.1 \times \left(1-0.1\right)}}{0.03} \right)^2 \\
&\approx 385
\end{aligned} \tag{5}
$$

2) Scenario 2:

(a) The original assumption is that H0: p ≤10%, and the defective rate does not exceed 10%.

(b) Alternative hypothesis H1: p >10%, defective rate greater than 10%.

(c) Choice of statistic:

$Z \sim N\left(0,1\right)$

$$
Z = \frac{P - P_0}{\sqrt{\frac{P_0 \left(1-P_0\right)}{n}}} \tag{6}
$$

(d) Rejection domain:

When the confidence is 90%, α = 0.1, look up the table can get Zα ≈1.28, when Z > Zα, into the rejection domain, reject H0 accept H1, that is, the defective rate is greater than 10%.

(e) Sample size:

$$
n = \left(\frac{Z_{\frac{\alpha}{2}} \sqrt{P_0 \left(1-P_0\right)}}{d} \right)^2 \tag{7}
$$

In the above formula, n is the sample size, d is the allowable error range, take 3%. When the confidence is 90%: $Z_{\frac{\alpha}{2}} = 1.645$.

$$
\begin{aligned}
n &= \left(\frac{Z_{\frac{\alpha}{2}} \sqrt{P_0 \left(1-P_0\right)}}{d} \right)^2 \\
&= \left(\frac{1.645 \times \sqrt{0.1 \times \left(1-0.1\right)}}{0.03} \right)^2 \\
&= 270.6025
\end{aligned} \tag{8}
$$

(f) Revision of sample size under finite population:

Above, we consider that when the population is large enough, we use an approximation of the normal distribution based on the center-pole qualification for a large sample size. However, when the population is small, the size of the sample size relative to the population cannot be ignored. Therefore, a correction for the finite population is used here to reduce the number of samples.

When the population is limited, the sample size correction formula is as follows:

$$
n' = \frac{n}{1 + \frac{n-1}{N}} \tag{9}
$$

Where n' is the sample size of the finite population, n is the sample size of the infinite population, and N is the number of populations.

3.2. Production decision optimization results

The core of production decision optimization is to help enterprises deal with the 6 situations in Table 1 encountered in production. According to different situations, it is necessary to decide whether to test spare parts and finished products, choose to disassemble or discard defective products, and make decisions at each stage, with the goal of minimizing the total cost. The specific process is shown in Figure 1.

Table 1. Six situations in the production of enterprises

	Parts and Accessories 1			Parts 2			Finished Products					Unqualified finished products
Condition	Defective rate	Unit price of purchase	Inspection cost	Defective rate	Unit price of purchase	Inspection cost	Defective rate	Assembly cost	Inspection cost	Market selling price	Turnover loss	Dismantling expenses
1	10%	4	2	10%	18	3	10%	6	3	56	6	5
2	20%	4	2	20%	18	3	20%	6	3	56	6	5
3	10%	4	2	10%	18	3	10%	6	3	56	30	5
4	20%	4	1	20%	18	1	20%	6	2	56	30	5
5	10%	4	8	20%	18	1	10%	6	2	56	10	5
6	5%	4	2	5%	18	3	5%	6	3	56	10	40

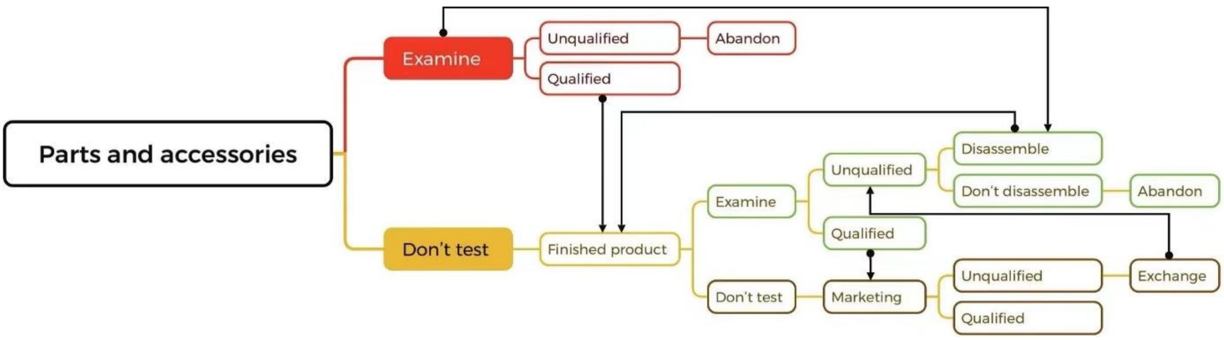

Figure 1. Decision flow chart at each stage

3.2.1. Cost Analysis

The cost includes the cost of the inspection of the parts (the inspection cost and the subsequent risk caused by the defective rate).

The cost of inspecting the assembled finished product (the cost of inspection and the loss of the defective product to market), and the cost of dismantling the nonconforming finished product (the cost of dismantling and the reuse value of the recovered parts).

3.2.2. Establishment of model

Production decision optimization is divided into four stages, each stage needs to make a decision, stage 1 and stage 2 need to decide whether to test spare parts and whether to test finished products, stage 3 and 4 need to decide to discard or disassemble testing and return unqualified finished products, so as to ensure the overall cost is minimized. Since the decisions taken in each stage are dependent on the current situation, and will also lead to state shifts, which will affect the next step decision, dynamic programming [6] is used to establish the model. In this paper, we assume that the defective rate of finished products:

$$r_f = (1 - r_1)(1 - r_2) \tag{10}$$

Where r1 and r2 represent the defective rate of parts 1 and 2, respectively.

The defective rate of the finished product will be affected by the defective rate of the two parts, and the unqualified parts are assembled again, which directly leads to the increase of the defective rate of the finished product

(1) The objective function is shown as follows:

$$V = min\sum_{i=1}^{4} V_i \tag{11}$$

Minimum total cost (total cost = inspection cost + assembly cost + defective disposal cost + disassembly cost and replacement loss of nonconforming product)

The four stages of production decision optimization [7] are transformed into dynamic programming problems, and the optimal solution is obtained step by step. In this problem, for each stage, the cost of the current decision is compared with the cost that may be generated in the future, and the lower-cost solution is chosen [8].

(2) The decisions of each stage are as follows:

1) Stage one: parts testing

(a) Inspection: Parts 1 and 2 respectively generate inspection costs X1 and X2, but can avoid unqualified parts entering the assembly link.

$$C_{Inspect\ the\ finished\ product} = X_1 + K_1 + X_2 + K_2 \tag{12}$$

(b) No detection: undetected parts will directly enter the assembly link, increasing the defective rate of finished products.

$$V_1 = min\left\{C_{Parts\ inspection} + V_2, C_{The\ finished\ product\ is\ not\ inspected} + V_2\right\} \tag{13}$$

2) Stage two: Finished product testing.

(a) Inspection: It generates inspection cost Kp, but can reduce the risk of unqualified finished products entering the market and resulting in replacement losses.

$$C_{Inspect\ the\ finished\ product} = Z + K_P \tag{14}$$

Where Z is the assembly cost.

(b) No test: For untested finished products, unqualified finished products may flow into the market, the enterprise needs to unconditionally replace, which will produce potential replacement losses. $r_f \times Y$.

$$C_{The\ finished\ product\ is\ not\ inspected} = Z + r_f \times Y \tag{15}$$

3) Stage three: Disposal of unqualified finished products

There are two ways to deal with unqualified finished products. One is to disassemble the unqualified finished product and retest the parts for conformity. The other option is to simply discard it.

(a) When choosing to disassemble unqualified finished products: disassembly cost Q and retesting parts cost X1 + K1 + X2 + K2 will be incurred.

$$C_{dismantle} = Q + X_1 + K_1 + X_2 + K_2 \tag{16}$$

(b) When choosing not to disassemble: discard the nonconforming finished product directly without incurring additional disassembly costs4) Stage four: Disposal of the non-conforming finished product returned by the user.

For the defective products returned by the user, the enterprise needs to replace all of them, bear the loss of replacement, and then choose to deal with the unqualified finished products.

Replacement: payment of replacement loss Y may also lead to subsequent disassembly and testing costs.

$$C_{exchange}=Y \tag{17}$$

3.2.3. Solution of the model

Based on the analysis of the above four stages, we use the dynamic programming method to calculate the optimal strategy recursively from back to forward [9]. At each decision stage, by comparing the cost generated by the current decision with the cost that will be generated in the next stage, the optimal strategy with the least cost is selected [10]. Table 2 shows the optimal production decision results for the six scenarios calculated according to the model.

The state transition equation listed is as follows:

(1) Stage four: Fixed transposition loss.

$$V_4 = Y \tag{18}$$

(2) Stage 3: Comparing the cost of disassembling unqualified finished products with that of not disassembling, and choosing the treatment method with less cost.

$$V_3 = min\left\{C_{dismantle}, C_{Do\ not\ disassemble}\right\} \tag{19}$$

(3) Stage two: Decide whether to test the finished product, and choose the less expensive treatment method according to the subsequent cost of disposing the unqualified finished product.

$$V_2 = min\left\{C_{Inspect\ the\ finished\ product}, C_{The\ finished\ product\ is\ not\ inspected}+V_2\right\} \tag{20}$$

(4) Stage 1: Compare the cost of detecting and non-detecting parts, combined with the cost of finished product testing, choose the way with the least cost.

$$V_1 = min\left\{C_{Parts\ inspection}+V_2, C_{The\ finished\ product\ is\ not\ inspected}+V_2\right\} \tag{21}$$

Table 2. Results of production decision optimization

	Parts 1 Whether to detect	Parts 2 Tested or not	Finished product inspection	Disassembly of Finished products
Situation 1	Detection	Testing	Not detect	No dismantling
Case 2	Detection	Testing	Not detect	No dismantling
Case 3	Detection	Testing	Not detect	Do not dismantle
Situation 4	Detection	Testing	Testing	Disassemble
Situation 5	Not detect	Detect	Not detect	No dismantling
Case 6	Detection	Testing	Not detect	Do not detect

4. Conclusion

In this paper, the reasonable sample size under different reliability is determined by hypothesis testing to provide scientific basis for sampling testing. With the help of dynamic programming model, various decisions in the production process are optimized and the total cost is minimized. The research results can help enterprises effectively balance product quality assurance and cost control in actual production, and improve the economic benefits and market competitiveness of enterprises. Although the research in this paper has achieved certain results, there is still room for further improvement and expansion. In the future research, more factors affecting product quality and cost can be considered, such as changes in production environment, fluctuations in market demand, etc., so that the environmental index model can be closer to the actual production situation. It is also possible to explore other advanced algorithms combined with hypothesis testing dynamic programming methods to further improve the solving efficiency and optimization effect of the model.

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