Single-objective optimization model based on spiral motion in bench dancing dragon

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Abstract. The Bench Dragon dance is a traditional folk cultural activity in Zhejiang and Fujian, China, representing a unique form of collective behavior. A successful performance requires precise path planning and speed control to ensure safety and aesthetic appeal. This study develops a single-objective optimization model based on spiral motion to address these challenges. Using geometric analysis, velocity decomposition, vector analysis, traversal search, and dichotomous search, we derive and prove various geometric relationships. The model provides optimal motion parameters, including positions and velocities of the dragon dance team, and investigates the trends of speed changes and potential collisions. Results indicate that the speed difference between the dragon's head and tail remains minimal (0.0035 m/s), ensuring a stable and coordinated performance. The study concludes that maintaining a consistent speed profile is crucial for reducing the risk of accidents and enhancing the visual impact of the dance. This research not only offers practical guidance for Bench Dragon performances but also introduces a novel quantitative approach for analyzing traditional cultural activities, highlighting the potential for mathematical modeling to preserve and enhance cultural heritage.

Keywords: Bench Dragon Dance, Collision Detection, Spiral Motion, Dynamic Simulation.

1. Introduction

The Bench Dragon dance is a traditional folk cultural activity widely popular in Zhejiang and Fujian provinces of China. It represents a unique form of collective behavior where participants work together to create intricate and dynamic movements. A successful performance requires not only artistic skill but also precise coordination among participants, involving reasonable path planning and speed control to ensure both safety and aesthetic appeal (Figure 1). This study aims to provide a quantitative approach to optimize the performance of the Bench Dragon dance by developing a motion optimization model based on mathematical modeling and simulation.

Figure 1. Bench dragon dance art at Zhejiang and Fujian areas in China

Currently, research on collective behavior is prevalent in various fields, including psychology and sociology, focusing on understanding the behavioral logic of individuals within a group and formulating risk-mitigation policies [1,2]. However, few studies have applied mathematical modeling to traditional cultural activities like the Bench Dragon dance. By abstracting the performance into a mathematical model, we can identify optimal individual decision-making strategies within collective actions and provide practical guidance for enhancing the performance [3,4].

In the Bench Dragon dance, the dragon head moves along an Archimedean spiral at a constant speed, driving the other members. Each board's position and direction differ instantaneously, causing the members' speeds to change constantly. To prevent stalls or grip loss during coiling and uncoiling, it is essential to set standard speeds for each member using mathematical modeling [5]. This study simulates the performance and addresses two key questions: (1) determining the position and speed of the entire dragon dance team within 0-300 seconds, and (2) identifying the termination time when collisions between benches occur and setting the corresponding positions and speeds for key sections of the dragon body.

To achieve these goals, we establish a comprehensive motion optimization model [6,7] that integrates geometric analysis, velocity decomposition, vector analysis, traversal search, and dichotomous search [8,9]. This model derives and proves various geometric relationships, providing optimal motion parameters that align with the actual performance dynamics. Additionally, we develop a collision detection model to predict the termination time of the performance when benches collide. The results offer valuable insights into the safety and aesthetic aspects of the Bench Dragon dance, contributing to the preservation and enhancement of this traditional cultural activity.

In summary, this study innovates by applying mathematical modeling to a traditional cultural performance, offering a novel approach to optimizing collective behavior. The findings not only provide practical guidance for Bench Dragon dance performances but also highlight the potential for interdisciplinary research to preserve and enhance cultural heritage [10].

2. Literature review

During the performance, the dragon head moves at a constant speed along an Archimedean spiral, driving the other members. As each board's position and direction differ instantaneously, members' speeds change constantly. To prevent stalls or grip - loss during coiling and uncoiling, this paper uses math modeling to set standard speeds for each member. So, this paper simulates the performance and pose the following questions. All data in this paper was obtained from the website https://github.com/memory395/bench-dragon-data.git.

3. Methodology

3.1. Establishment of velocity model and Solving

3.1.1 Derivation of the solenoidal equation and expression of the coordinates

Since the pitch of the screw thread is 55cm and the initial position is located at the point A of the 16th circle of the screw thread, based on the above conditions, the expression under the polar coordinate system of the screw thread can be listed as:

$$r_n = \frac{0.55}{2\pi}\left(32\pi - \theta_n\right) \quad \left(n = 1, 2, \cdots 223\right) \tag{1}$$

where r_n and θ_n represent the polar diameter and polar angle corresponding to the center coordinates of the front handle of the n th bench, respectively. This paper defines the center coordinate of the front handle of the n th bench as $Q_n\left(x_n, y_n\right)$, where $n = 1, 2, \cdots 223$. The center coordinate of the front handle of the next bench (i.e., the back handle of the bench) is $Q_{n+1}\left(x_{n+1}, y_{n+1}\right)$. From the conditions of Problem 1, this paper obtains that the centers of the handles are on the solenoids, and thus we can establish the relationship between the coordinates and the polar angles:

$$\begin{cases} \sqrt{{x_n}^2 + {y_n}^2} = \dfrac{0.55}{2\pi}\left(32\pi - \theta_n\right) \\ x_n = r_n \cos\theta_n \\ y_n = -r_n \sin\theta_n \end{cases} \tag{2}$$

3.1.2 Bench Direction Vector Solver

To determine the direction vectors of the benches, it is first necessary to determine the exact position of the front and rear handles of each bench in the solenoid, and this paper establishes the connection between the two by the angle between the front and rear handles and the line connecting them to the origin:

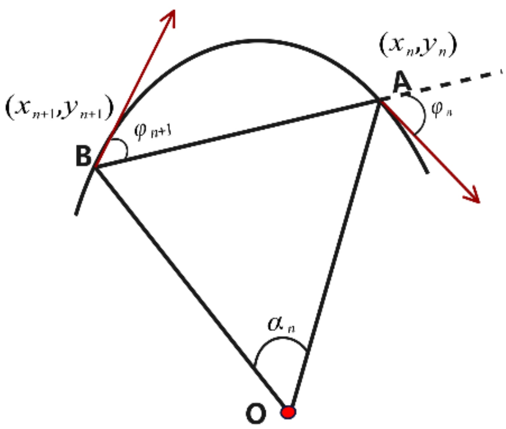

Figure 2. Front and rear handles of the bench in line with the origin

As shown in the Figure 2, r_n and r_{n+1} denote the polar diameters of the front and rear handles of the n th bench, and a_n denotes the angle between the front and rear handles of the n th bench and the origin, then there is the following correspondence:

$$\begin{cases} \theta_n - \alpha_n = \theta_{n+1} \\ r_{n+1} = \dfrac{0.55}{2\pi}\left(32\pi - \theta_{n+1}\right) \\ x_{n+1} = r_{n+1}\cos\theta_{n+1} \\ y_{n+1} = -r_{n+1}\sin\theta_{n+1} \end{cases} \tag{3}$$

As shown in the figure above, in the triangle ABO, the lengths of AO, BO and the angle of entrapment have completed the functional representation, specifying that the length of AB is l_n , based on the conditions of the topic:

$$l_n = \begin{cases} 3.41 - 0.55 = 2.86 & n = 1 \\ 2.2 - 0.55 = 1.65 & n \neq 1 \end{cases} \tag{4}$$

From the cosine theorem, the relationship between the angle of pinch and the two polar diameters is given as:

$$\cos\alpha = \frac{r_n^{\;2} + r_{n+1}^{\;2} - l_n^{\;2}}{2r_n r_{n+1}} \tag{5}$$

By calculating the angle α_n corresponding to each bench and solving for the polar angle θ_n corresponding to each bench handle, the coordinate value of the center of each handle can be obtained, thus obtaining the expression of the direction vector of the bench.

3.1.3 Solving for Front and Rear Handle Velocity Relationships

For the n th bench, this paper defines its direction vector to be $\overrightarrow{v_n}\left(x_n - x_{n+1}, y_n - y_{n+1}\right)$. Let the velocity of the front handle of the bench be V_n , and the velocity of the back handle be V_{n+1}.

If the coordinates of the center of either handle are known to be $Q_n\left(x_n, y_n\right)$, then the direction vector of the tangent line at the center of the handle is $\overrightarrow{r_n}\left(dx_n, dy_n\right)$. According to the formula for the angle of a vector, the angle between the tangent direction vector of the front and rear handles of the bench and the direction vector of the bench is:

$$\begin{cases} \cos\varphi_n = \dfrac{\overrightarrow{v_n}\cdot\overrightarrow{r_n}}{\left|\overrightarrow{v_n}\right|\cdot\left|\overrightarrow{r_n}\right|} \\ \cos\varphi_{n+1} = \dfrac{\overrightarrow{v_n}\cdot\overrightarrow{r_{n+1}}}{\left|\overrightarrow{v_n}\right|\cdot\left|\overrightarrow{r_{n+1}}\right|} \end{cases} \tag{6}$$

where φ_n denotes the angle between the tangent vector of the handle in front of the n th bench and the direction vector of the bench.

In order to facilitate the explanation, this paper draws the Figure 3:

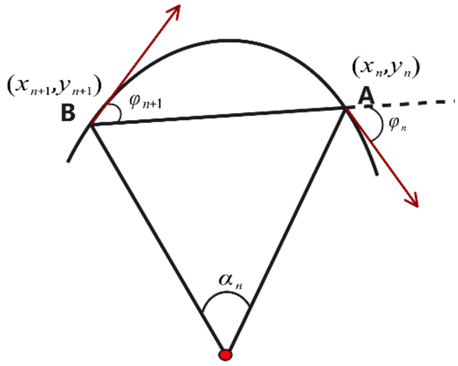

Figure 3. Front and rear handle speed relationship

This paper treats the bench as a rigid body, i.e., we consider that the velocity components of the front and rear handles along the direction of the bench remain the same at all times, and we can obtain the following equation:

$$V_n \cos\varphi_n = V_{n+1}\cos\varphi_{n+1} \tag{7}$$

which is known according to the conditions of the topic: $V_1 = 1m/s$. This completes the derivation of the velocity relationship between the front and rear handles.

3.1.4 Solving for the initial condition of the dragon head

The recursive equation for the coordinates of the center position of each handle and the angle between the tangent direction and the direction of the bench has been completed above, and now the recursive solution is completed by obtaining the values of the initial conditions by modeling and solving the relationship between the time and angle of the faucet's motion.

First derive the formula for the arc length of a solenoid, for solenoid $r = r(\varphi), \varphi_1 \le \varphi \le \varphi_2$, which is transformed into a parametric equation:

$$\begin{cases} x = r(\varphi)\cos\varphi \\ y = r(\varphi)\sin\varphi \end{cases} \tag{8}$$

Derive for x, y , respectively.

$$\begin{cases} \dfrac{dx}{d\varphi} = \dfrac{dr}{d\varphi}\cos\varphi - r(\varphi)\sin\varphi \\ \dfrac{dy}{d\varphi} = \dfrac{dr}{d\varphi}\sin\varphi + r(\varphi)\cos\varphi \end{cases} \tag{9}$$

Substituting Equation 4.8 into the expression for the line element in the Cartesian coordinate system:

$$dl = \sqrt{\left(\frac{dx}{d\varphi}\right)^2 + \left(\frac{dy}{d\varphi}\right)^2}\, d\varphi = \sqrt{\left(\frac{dr}{d\varphi}\right)^2 + r^2(\varphi)}\, d\varphi \tag{10}$$

Thus this paper derives the formula expression for the length of the solenoidal arc in the polar coordinate system as：

$$dl^2 = dr^2 + r^2 d\varphi^2 \tag{11}$$

Converting the derived arc length formula into integral form, the following expression relation can be obtained:

$$\int_{\theta_1}^{\theta_2} \sqrt{r^2 + r'^2}\, d\theta = l = vt \tag{12}$$

where, v is the speed of the dragon head movement, in this problem is constant 1m/s. According to formula 3.12, substituting known conditions, you can find out the moment of the dragon head before the handle of the polar angle $\theta_1(t)$, the moment of the initial value into formula 3.4 and 3.6 can complete the recursion, so as to solve for the “bench dragon” bench handles corresponding to the speed and position of the moment.

3.2. Establishment of collision model and Solving

This paper abstracts the benches as rectangles moving along a solenoid, and according to the question, the moment when the benches collide with each other is the termination moment of the dragon dance team. Thus the problem can be thought of as solving for when the rectangle has an intersection with the surrounding neighboring rectangles by modeling.

3.2.1 Vertex collision judgment

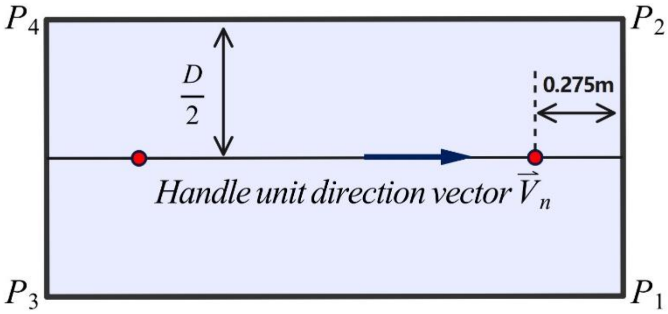

Figure 4. Schematic diagram of vertex coordinate solution

As the coordinates of the four vertices of the colliding bench and Q_n、Q_{n+1} as the coordinates of the front and rear handles of this colliding bench, the unit direction vector of the bench handles is:

$$\overrightarrow{V_n} = \frac{\overrightarrow{Q_n} - \overrightarrow{Q_{n+1}}}{\left|\overrightarrow{Q_n} - \overrightarrow{Q_{n+1}}\right|} \tag{13}$$

Based on the translation relations of plane geometry coordinates, the derivation of the coordinates of the four vertices $P_n(n = 1,2,3,4)$ which are marked in Figure 4 is:

$$\begin{cases} P_1 = Q_n + 0.275\overrightarrow{V_n} + \dfrac{D}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\overrightarrow{V_n} \\ P_2 = Q_n + 0.275\overrightarrow{V_n} + \dfrac{D}{2}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\overrightarrow{V_n} \\ P_3 = Q_{n+1} - 0.275\overrightarrow{V_n} + \dfrac{D}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\overrightarrow{V_n} \\ P_4 = Q_{n+1} - 0.275\overrightarrow{V_n} + \dfrac{D}{2}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\overrightarrow{V_n} \end{cases} \tag{14}$$

Assume that section n bench collides with the surrounding neighboring benches, which have corresponding vertices P_n. Let the direction vector of the collided bench be $\overrightarrow{v_j}$. At the time of the collision, there must be:

$$dis\left(P_n,\overrightarrow{v_j}\right)<\frac{D}{2} \tag{15}$$

where D denotes the width of the bench, which is a fixed value of 30 cm, and since the collision point is on the side of the bench, the collision point P_n must satisfy the requirement that it is within the rectangular side of the bench being touched.

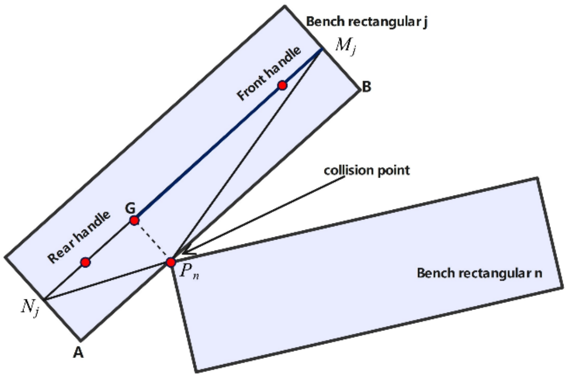

Figure 5. Schematic diagram of collision

According to the vector dot product formula and the points in the formula are all marked in Figure 5:

$$L_{M_jG}=\frac{\overrightarrow{M_j-P_n}\cdot\overrightarrow{M_j-N_j}}{\left|\overrightarrow{M_j-P_n}\right|}\in\left[0,\ L\right] \tag{16}$$

In summary, the stopping time of the dragon dance team can be obtained by determining the collision of each vertex of each bench through the programming solution.

4. Result Analysis and Discussion

4.1. The establishment of simulation model

According to the established mathematical model, the initial speed of the dragon head is 1m/s, the pitch is 55cm, the initial position of the dragon head is the 16th circle of the spiral at point A and other parameter values are substituted into our model, and the MATLAB programming is used to solve the corresponding position and speed of the dragon head of the “bench dragon” and other parts , the final results are shown in the table 1and table 2:

Table 1 Question 1 position results (retain six decimals)

	0 s	60 s	120 s	180 s	240 s	300 s
Mixer x (m)	8.800000	5.799212	-4.084887	-2.963609	2.594494	4.420273
Mixer y (m)	0.000000	-5.771090	-6.304480	6.094780	-5.356742	2.320430
Dragon's tail (back) x(m)	-5.306205	7.363948	10.974398	7.384402	3.241721	1.785606
Dragon's tail (back) y(m)	-10.676199	-8.798494	0.842744	7.491863	9.469101	9.301048

Table 2 Question 1 speed results (retain six decimals)

	0 s	60 s	120 s	180 s	240 s	300 s
Mixer (m/s)	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
Dragon's tail (back) (m/s)	0.999311	0.999136	0.998883	0.998489	0.997816	0.996478

The moment of collision is schematized in the figure 6:

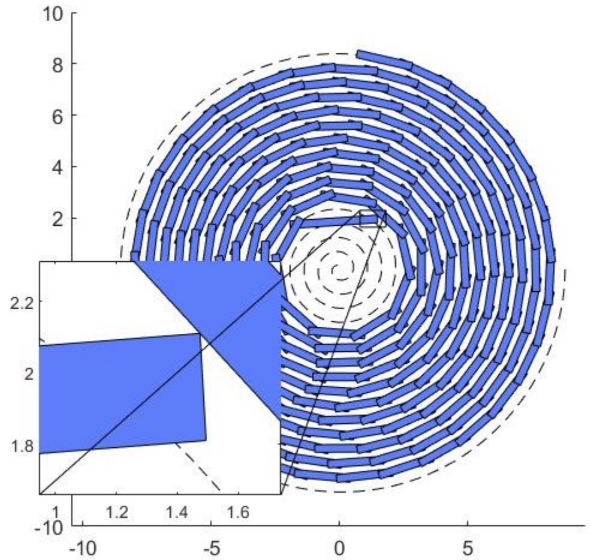

Figure 6. Schematic diagram of the moment of collision

The final solved corresponding bench handle position velocity results are shown in the table 3:

Table 3 Problem 2 position results (retain six decimals)

	terminal moment
Mixer x (m)	1.246984
Mixer y (m)	1.917157
Dragon's tail (back) x(m)	1.000212
Dragon's tail (back) y(m)	8.317097
	terminal moment
Mixer (m/s)	1.000000
Dragon's tail (back) (m/s)	0.972928

4.2. Analysis of experimental results

Analyzing the velocities this paper finds that at the same moment, using the leading bench handle as the starting velocity, the handle velocity decreases as the bench number n increases, but the trend of velocity reduction gradually becomes slower. For the same bench, as the time increases, the speed of the handles also decreases.

For the whole process, it is not difficult to see that the speed of the whole dragon dance team is similar from the dragon's head just entering the starting point to the movement until 300s. The maximum speed difference from the tail to the head of the dragon is only about 0.0035 m/s. Combined with the reality of life, as a whole, the traveling speed of each handle can not vary too much. If the difference in traveling speed is too large, it will lead to a large inertial force on the dragon dancers during the movement, which is prone to fall, congestion and other unsafe accidental conditions, so this result is reasonable.

5. Conclusions

The study has conducted a comprehensive analysis of the motion process of the Bench Dragon dance, considering various influential factors such as running speed, the pitch of the spiral path, and potential bench collisions. By establishing a mathematical model based on the physical laws of rigid body motion, we have successfully simulated the complex dynamics of the Bench Dragon dance. Our work involved extensive geometric analysis, velocity decomposition, vector calculations, and collision detection, culminating in the derivation of optimal motion parameters that accurately reflect the actual performance dynamics. The results demonstrate that the speed of the dragon dance team remains stable throughout the performance, with a minimal speed difference of only 0.0035 m/s between the head and tail. This finding underscores the feasibility of our model in ensuring both the safety and aesthetic appeal of the Bench Dragon dance. By using single objective optimization model, this study can further discuss the optimal strategies for independent individuals in different collective cooperative activities, in order to achieve the safe and smooth progress of collective activities.

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