

Research on the Location Selection Problem of Urban Garbage Stations Based on the Epsilon Constraint Method

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Abstract. This paper takes Zhenjiang City as an example and proposes a garbage station location framework that integrates spatial clustering and multi-objective optimization to address issues such as the imbalance in waste treatment facility layout and insufficient terminal treatment capacity in small and medium-sized cities. Firstly, a weighted K-Means clustering model is applied to spatially aggregate garbage generation points, identify high-demand areas, and reduce the problem scale. Secondly, a multi-objective integer programming model is constructed, which comprehensively considers key indicators such as transportation costs, operational costs, environmental impacts, and coverage rates. Pareto frontier analysis is used to reveal the trade-offs among different objectives. Finally, by incorporating practical constraints such as avoiding sensitive areas and capacity limitations, a location plan that balances economic efficiency and sustainability is generated. The research results demonstrate that this method can effectively balance costs and environmental impacts, providing a data-driven decision-making approach for garbage station planning in Zhenjiang City and other small and medium-sized cities.

Keywords: Garbage Classification, Multi-objective Optimization, K-Means Clustering, Pareto Frontier Analysis.

1. Introduction

With the acceleration of urbanization in China, the volume of domestic waste has been continuously increasing. In 2022, the amount of urban garbage cleared nationwide exceeded 250 million tons. The traditional mixed treatment model is facing dual pressures from land resource shortages and environmental pollution. Driven by national policies, 46 key cities have initially established classified treatment systems. However, small and medium-sized cities still generally face problems such as imbalanced facility layout and insufficient end-treatment capacity. Currently, the waste management challenges in small and medium-sized cities are not only reflected in the mismatch between facility layout and demand but also expose the limitations of traditional planning methods. On the one hand, most small and medium-sized cities rely on experience-based site selection, lacking systematic analysis of the spatial distribution of waste generation points, population density, and environmentally sensitive areas. This leads to low facility utilization rates and prominent risks of secondary pollution. On the other hand, existing studies often focus on single-objective optimization (e.g., minimizing costs), neglecting the dynamic balance among transportation efficiency, environmental impact, and coverage. In previous research, Na Li and Zhenping Li (2011) established a bi-objective site selection mathematical model to study the impact of garbage stations on residents and the influence of construction and operational costs on garbage station location decisions [1]. Fang Liu (2021) used analytical hierarchy process (AHP) and ArcGIS spatial analysis to analyze factors affecting garbage transfer station location decisions, scientifically deriving optimal site selection plans [2]. Weiling Zhang and Tao Liu (2022) applied an improved genetic algorithm combined with AHP to address garbage station location problems [3]. Jiajun Liu (2023) utilized a mixed-integer programming model and K-means clustering algorithm to conduct garbage transfer station location work for Hohhot City [4]. In this study, we propose a novel site selection framework that integrates spatial clustering with multi-objective optimization. By clustering multiple small waste generation points into a single large waste generation point, the framework combines a multi-objective optimization model to explore the influence of various factors on garbage station location decisions.

The main contributions of this paper are as follows: 1) A garbage station location framework is proposed, which integrates spatial clustering and multi-objective optimization. The weighted K-Means clustering model is used to reduce the problem scale and identify high-demand areas. 2) A multi-objective integer programming model is constructed to comprehensively optimize key indicators such as transportation costs, operational costs, environmental impacts, and coverage rates. Pareto frontier analysis is employed to reveal the trade-offs among objectives. 3) Realistic constraints, such as the avoidance of sensitive areas and capacity limitations, are incorporated to generate location schemes that balance economic efficiency and sustainability. The effectiveness of the proposed method is verified through a case study.

The structure of this paper is as follows: The first part is the introduction, which presents the background of current waste sorting and treatment, the research status, and the innovative points of this paper regarding waste station location selection. The second part covers relevant theories, elaborating on the content of k-means clustering and multi-objective linear programming. The third part focuses on the establishment of the model, describing the process of constructing the problem model and providing a detailed explanation of the composition of each constraint condition. The fourth part discusses the model solving and experimental results, presenting the algorithm designed to solve the model and the experimental outcomes obtained. The fifth part presents the experimental conclusions, summarizing the final findings based on the aforementioned process.

2. Related Theories

The K-means clustering algorithm is a simple iterative clustering algorithm that performs clustering on a dataset $D = \{x_i | i = 1, \dots, n\}$ of n -dimensional vector data points, where x_i represents the i data point. The goal is to divide the set D into k clusters. The grouping is primarily based on "closeness" or "similarity," aiming to maximize similarity within clusters and maximize differences between clusters. Common distance metrics include Euclidean distance, Manhattan distance, and Chebyshev distance. Clustering algorithms typically use Euclidean distance as the similarity measure and employ the sum of squared errors as the objective function to evaluate clustering quality. By minimizing the objective function, the data points are grouped into k clusters based on their proximity to the cluster centers [5].

Multi-objective optimization is a core tool for solving complex decision-making problems in the real world. Its core idea lies in balancing multiple conflicting objectives rather than pursuing the optimum of a single objective. Through Pareto optimality theory, it provides a set of "best trade-off solutions" (i.e., the Pareto front), where these solutions are non-dominated in the objective space. Based on practical needs, the most suitable solution can be selected. Multi-objective optimization is commonly applied in engineering design, financial investment, and urban planning. It primarily involves two conflicting objective functions, decision variables, and constraints. Methods such as weighted sum method, particle swarm optimization, and ant colony algorithm can be used. Different algorithms are suitable for different application scenarios [6].

The Epsilon Constraint Method is the most commonly used exact algorithm for solving multi-objective optimization problems. Its basic idea is to optimize one primary objective function while transforming the other objective functions into constraints. By gradually reducing the Epsilon value, the method establishes a connection between the two objectives. Through multiple model optimizations, the Pareto optimal solution set can be obtained [7].

3. Experiments

3.1. Clustering model

Urban waste generation points are widely distributed, numerous, and densely located. Directly conducting site selection planning for all points would result in extremely high computational

complexity. To address this issue, the K-Means clustering model is first introduced to spatially aggregate the urban waste generation points. This process yields candidate locations for waste station site selection.

First, define the Euclidean distance between data points:

$$d(x, y) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2} \quad (1)$$

Secondly, define the Euclidean distance between data points and cluster centroids:

$$d(x, c_i) = \sqrt{\sum_{j=1}^n (x_j - c_{i,j})^2} \quad (2)$$

In the formula: c_i is the cluster center of the i -th class; x is a data object in the dataset D

Finally, define the calculation formula for the sum of squared errors:

$$SSE = \sum_{i=1}^k \sum_{x \in C_i} d(x, c_i)^2 \quad (3)$$

Where: C_i represents the i -th cluster; $x \in C_i$ refers to the data object belonging to that cluster; k is the number of clusters.

The K-means clustering algorithm is a type of dynamic clustering algorithm that requires continuous iterative processing. First, select K data points as the initial clustering centers. Calculate the distance from each data point to the respective initial clustering centers, and assign the data points to the nearest cluster based on these distances, forming a cluster. Then, update the center of each cluster based on the points within it. Repeat the steps of assignment and updating until the clusters no longer change. Eventually, the centers of the clusters will serve as the candidate locations for waste collection sites.

3.2. Multi-objective integer programming model

Based on the candidate sites generated through clustering, a multi-objective integer programming model is further constructed to comprehensively optimize four major objectives: construction cost, transportation cost, operational cost, and environmental impact, while incorporating realistic constraints.

When selecting a site for the waste transfer station, the first consideration is the total construction cost. The total cost includes the construction cost of the waste transfer station F_{11} , the operating costs of the transfer station F_{12} , as well as the transportation costs between network facilities F_{13} .

Regarding the construction cost, the construction cost of each waste transfer station is fixed and known. Thus, the total construction cost can be obtained.

$$F_{11} = \sum_{j \in M} f_j x_j \quad (4)$$

Regarding the operating costs, in order to avoid wasting the capacity of the transfer station, the operating costs of the transfer station are related to the capacity of the transfer station. Thus, the total operating costs can be obtained as:

$$F_{12} = \sum_{j \in M} c_j q_j x_j \quad (5)$$

Regarding transportation costs: Considering the different characteristics of dry garbage and wet garbage, two types of domestic waste, their unit transportation costs vary. The total transportation

cost consists of two parts. The first part is the transportation cost incurred in transporting garbage from the collection points to the garbage transfer stations, it is related to the transportation distance d_{ij} and the waste transportation volume s_{ij}^g of type g . The second part is the transportation cost from the waste transfer station to the class $-p$ end - treatment plant, which is related to the transportation distance d_{jp} and the waste transportation volume u_{jp} . The total transportation cost calculation formula is:

$$F_{13} = \sum_{i \in N} \sum_{j \in M} \sum_{g \in G} t_{ij}^g s_{ij}^g d_{ij} + \sum_{j \in M} \sum_{p \in P} t_{jp} u_{jp} d_{jp} \quad (6)$$

Finally, we can obtain the total cost:

$$F_1 = F_{11} + F_{12} + F_{13} \quad (7)$$

The construction of garbage stations will have a negative impact on the surrounding area. This paper will adopt the negative effect function $(q_j)^\alpha (d_{ij})^{-\beta}$ [8]. This function is used to measure the negative effects generated by waste transfer facilities, Assuming that the negative effect is directly proportional to the facility capacity q_j , Proportional to the distance d_{ij} between the facility and the impact point. Then the environmental negative effect objective function of the transfer station is:

$$F_2 = \sum_{i \in N} \sum_{j \in M} (q_j)^\alpha (d_{ij})^{-\beta} x_j \quad (8)$$

Where α is the impact coefficient of transfer station capacity on negative effects, and β is the impact coefficient of the distance between the transfer station and the collection point on negative effects. In the literature, the values of α and β are generally set to 1.

3.3. Restrictive condition

Ensure that the transfer station is constructed before it can serve the waste collection points:

$$x_j \geq y_{ij}, \forall i \in N, j \in M \quad (9)$$

This constraint ensures that if a collection point i is served by a transfer station j (i.e., $y_{ij} = 1$), then the transfer station j must be selected for construction (i.e., $x_j = 1$).

Each collection point must be served by a garbage station:

$$\sum_{j \in M} y_{ij} = 1, \forall i \in N \quad (10)$$

Each collection point i must and can only be served by one transfer station j .

The amount of garbage collected at the collection points does not exceed the processing capacity of the garbage station:

$$S_{ig} \leq \sum_{j \in M} s_{ij}^g, \forall i \in N, g \in G \quad (11)$$

Ensure that the amount of waste generated at collection point i , S_{ig} , does not exceed the total amount of waste processed by all transfer stations j at that collection point.

The reasonable range of garbage transportation volume:

$$0 \leq s_{ij}^g \leq M y_{ij}, \forall i \in N, j \in M, g \in G \quad (12)$$

This constraint ensures that the waste transportation volume from collection point i to transfer station j , denoted as s_{ij}^g , is within a reasonable range, i.e., it cannot be negative and must not exceed the maximum possible transportation volume that transfer station j can handle for that collection point.

The processing capacity limitation of the transfer station:

$$\sum_{i \in N} \sum_{g \in G} s_{ij}^g \leq q_j, \forall j \in M \quad (13)$$

This constraint ensures that the total amount of waste from all collection points handled by the transfer station j does not exceed its maximum capacity q_j .

Restrictions on garbage transportation volume after considering the reduction rate:

$$(1 - \gamma_g) \sum_{i \in N} s_{ij}^g \leq u_{jp}, \forall j \in M, p \in P, g \in G, p = g \quad (14)$$

Ensure that the amount of waste transported from transfer station j to the terminal treatment plant p , u_{jp} , does not exceed the total amount of waste treated at transfer station j after considering the reduction rate γ_g .

Whether the transfer station j has been selected for construction:

$$x_j \in \{0, 1\}, \forall j \in M \quad (15)$$

While $x_j = 1$, the transfer station is selected for construction; otherwise, the transfer station is not selected for construction.

Whether the collection point i is served by the transfer station j :

$$y_{ij} \in \{0, 1\}, \forall i \in N, j \in M \quad (16)$$

While $y_{ij} = 1$, the collection point i is served by the transfer station j ; otherwise, the collection point i is not served by the transfer station j .

The non-negativity of garbage transportation volume:

$$cc u_{jp} \geq 0, \forall j \in M, p \in P \quad (17)$$

This constraint ensures that the waste transportation volume u_{jp} from transfer station j to the end processing plant p cannot be negative.

To sum up, the mathematical model can be obtained as follows:

$$\min F_1 = \sum_{j \in M} f_j x_j + \sum_{j \in M} c_j q_j x_j + \sum_{i \in N} \sum_{j \in M} \sum_{g \in G} t_{ij}^g s_{ij}^g d_{ij} + \sum_{j \in M} \sum_{p \in P} t_{jp} u_{jp} d_{jp} \quad (18)$$

$$\min F_2 = \sum_{i \in N} \sum_{j \in M} (q_j)^\alpha (d_{ij})^{-\beta} x_j \quad (19)$$

$$\left\{ \begin{array}{l} x_j \geq y_{ij}, \forall i \in N, j \in M \\ \sum_{j \in M} y_{ij} = 1, \forall i \in N \\ S_{ig} \leq \sum_{j \in M} s_{ij}^g, \forall i \in N, g \in G \\ 0 \leq s_{ij}^g \leq M y_{ij}, \forall i \in N, j \in M, g \in G \\ \sum_{i \in N} \sum_{g \in G} s_{ij}^g \leq u_j, \forall j \in M \\ (1 - \gamma_g) \sum_{i \in N} s_{ij}^g \leq u_{jp}, \forall j \in M, p \in P, g \in G, p = g \\ x_j \in \{0, 1\}, \forall j \in M \\ y_{ij} \in \{0, 1\}, \forall i \in N, j \in M \\ u_{jp} \geq 0, \forall j \in M, p \in P \end{array} \right. \quad (20)$$

3.4. Epsilon Constraint Method

To introduce the Epsilon constraint method for solving multi-objective models, we first need to explain the concept of Pareto dominance. For a minimization multi-objective model, Pareto dominance can be defined as follows: if a feasible solution X dominates another feasible solution Y (where X, Y are vectors), then it must be true that $f_1(X) \leq f_1(Y)$ and $f_2(X) \leq f_2(Y)$, and at least one of these inequalities must be strict. In this case, we consider feasible solution X to be paired with feasible solution Y , and X is called a non-dominated solution, while Y is called a dominated solution. All non-dominated solutions in the feasible solution space of the objective function form the Pareto frontier. On this frontier, there is no dominance relationship between any two points. This solution set contains a series of different points, which are used by decision-makers to balance the values of the multi-objective functions. The Epsilon constraint method requires the calculation of the following three sets of points:

First of all, it is the ideal point:

$$f^1 = (f_1^1, f_2^1) \quad (21)$$

$$f_1^1 = \min\{f_1(X)\} \quad (22)$$

$$f_2^1 = \min\{f_2(X)\} \quad (23)$$

Secondly, the weakest point:

$$f^N = (f_1^N, f_2^N) \quad (24)$$

$$f_1^N = \min\{f_1(X), f_2(X) = f_2^1\} \quad (25)$$

$$f_2^N = \min\{f_2(X), f_1(X) = f_1^1\} \quad (26)$$

Finally, it is the highest point:

$$f^E = \{(f_1^1, f_2^N), (f_1^N, f_2^1)\} \quad (27)$$

Based on the previously established bi-objective integer optimization model, since both objectives are minimization problems, the exact Epsilon constraint algorithm can be expressed as follows: Calculate the ideal point $f^1 = (f_1^1, f_2^1)$ and the worst point $f^N = (f_1^N, f_2^N)$. Choose $f_1(x)$ as the main objective, and obtain the range of ε for $f_2(x)$ as $[f_2^N, f_2^E]$. Set $F' = \{(f_1^1, f_2^N)\}$, let $\varepsilon = f_2^N - \Delta$, while $\varepsilon \geq f_2^1$, solve the Epsilon constraint problem, where the Epsilon constraint is $f_1 \leq \varepsilon$ and the

optimization single objective is $\min f_2$. Solve this single objective optimization problem to optimality, and add the optimal solution (f_1^*, f_2^*) to the set F' . Remove the dominated points from the set F' to obtain the Pareto frontier F .

4. Results

Taking Dantu District of Zhenjiang City as an example, the map of Dantu District is in a "T" shape. There are 6 towns and 2 subdistricts under the district. Residential areas are clustered around the towns and subdistricts. Due to the large number of garbage points in the region, for simplification, we select the relatively large-scale garbage points and assume that the amount of garbage generated at each garbage point is the same. Through the clustering model, we can obtain the schematic diagram of the distribution of garbage - stations as shown in Figure 1:

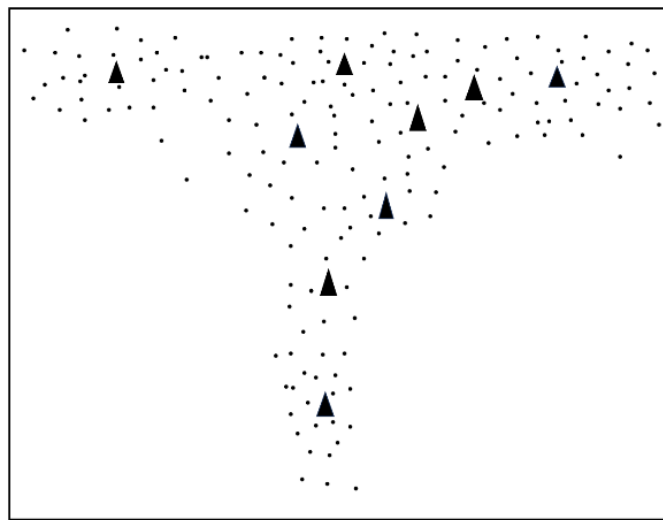


Figure 1. Candidate Garbage Station Distribution Map

Among them, the triangles are the candidate points for the garbage station, and the origin represents the garbage generation points.

Through the multi-objective integer programming model, we can observe that the site selection for waste stations is constrained by multiple factors and requires simultaneous consideration of costs and environmental impacts. The Epsilon Constraint Method can be used to re-evaluate and screen the locations of candidate stations. According to data from the Zhenjiang Environmental Protection Bureau, the daily amount of domestic garbage generated in Zhenjiang is approximately 3,400 tons, and the total population of Zhenjiang is about 3.2 million, resulting in a per capita daily output of approximately 1.06 kg [9]. Among this, wet waste accounts for about 35%, which is 0.37 kg, while dry waste accounts for 0.69 kg, with the remainder being recyclable and hazardous waste. Therefore, this study assumes that the dry waste output at each collection point is randomly generated within the interval (0, 7000], and the wet waste output is randomly generated within the interval (0, 4000]. The capacity of transfer stations is 60 tons, the construction cost is 260 yuan/day, and the operational cost is 130 yuan/ton[10].

For the multi-objective location model of waste transfer stations, the Pareto front can be obtained. Regarding the total network cost and environmental negative impact, it can be seen that they are two conflicting objectives. Specifically, the total network cost is related to the number of transfer stations constructed and the distances between facilities in the network, while the environmental negative impact is related to the capacity of the transfer stations and the distances between the transfer stations and the collection points. A lower total cost indicates a smaller total capacity of the constructed transfer stations, fewer transfer stations, and closer facility distances, which will inevitably sacrifice part of the environmental performance. Through model solving, the waste station coverage map as

shown in the Figure 2 can be obtained. In Figure 2, according to the solution results, filter out the redundant candidate points for garbage stations, keep the required garbage stations, and demarcate the garbage - collection range for each garbage station.

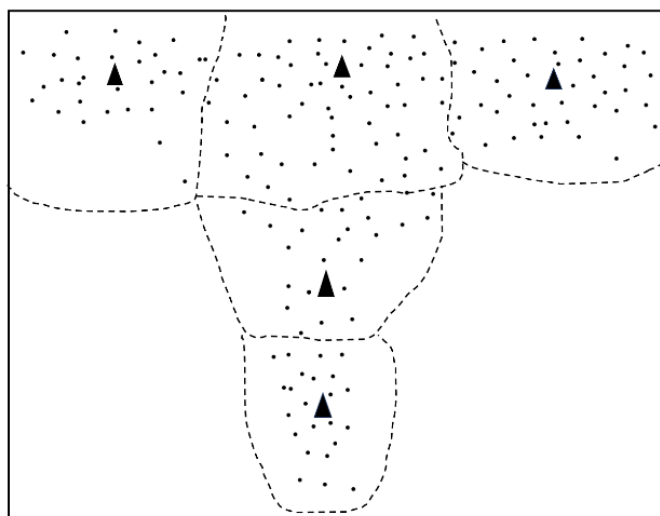


Figure 2. Garbage station coverage area

5. Conclusion

This paper addresses the issue of imbalanced layout of waste treatment facilities in small and medium-sized cities and proposes a novel site selection framework that integrates spatial clustering and multi-objective optimization. Taking Zhenjiang City as a case study, the research demonstrates that the combination of a K-Means clustering model and a multi-objective integer programming model can effectively identify high-demand areas, optimize the layout of waste stations, improve facility utilization, reduce transportation costs, and minimize the environmental impact on residents' lives. Specifically, the proposed method uses the clustering model to spatially aggregate waste generation points, identify core demand areas, and thus narrow down the site selection scope. The multi-objective integer programming model comprehensively considers key indicators such as transportation costs, operational costs, environmental impact, and coverage rate. By leveraging Pareto frontier analysis, it reveals the trade-offs among different objectives and generates site selection solutions that balance economic efficiency and sustainability. The research results show that this method not only provides a data-driven decision-making pathway for Zhenjiang City but also offers a modular design that serves as a replicable technical template for waste station planning in other small and medium-sized cities. The proposed method effectively balances costs and environmental impacts, optimizes the layout of waste stations, improves service coverage rates, and supports the efficient implementation of the "14th Five-Year Plan" waste sorting goals and the in-depth advancement of ecological civilization construction.

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