Optimal design of production decisions based on probabilistic and statistical models

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Abstract. This paper conducts an in-depth study on a series of decision-making problems faced by a certain enterprise in the production process of electronic products, such as the defective rate, inspection and disassembly. By optimizing the sampling, inspection and disassembly strategies in the production process, the enterprise's profit and production efficiency are maximized. Firstly, a sampling inspection plan is designed. For the two cases of 95% confidence and 90% confidence, a hypothesis testing problem is established. By utilizing the mutually restrictive relationship between inspection cost and hypothesis testing effect, a decision function is established, and the optimal sample size is obtained. Secondly, for four decision-making problems: (1) whether spare parts 1 are inspected; (2) whether spare parts 2 are inspected; (3) whether finished products are inspected; (4) whether unqualified finished products are disassembled, the properties of binomial distribution and geometric distribution are used. Based on parameters such as the defective rate, purchase unit price and inspection cost, a cost-benefit analysis model is established. For six different situations, specific decision-making plans, decision-making basis and corresponding index results are given with the standard of maximizing expected profit. The experimental results show that the method proposed in this paper can provide the enterprise with a set of decision-making plans based on probability and statistical models, helping it to reasonably select sampling, inspection and disassembly strategies when facing different production conditions, and improve resource utilization and economic benefits.

Keywords: Production Process Decision Making, Sampling Testing, Hypothesis Testing, Binomial Distribution, Geometric Distribution.

1. Introduction

An enterprise producing a best-selling electronic product needs to purchase two kinds of spare parts, respectively, parts 1 and parts 2, and assemble the two spare parts into finished products in the enterprise. In the assembly of the finished product, as long as one of the parts is unqualified, the finished product must be unqualified; If both parts are qualified, the finished product may not be qualified. For unqualified finished products, the enterprise can choose to scrap, or disassemble it, the disassembly process will not cause damage to the parts, but it needs to spend the disassembly cost.

Therefore, the enterprise needs to consider each step of the production process inspection and disassembly strategy, in order to expect to achieve the goal of profit maximization. However, the existing researches mainly focus on the allocation and scheduling of manpower, equipment and other resources in the production process [1-3], and there are few researches on the hypothesis testing and cost-benefit analysis based on mathematical models. This paper aims to establish a hypothesis testing model, design decision function, and get the optimal sample size. At the same time, the paper discusses four decision-making problems faced by the enterprise in the production process: (1) Whether the spare parts 1 is tested. (2) Whether the spare parts 2 is tested. (3) Whether the finished products are tested. (4) Whether the unqualified finished products are disassembled. By establishing a cost-benefit analysis model, the concrete decision-making scheme is given based on the maximum profit expectation.

This paper focuses on the following two questions:

Question 1. Hypothesis testing questions:

The supplier claims that the defective rate of a batch of parts (Part 1 or part 2) will not exceed a certain nominal value. The enterprise intends to adopt sampling testing method to decide whether to accept the parts purchased from the supplier, and the testing cost shall be borne by the enterprise itself.

Please design a sampling inspection plan for the enterprise with as few inspection times as possible. If the nominal value is 10%, according to the designed sampling detection scheme, specific results are given for the following two cases:

Case (1): If it is determined that the defective rate of spare parts exceeds the nominal value with 95% reliability, the batch of spare parts will be rejected;

Case (2): If it is determined that the defective rate of spare parts does not exceed the nominal value with 90% confidence, the batch of spare parts is received.

Question 2. Production process decision problems:

In the production process, the enterprise is faced with 6 situations as shown in Table 1:

Table 1. Situations encountered by enterprises in production

Situation		1	2	3	4	5	6
Spare parts 1	Defective rate	10%	20%	10%	20%	10%	5%
	Purchase unit price	4	4	4	4	4	4
	Inspection cost	2	2	2	1	8	2
Spare parts 2	Defective rate	10%	20%	10%	20%	20%	5%
	Purchase unit price	18	18	18	18	18	18
	Inspection cost	3	3	3	1	1	3
Finished product	Defective rate	10%	20%	10%	20%	10%	5%
	Assembly cost	6	6	6	6	6	6
	Inspection cost	3	3	3	2	2	3
	Market price	56	56	56	56	56	56
Unqualified Finished Product	Loss on replacement	6	6	30	30	10	10
	Dismantling cost	5	5	5	5	5	40

In Table 1, the defective rate of spare parts and finished products is known, and the production process of the enterprise is as follows:

(1) Whether the spare parts (parts 1 and/or parts 2) are tested. If the spare parts are not tested, the spare parts will be directly entered into the assembly link; Otherwise, the detected unqualified parts will be discarded. (2) Whether to test each assembled finished product. If not tested, the assembled finished product directly enters the market; Otherwise, only qualified finished products enter the market. (3) Whether to disassemble the detected unqualified products. If not, directly discard the unqualified products; Otherwise, repeat (1) and (2) for disassembled parts. (4) For the unqualified products purchased by users, the enterprise will unconditionally replace them, and generate certain exchange losses. Repeat (3) for returned nonconforming products.

This paper aims to discuss, under the above four production stages, when the enterprise is faced with 6 different situations as shown in Table 1, which decision scheme can be adopted respectively to maximize the expected profit of each situation.

2. The main research methods

The main steps of the research method in this paper are as follows: (1) Hypothesis testing methods. By setting null hypothesis and alternative hypothesis, test statistics are calculated using sample data, and rejection domains are determined according to the significance level, so as to determine whether the defective rate of parts exceeds the nominal value [4]. (2) Establish the decision function. By constructing a decision function containing detection cost and inspection effect, and solving the optimal solution of the function, a sampling detection scheme with as few detection times as possible can be obtained while meeting the reliability requirements [5]. (3) Establish a cost-benefit analysis model. Through establishing the cost-benefit analysis model, the profit expectation under different decision schemes is calculated according to the defective rate, purchase price, inspection cost and other parameters, so as to choose the optimal decision scheme. (4) Application of binomial distribution and geometric distribution properties [6-7]. When establishing the cost-benefit analysis model, the expected cost and expected benefit under different decision schemes are calculated according to the defective rate of parts and finished products (subject to binomial distribution or geometric distribution), so as to compare and optimize.

3. The establishment and solution of the mathematical model

3.1. Symbol specification

The unit symbols used in this paper are shown in Table 2:

Table 2. The symbol description of this paper

Mathematical Symbol	Paraphrase	Units
n	Sample size for sampling detection	-
V	Profit per unit sold of finished products	¥
P	The market price of the finished product	¥
C	Unit cost of qualified finished goods	¥

3.2. Establishment and solution of hypothesis testing problem

Under the theoretical system of hypothesis testing of the frequency school, the original hypothesis and the alternative hypothesis have different positions, and the enterprises also have different attitudes towards rejection and acceptance in this problem. Therefore, the idea of hypothesis testing can be used to deal with this problem [8]. In the two cases (1) and (2), there is not only the difference in significance level, but also the opposite of the original hypothesis and the alternative hypothesis. Therefore, the two scenarios are discussed separately in the following.

3.2.1. Case (1)

Simple random sampling is used to test the defective rate. It is assumed that a total of n samples is taken and whether each sample is independently and equally distributed as defective products, then the problem can be expressed in the following mathematical language:

$$X_1, X_2, \dots\dots, X_n i,i,d. \sim B(1,p) \tag{1}$$

$$X_i = \begin{cases} 1, The\ i\ th\ sample\ is\ defective \\ 0, The\ i\ th\ sample\ is\ qualified \end{cases} \tag{2}$$

Where $i,i,d.$ represents independent identically distribution, $B(1,p)$ represents binomial distribution [9], and p represents the defective rate of parts.

According to the 10% nominal value and the program requirement, "If the defective rate of parts is determined to exceed the nominal value with 95% confidence, the batch of parts will be rejected", the hypothesis test problem can be obtained:

$$H_0: p \leq 0.1 \tag{3}$$

$$H_1: p > 0.1 \tag{4}$$

The significance level of the hypothesis testing problem $\alpha = 0.05$. According to the idea of hypothesis testing, the probability that the defective rate does not exceed 10% of the nominal value but rejects the null hypothesis (that is, the probability of making the first type of error) must be controlled below $\alpha = 0.05$. Under the premise of meeting the above requirements, try to control the probability that the defective rate exceeds 10% of the nominal value, but accept the original hypothesis (that is, the probability of making the second type of error).

If the number of tests is larger, the cost of the enterprise is larger; If the number of tests is small, although the cost is saved, the probability of making the second type of error is greater. It can be seen that choosing the right number of tests is actually a trade-off between the cost of testing and the probability of making a type II error.

Since the value of n is large, the test statistic can be constructed in the following way and its asymptotic normal property can be utilized [10]:

$$Z = \frac{\overline{X}-p}{\sqrt{\frac{p(1-p)}{n}}} \to N(0,1) \tag{5}$$

In this formula, $\overline{X}$ is the sample mean:

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{6}$$

The boundary condition for the null hypothesis is $p = 0.1$, under which:

$$Z = \frac{\overline{X}-0.1}{\sqrt{\frac{0.09}{n}}} = \frac{\sqrt{n}(\overline{X}-0.1)}{0.3} \to N(0,1) \tag{7}$$

Therefore, the rejection domain for $\alpha = 0.05$ is:

$$\{\overline{X} \mid \frac{\sqrt{n}(\overline{X}-0.1)}{0.3} > \mu_{0.95}\} \tag{8}$$

That is:

$$\{\overline{X} \mid \overline{X} > \frac{0.3\mu_{0.95}}{\sqrt{n}} + 0.1\} \tag{9}$$

Where $\mu_{0.95}$ represents the upper 0.05 quantile of the standard normal distribution.

This formula is a judgment method. If the $\overline{X}$ calculated based on the sample falls into the above set, the null hypothesis will be rejected and this batch of parts will be rejected. Conversely, if the $\overline{X}$ falls into the complement of the above set, the null hypothesis is accepted and the parts are received.

To measure the quality of the hypothesis test, the power function $K(n,p)$ of the hypothesis test is calculated as follows [11]:

$$K(n,p) = P\left(\overline{X} > \frac{0.3\mu_{0.95}}{\sqrt{n}} + 0.1 \,\middle|\, p\right) = P\left(\frac{\overline{X}-p}{\sqrt{\frac{p(1-p)}{n}}} > \frac{\frac{0.3\mu_{0.95}}{\sqrt{n}}+0.1-p}{\sqrt{\frac{p(1-p)}{n}}} \mid p\right)$$
$$= 1 - \Phi\left(\frac{\frac{0.3\mu_{0.95}}{\sqrt{n}}+0.1-p}{\sqrt{\frac{p(1-p)}{n}}}\right) \tag{10}$$

Where $\Phi(\cdot)$ represents the distribution function of the standard normal distribution.

3.2.2. Case (2)

In the case of (2), the scheme requirement is "if it is believed that the defective rate of parts does not exceed the nominal value with 90% reliability, this batch of parts will be accepted", which can be seen that the null hypothesis and alternative hypothesis in the case of (1) are reversed as follows:

$$H_0: p > 0.1 \tag{11}$$

$$H_1: p \leq 0.1 \tag{12}$$

Since the reliability is 90%, the significance level $\alpha = 0.1$, and under the boundary condition $p = 0.1$ for the null hypothesis, the asymptotic normal property of the test statistic still holds, as follows:

$$Z = \frac{\overline{X}-0.1}{\sqrt{\frac{0.09}{n}}} = \frac{\sqrt{n}(\overline{X}-0.1)}{0.3} \to N(0,1) \tag{13}$$

The difference from (1) is that the rejection field is constructed as follows:

$$\{\overline{X} \mid \frac{\sqrt{n}(\overline{X}-0.1)}{0.3} < \mu_{0.1}\} \tag{14}$$

That is:

$$\{\overline{X} \mid \overline{X} < \frac{0.3\mu_{0.1}}{\sqrt{n}} + 0.1\} \tag{15}$$

Where $\mu_{0.1}$ represents the lower 0.1 quantile of the standard normal distribution.

If the $\overline{X}$ calculated based on the sample falls into the above set, the null hypothesis will be rejected and the parts will be rejected. Conversely, if $\overline{X}$ falls into the complement of the above set, the null hypothesis is accepted and the parts are received.

The power function $K(n,p)$ in case (2) is derived below:

$$\begin{aligned} K(n,p) &= P\left(\overline{X} < \frac{0.3\mu_{0.1}}{\sqrt{n}} + 0.1 \,\middle|\, p\right) = P\left(\frac{\overline{X}-p}{\sqrt{\frac{p(1-p)}{n}}} < \frac{\frac{0.3\mu_{0.1}}{\sqrt{n}}+0.1-p}{\sqrt{\frac{p(1-p)}{n}}} \mid p\right) \\ &= \Phi\left(\frac{\frac{0.3\mu_{0.1}}{\sqrt{n}}+0.1-p}{\sqrt{\frac{p(1-p)}{n}}}\right) \end{aligned} \tag{16}$$

3.2.3. Selection of sample size n in cases (1) and (2)

As mentioned above, the selection of n is actually a trade-off between the cost of testing and the effect of hypothesis testing. To measure the effect of hypothesis testing, the power function $K(n,p)$ calculated above is used. Obviously, for a given p, the larger n is, the larger $K(n,p)$ is, and the larger $K(n,p)$ is, the lower the probability of making the second type of error is, and the better the effect of hypothesis testing is. The expected $q(n)$ of $K(n,p)$ is calculated below as a measure of the effect of hypothesis testing when the sample size is n:

$$q(n) = E(\,K(n,p) \mid n\,) = \int_{p\in\theta_1} f(p)K(n,p)dp \tag{17}$$

Where θ_1 represents the parameter space of the alternative hypothesis, and $f(p)$ is the prior distribution density function of p, which in this paper can be assumed to be a uniform distribution $U(0,1)$.

On the other hand, more n means more money. Assuming that the cost is proportional to the sample size n, the following function $g(n)$ can be listed as the decision function for choosing the value of n:

$$g(n) = w_1cn - w_2q(n) = w_1cn - w_2\int_{p\in\theta_1} f(p)K(n,p)dp \tag{18}$$

Where, w_1 and w_2 are the given weights, satisfying $w_1 + w_2 = 1$, $w_1 > 0$, $w_2 > 0$, which are determined by the importance of detection cost and detection effect considered by the enterprise, and c is the cost of each sample detected. If the enterprise thinks that the detection effect is above, w_2 should be larger; If the enterprise believes that reducing the cost of testing is desirable, w_1 should be larger.

For example, if $w_1 = 0.001$ and $w_2 = 0.999$ are assumed, in case (1), the decision function $g(n)$ is as follows:

$$g(n) = w_1cn - w_2\int_{0.1}^{1} f(p)K(n,p)dp \tag{19}$$

The optimal value of $n = 43$ is obtained through R language calculation. That is, when $n = 43$, the function $g(n)$ takes the minimum value.

3.3. Establishment and solution of decision-making problems in production process

The essence of decision making is the expectation of maximizing returns. In practice, the manufacturer usually has a certain expected output n, and the enterprise needs to maximize the expected revenue of producing each qualified finished product. Due to the constant selling price, the above problem is equivalent to the expectation of minimizing the cost of producing each qualified finished product.

There are 16 kinds of decisions for enterprises to consider, as shown in Table 3:

Table 3. 16 decision schemes and corresponding numbering tables

Whether the unqualified products are disassembled	Whether to test spare parts 1	Whether to test spare parts 2	Whether to test the finished product	ID
Yes	Yes	Yes	Yes	001
			No	002
		No	Yes	003
			No	004
	No	Yes	Yes	005
			No	006
		No	Yes	007
			No	008
No	Yes	Yes	Yes	009
			No	010
		No	Yes	011
			No	012
	No	Yes	Yes	013
			No	014
		No	Yes	015
			No	016

In the following, the expected cost of producing each qualified finished product under 16 different decisions is calculated in relation to the defective rate of each product, the unit price of purchase, the inspection cost, the assembly cost, and the replacement loss and dismantling cost. A unified expression can be given for the 8 decision cases of discarding unqualified products, while the 8 decision cases of disassembling unqualified products are more complicated. For the reason of space, the results of disassembling the unqualified product are presented directly at the end.

3.3.1. Discard the solution of the expression under the unqualified product

Under the premise of discarding unqualified finished products, three indicator variables I_1, I_2 and I_3 are established according to whether parts 1, parts 2 and finished products are detected, which are defined as follows:

$$I_1 = \begin{cases} 1, Do\ not\ test\ spare\ parts\ 1 \\ 0, Test\ spare\ parts\ 1 \end{cases} \tag{20}$$

$$I_2 = \begin{cases} 1, Do\ not\ test\ spare\ parts\ 2 \\ 0, Test\ spare\ parts\ 2 \end{cases} \tag{21}$$

$$I_3 = \begin{cases} 1, Do\ not\ test\ finished\ products \\ 0, Test\ finished\ products \end{cases} \tag{22}$$

According to whether to inspect the parts are divided into the following two situations ① and ②:

① If parts i are inspected, the number of production times ξ_i required to produce parts that can enter the next process follows the following distribution (for inspection parts, it is necessary to ensure that the parts are qualified before entering the next process):

$$\xi_i \sim G(1-p_i)(i=1,2) \tag{23}$$

Where G represents geometric distribution [12] and p_i is the defective rate of parts i. Therefore:

$$E(\xi_i) = \frac{1}{1-p_i} \tag{24}$$

If the parts are not inspected, the number of production times required to produce the parts that can enter the next process $\xi_i = 1$

By combining ① and ②, it can be concluded that:

$$E(\xi_i) = \frac{1}{1-(1-I_i)p_i} \tag{25}$$

Considering the probability of qualified finished products, it is not difficult to find:

$$P(Product\ qualification) = (1 - I_1p_1)(1 - I_1p_2)(1 - p_3) \tag{26}$$

Where p_1 and p_2 represent the defective rate of parts 1 and parts 2 respectively, and p_3 represents the defective rate of finished products.

Obviously, the number of finished products λ_1 produced when the first qualified product appears in the finished product follows the geometric distribution [13]:

$$\lambda_1 \sim G((1 - I_1p_1)(1 - I_1p_2)(1 - p_3)) \tag{27}$$

The mean $E(C)$ of C_0 cost per finished product is:

$$E(C_0) = (a_1 + (1 - I_1)b_1)E(\xi_1) + (a_2 + (1 - I_2)b_2)E(\xi_2) + v = \frac{a_1+(1-I_1)b_1}{1-(1-I_1)p_1} + \frac{a_2+(1-I_2)b_2}{1-(1-I_2)p_2} + v + (1 - I_3)b_3 \tag{28}$$

Where a_1 and a_2 represent the purchase unit price of parts 1 and parts 2 respectively, b_1, b_2 and b_3 respectively represent the inspection cost of parts 1, parts 2 and finished products, and v represents the assembly cost of finished products.

Therefore, the mean $E(C)$ of the cost C required to produce a qualified finished product is expressed as follows:

$$E(C) = E(C_0)E(\lambda_1) + eI_3(E(\lambda_1) - 1) = \frac{\frac{a_1+(1-I_1)b_1}{1-(1-I_1)p_1} + \frac{a_2+(1-I_2)b_2}{1-(1-I_2)p_2} + v + (1-I_3)b_3}{(1-I_1p_1)(1-I_1p_2)(1-p_3)} + eI_3\left(\frac{1}{(1-I_1p_1)(1-I_1p_2)(1-p_3)} - 1\right) \tag{29}$$

Where e stands for replacement loss.

3.3.2. Decision No. 001-016 expected solution of the cost of producing each qualified finished product

When the market price P of the finished product is a fixed value, the decision is based on the maximization of profit V, and $\text{profit} = \text{price} - \text{cost}$, so there are:

$$max\,E(V) = max\,E(P - C) = max((P - E(C)) \tag{30}$$

Since the values of defective rate, purchase price, assembly cost, inspection cost, replacement loss and disassembly cost in P and $E(C)$ are all known in 6 cases, the $E(V)$ expression corresponding to 16 decision schemes can be traversed in R software for 6 cases respectively, and the maximum value of the corresponding results of 16 decision schemes can be screened out. The decision scheme corresponding to the maximum value of $E(V)$ in each case is taken as the final decision scheme in this case. $E(V)$ traversal results are shown in Table 4:

Table 4. Table of profit expectations corresponding to 6 situations and 16 decision plans

Decision number	Situation 1	Situation 2	Situation 3	Situation 4	Situation 5	Situation 6
001	15.44	9.75	15.44	14.75	9.47	16.00
002	18.11	12.00	15.44	9.75	10.58	18.63
003	16.47	8.87	16.47	12.79	7.59	16.26
004	18.84	10.52	13.77	2.19	7.10	18.54
005	15.66	8.23	15.66	12.79	14.65	15.36
006	18.03	9.88	12.96	2.19	14.96	17,64
007	16.43	7.27	16.43	11.09	11.66	15.51
008	18.53	8.44	11.30	-3.99	10.54	17.46
009	12.67	2.56	12.67	8.50	5.91	16.61
010	15.33	4.81	12.67	3.50	7.02	19.24
011	14.44	2.09	14.44	5.61	1.37	19.09
012	16.73	3.40	11.10	-8.14	0.26	21.33
013	11.14	-5.33	11.14	0.14	11.86	17.10
014	13.44	-4.02	7.81	-13.61	11.99	19.35
015	13.48	-4.55	13.48	-2.60	9.70	19.84
016	15.36	-4.41	6.44	-27.28	7.36	21.68

The bold part in the table represents the maximum profit expectation of a single qualified finished product in each case, and its corresponding decision number is the final decision scheme in this case.

With the maximum profit expectation of a single piece of qualified finished product as the decision-making basis and the expected profit expectation of a single piece of qualified finished product as the decision-making index, the final decision-making scheme of each situation can be sorted out as shown in Table 5:

Table 5. Final decision scheme and decision index value table for 6 situations

Situation	Option	Final value of decision index
1	Test spare parts 1, do not test spare parts 2, do not test finished products, disassemble unqualified finished products	18.84
2	Test spare parts 1, test spare parts 2, do not test finished products, disassemble unqualified finished products	12.00
3	Test spare parts 1, do not test spare parts 2, test finished products, disassemble unqualified finished products	16.47
4	Test spare parts 1, test spare parts 2, test finished products, disassemble unqualified finished products	14.75
5	Do not test spare parts 1, test spare parts 2. Do not test finished products, disassemble unqualified finished products	14.96
6	Do not test spare parts 1, do not test spare parts 2, do not test finished products, do not disassemble unqualified finished products	21.68

The data unit corresponding to the final value column of the decision indicator in the table is yuan/piece.

4. Conclusion

This paper aims to explore the challenges of defective rate management, inspection process and disassembly decision of an electronic product manufacturer. The core purpose of this study is to optimize sampling, testing and disassembly strategies in production processes through scientific methods, in order to achieve the goal of improving enterprise profits and production efficiency. In terms of method, first of all, this paper designs a set of sampling detection scheme, especially for the 95% and 90% reliability requirements, and constructs a hypothesis testing model. The scheme cleverly balances the relationship between testing cost and testing effect, and determines the optimal sample size under different reliability requirements by establishing the decision function. Secondly, for the four key decision points in the production process, this paper uses the mathematical characteristics of binomial distribution and geometric distribution, combined with the actual parameters such as defective rate, purchase cost, inspection cost, etc., to build a comprehensive cost-benefit analysis model. Based on the analysis of six different production situations, this paper puts forward the decision scheme based on profit expectation maximization, and gives the detailed decision basis and quantitative index. The results show that the method system proposed in this paper can provide enterprises with a set of accurate mathematical modeling tools, so that they can quickly and accurately choose the optimal sampling, detection and dismantling strategies in the face of complex and changeable production conditions. This not only significantly improves the resource utilization efficiency of enterprises, but also effectively enhances their economic benefits and market competitiveness. To sum up, through the comprehensive application of mathematical modeling, hypothesis testing and cost-benefit analysis, this paper successfully provides a set of feasible decision optimization scheme for electronic product manufacturers, which has important practical significance and theoretical value for guiding the actual production operation of enterprises and improving the overall operation efficiency.

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