Evaluating and predicting future Olympic Summer Games based on the combination of multiple mathematical models

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Abstract. The Olympic Summer Games is a symbol of global sportsmanship and thus the world's finest athletes attend the Olympic Summer Games. However, the selection of sports, disciplines, or events (SDEs) is a complex process that requires modern value resonance and global audience attractiveness. Therefore, a future estimation for new additions or reintroductions of SDEs may lower the burden of the International Olympic Committee (IOC) and thus allowing the IOC to make quantitatively informed decisions. The article presents an evaluation model for sports programs by employing the Analytic Hierarchy Process (AHP) and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) together to assess sports based on six IOC criteria: Popularity and Accessibility, Sustainability, Relevance and Innovation, Safety and Fair Play, Gender Equity, and Inclusivity. AHP is used to compare these criteria and give each criterion a suitable weight while TOPSIS ranks alternatives, which are the sports, by calculating weighted sums and thus attaining the sums' proximity to an ideal solution. This team then uses the results obtained by TOPSIS as input data to preform predictions through Gray Forecast Model. Combining the predictions attained with randomness predictions by Markov Chain, a comprehensive overall prediction model presented. In the end, this article concludes that most sports included in both 2024 and 2028 are likely to remain in 2032 and 2036. It suggests Boxing, Polo, and Roque as potential reintroductions for 2032 and 2036 based on overall predictions. The study may provide the future developments of the Olympic Summer Games with a 'grand new design' and technical support.

Keywords: AHP, TOPSIS, Gray Forecast, Olympic, SDEs.

1. Introduction

The Olympic Games have always been the most attractive event in the global sports area, and the sports events of each Olympic Game are highly anticipated by athletes around the world. The selection of SDEs is a complex process that requires consideration over multiple factors, namely, the six IOC criteria: Popularity and Accessibility, Sustainability, Relevance and Innovation, Safety and Fair Play, Gender Equity, and Inclusivity.

According to The Olympic Studies Centre, the continental criteria and numeric requirements increased for both gender and Games type over time [1]. Jean-Loup Chappelet uses longitudinal data, including IOC data, to outline the evolution of the Olympic programme[2]. The IOC stated that SDEs are determined by 3 steps: first, original proposals are submitted 7 years prior to the Games; then, proposals are reviewed 4~6 years ahead of the game; finally, IOC EB votes whether an SDE should be included or excluded based on recommendations made considering the proposals [3].

It can be concluded that, even though a few numerical standards exist, the process for determining whether an SDE should be included or excluded in the Olympic Summer Games is subjective in general and thus limited. This study intends to present a method of SDE selection with higher level of objectiveness. Such an evaluation model utilizes AHP to compare 6 IOC criteria and give each criterion a suitable weight while TOPSIS ranks alternatives, which are the sports, by calculating weighted sums and thus attaining the sums' proximity to an ideal solution. Then, the results obtained by TOPSIS are used as input data to preform predictions through Gray Forecast Model.

2. Research Methodology

2.1. Data Acquisition and Preprocessing

This study obtains its original data regarding the 6 IOC criteria from the following sources: Popularity and Accessibility is measured and represented by Google Trends, as Google is the largest search engine in the world and Google Trends is the official tool provided by Google that allows users to explore the relative popularity of search queries over time[4]; sustainability of each SDE can be considered by calculating the costs and the costs are attained from Amazon data[5], the lower the costs spent on an SDE, the more socially responsible and thus the more sustainable that SDE is; if an Olympic SDE can be participated by athletes from all genders equally and is participated by athletes from at least 75 countries across four continents[6,7], this study considers that SDE achieved Gender Equity and Inclusivity; Relevance and Innovation of an SDE can be reflected through the number of Instagram posts[8], since Instagram is the largest social media that represents young population in general; Safety and Fair Play can be measured through the amount of news regarding incidents of each and every sport[9], the lesser the amount of news regarding incidents of a particular sport, the safer and fairer that sport is.

2.2. Methodology Description

2.2.1. Analytic Hierarchy Process

The six IOC criteria for modeling future Summer Olympics are subjective and influenced by ideological trends, making the Analytic Hierarchy Process (AHP) particularly suitable. The Analytical Hierarchy Process is a reliable, rigorous, and robust method for eliciting and quantifying subjective judgments in multi-criteria decision-making[10]. AHP decomposes a complex multi-objective decision-making problem into multiple criteria, and further decomposes them into several levels of multiple indicators or constraints. All these indicators and constraints will be compared to obtain suitable weights for each indicator or constraint. Because of its simplicity, ease of use, and great flexibility, AHP has been studied extensively and used in nearly all applications related to multiple criteria decision-making[11]. Both academia and industry have witnessed a huge application of Analytic Hierarchy Process (AHP). The technique has gained significant attention in the various domains such as machine selection, supplier selection, ambulance allocation, nurse and other resource prioritization in healthcare, prioritization of academic policies[12].

2.2.2. Technique for Order Preference by Similarity to Ideal Solution

AHP alone is often considered insufficiently rigorous. AHP and TOPSIS techniques are commonly combined[13]. Technique for Order of Preference by Similarity to Ideal Solution, also known as TOPSIS, is based on the fundamental premise that the best solution has the shortest distance from the positive-ideal solution, and the longest distance from the negative-ideal one. Alternatives are ranked with the use of an overall index calculated based on the distances from the ideal solutions[14]. After attaining the AHP weights, a weighted 'distance' for each alternative plan relative to the overall objective can be calculated. The weighted 'distances' will be sorted and thus the optimal solution is obtained.

2.2.3. Gray Forecast Model

The Gray Forecast Model is a type of mathematical modeling technique that deals with uncertainty and partial information. It is particularly useful for situations where data is incomplete, sparse, or has a high level of uncertainty. The basic principle of the Gray Forecast model is to dilute the randomness of the data sequence by accumulating the original data[15] to generate approximate exponential law, and then the modeling is carried out[16]. The Gray Forecast Model was developed for time series forecasting with small samples and has good forecasting ability[17]. The model focuses on using limited data to predict future values or trends and thus this study uses the Gray Forecast Model as the prediction method for modeling future Olympic Summer Games.

3. Evaluation and Prediction

3.1. Evaluation

3.1.1. AHP

Assuming that there are n indicators and constraints, a pairwise comparison matrix $A=\left(a_{ij}\right)_{n\times n}$ is constructed. For two indicators or constraints $i-th$ and $j-th$, when $i-th$ is more significant than $j-th$, $2\le a_{ij}\le 9$ has to be satisfied where $a_{ij}\in Z^{+}$. The larger the a_{ij}, the bigger the difference between the significances of $i-th$ and $j-th$. $a_{ji}=\frac{1}{a_{ij}}$ can be defined, meaning that $a_{ji}\in\left\{\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},\frac{1}{6},\frac{1}{7},\frac{1}{8},\frac{1}{9}\right\}$ when $i-th$ is more significant than $j-th$.When $i-th$ and $j-th$ are of the same significance, $a_{ij}=a_{ji}=1$.

Supposing that all 6 IOC criteria are fairly important, then $\frac{1}{3}\le a_{ij}\le 3$ is satisfied for all $a_{ij}\in A=\left(a_{ij}\right)_{6\times 6}$. Since the number of indicators and constraints is 6, so $n=6$.

The entire pairwise comparison matrix is as follows:

$$A=\begin{bmatrix}1 & 2 & 3 & 2 & 3 & 1\\ \frac{1}{2} & 1 & 2 & 2 & 2 & 1\\ \frac{1}{3} & \frac{1}{2} & 1 & \frac{1}{2} & 3 & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & 2 & 1 & 3 & \frac{1}{2}\\ \frac{1}{3} & \frac{1}{2} & \frac{1}{3} & \frac{1}{3} & 1 & \frac{1}{3}\\ 1 & 1 & 2 & 2 & 3 & 1\end{bmatrix} \tag{1}$$

In order to calculate the weight for each criterion, it is required to normalize the pairwise comparison matrix first. This article defines $B=\left(b_{ij}\right)_{6\times 6}$ to be the normalized matrix of matrix A, then matrix B can be obtained using the following equation:

$$b_{ij}=\frac{a_{ij}}{\sum_{i=1}^{n}a_{ij}}=\frac{a_{ij}}{\sum_{i=1}^{6}a_{ij}} \tag{2}$$

The weight w_i for criterion $i-th$ is as follow:

$$w_i=\frac{1}{n}\sum_{j=1}^{n}b_{ij}=\frac{1}{6}\sum_{j=1}^{6}b_{ij} \tag{3}$$

This article obtained the weights for each criterion to 4 decimal places: $w_1=0.2700$, $w_2=0.1899$, $w_3=0.1081$, $w_4=0.1442$, $w_5=0.0656$, $w_6=0.2221$, as shown in Figure 1:

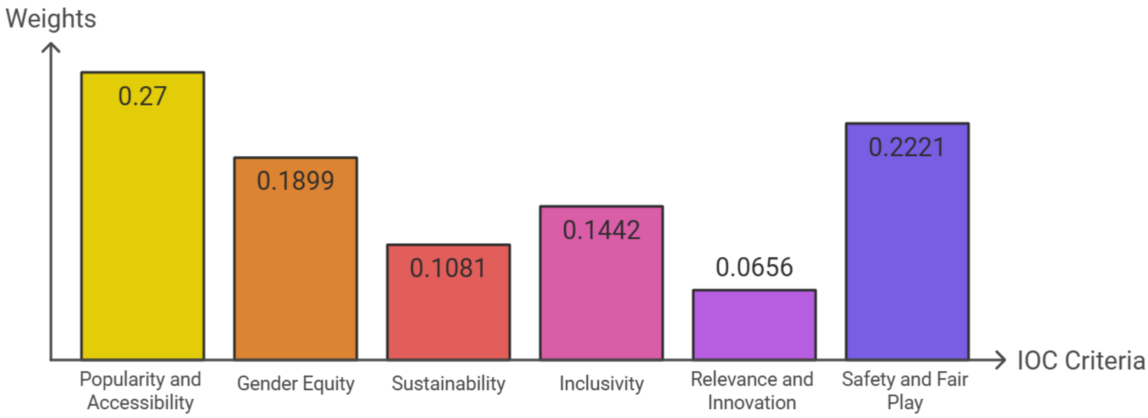

Figure 1. Weights of IOC criteria for Future Olympic Games

It is required to check the consistency of the pairwise comparison matrix and thus the weights for each criterion by calculating the consistency ratio CR. If $CR < 0.1$, then the pairwise comparison matrix and thus the weights for each criterion are considered consistent.

Let λ be the eigenvalues for pairwise comparison matrix A, then the maximum eigenvalue λ_{max} which satisfy $Ax = \lambda x$ where x is the corresponding eigenvector can be obtained.

The consistency index CI to 4 decimal places is computed as:

$$CI = \frac{\lambda_{max} - n}{n-1} = \frac{\lambda_{max} - 6}{5} = 0.0456 \tag{4}$$

Since $n = 6$, so the random consistency index $RI = 1.24$ according to Figure 2:

n	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
RI	**0**	**0**	**0.58**	**0.90**	**1.12**	**1.24**	**1.32**	**1.41**	**1.45**	**1.49**	**1.51**

Figure 2. Ramdom Index (RI)

Therefore, the consistency ratio CR to 4 decimal places is given by:

$$CR = \frac{CI}{RI} = \frac{0.0456}{1.24} = 0.0362 < 0.1 \tag{5}$$

The pairwise comparison matrix A and thus the weights w_i for criterion i are consistent.

3.1.2. TOPSIS

To preform TOPSIS, this article constructed an original decision matrix $C = \left(c_{ij}\right)_{m \times p}$ according to data gathered, where there are m alternatives to choose from and p indicators.

The first step is to transform matrix C and thus the influence of different types of indicators is eliminated, so that all indicators can be compared directly as they are turned into benefit indicators. This study defines the transformed matrix as $C' = \left(c_{ij}'\right)_{m \times p}$. After transformations, matrix C' needs to be normalized. The main goal of normalization is to eliminate the influence of different units of measurement, so that all criteria can be compared on the same scale, typically between the range $[0,1]$. This article defines matrix $D = \left(d_{ij}\right)_{m \times p}$ to be the normalized matrix of matrix C', then matrix D can be obtained using the following equation:

$$d_{ij} = \frac{c_{ij}'}{\sqrt{\sum_{k=1}^{m} {c_{kj}'}^2}} \tag{6}$$

For each indicator $j-th$, calculate the maximum value $d_{j_{max}}$ and the minimum value $d_{j_{min}}$ using the following set of equations:

$$\begin{cases} d_{j_{max}} = max(\, d_{1j}, d_{2j}, d_{3j}, \ldots, d_{mj}) \\ d_{j_{min}} = min(\, d_{1j}, d_{2j}, d_{3j}, \ldots, d_{mj}) \end{cases} \tag{7}$$

With these results, the authors can obtain the 'distance' from alternative $i-th$ to an ideal solution ${E_i}^+$ and the 'distance' to a negative ideal solution ${E_i}^-$ for the m alternatives to choose from by:

$$\begin{cases} {E_i}^+ = \sqrt{\sum_{j=1}^{p} w_j\left(d_{j_{max}} - d_{ij}\right)^2} = \sqrt{\sum_{j=1}^{6} w_j\left(d_{j_{max}} - d_{ij}\right)^2} \\ {E_i}^- = \sqrt{\sum_{j=1}^{p} w_j\left(d_{j_{min}} - d_{ij}\right)^2} = \sqrt{\sum_{j=1}^{6} w_j\left(d_{j_{min}} - d_{ij}\right)^2} \end{cases} \tag{8}$$

It is to be noted that w_j, the weights for each IOC criterion to 4 decimal places, are attained through Analytic Hierarchy Process. Therefore, the closeness coefficient F_i for alternative $i-th$ from the m alternatives can be determined:

$$F_i = \frac{E_i^-}{E_i^+ + E_i^-} \tag{9}$$

The alternative with the highest closeness coefficient is considered the best alternative, as it is the closest to the ideal solution.

Since the 6 indicators for modeling future Olympic Summer Games are the 6 IOC criteria and there are 46 sports for modeling future Olympic Summer Games, so $p = n = 6$ and $m = 46$. Therefore, the original decision matrix is $C = \left(c_{ij}\right)_{46\times 6}$.Standards of attaining the original decision matrix are stated in Data Acquisition and Preprocessing.

Obviously, every column representing measurements under a particular IOC criterion has its own unit and scale, not to mention that the 6 indicators are not of the same indicator type. These influences are caused by the 'rawness' of data and can be eliminated with transformations and normalizations as mentioned before.

The following table lists partial results acquired, what is to be noted is that there are more than 15 original decision matrices and more than 15 calculated closeness coefficients F_i for each sport, but the 8-page limit prohibits the authors from displaying full results and thus mean values are taken:

Table 1. average coefficient for 10 sports after TOPSIS calculations

Sport	average closeness coefficient F_i			
	2009~2012	2013~2016	2017~2020	2021~2024
Aquatics	0.046	0.0434	0.0414	0.0445
Archery	0.017	0.0187	0.0179	0.0181
Athletics	0.0418	0.0368	0.0345	0.0363
Badminton	0.0217	0.0216	0.0218	0.0241
Baseball and Softball	0.0184	0.0187	0.0214	0.0201
Basketball	0.0254	0.0278	0.0291	0.0266
……(40 more)	……(40 more)	……(40 more)	……(40 more)	……(40 more)

Aquatics, Archery, Athletics, Basketball, Canoeing, Cycling, Equestrian, Fencing, Field Hockey, Football, Gymnastics, Indoor Handball, Rowing, Sailing, Shooting, Table Tennis, Tennis, Indoor Volleyball, Weightlifting and Wrestling are sports that have continuously been in the Olympic Summer Games since the 1988 games or earlier. It can be easily concluded from table 1 that these sports have closeness coefficients $F_i \geq 0.017$ no matter the year. Also, the changes in their closeness coefficient F_i are small and thus can be considered negligible in particular circumstances, meaning that all these sports have stable closeness coefficients F_i regardless of time.

Breaking, Cricket, Flag football, Lacross, Squash, Skateboarding, Sport Climbing, Surfing and Karate are sports added or removed from recent Olympics, namely Olympic years 2020, 2024, and 2028. In general, these sports either have closeness coefficients $F_i \geq 0.015$ regardless of time or have increasing closeness coefficients over time. The average closeness coefficients F_i for Karate dropped significantly from 0.212 in 2021~2024 to 0.0162 in 2017~2020. Since it is highly possible for Karate's closeness coefficient to drop under 0.015, so this may be the reason that Karate was no longer included in the 2024 Paris Olympics.

In a word, either having closeness coefficients $F_i \geq 0.015$ regardless of time or having increasing closeness coefficients over time would allow an SDE to be considered as a candidate for inclusion in the Olympic Summer Games. Additionally, having stable closeness coefficients $F_i \geq 0.017$ with minor changes should ensure an SDE to be a continuous part of the Olympic Summer Games.

3.2. Predictions made by Gray Forecast

Since the original decision matrix C of each and every alternative, or SDE, varies every year in TOPSIS, so every SDE has a series of closeness coefficient F_i, one for each year. Let H be the number of different F_i utilized by the Gray Forecast Model that are from the same alternative or SDE.

The authors construct a series $Q = \{Q_1, Q_2, Q_3, Q_4, \ldots, Q_H\}$, such series is a rearrangement by sorting the H different F_i from the same alternative or SDE in time order, where Q_1 is the earliest of the H different F_i while Q_H is the latest. This article then defines the series $q = \{q_1, q_2, q_3, q_4, \ldots, q_H\}$ satisfying:

$$\begin{cases} q_i = \sum_{j=1}^{i} Q_j \\ 1 \leq i \leq H \\ i \in Z^+ \end{cases} \tag{10}$$

Let series q satisfy the following differential equation:

$$\frac{dq}{dt} + aq = u \tag{11}$$

The variable t represents time while a and u are constants to be determined.

This article obtains the following equation with the differential equation:

$$q_{i+1} - q_i = u - aq_i \tag{12}$$

Such an equation can be deduced into the following and thus a as well as u can be solved:

$$\begin{bmatrix} q_2 - q_1 \\ q_3 - q_2 \\ q_4 - q_3 \\ \vdots \\ q_H - q_{H-1} \end{bmatrix} = \begin{bmatrix} q_1 & 1 \\ q_2 & 1 \\ q_3 & 1 \\ \vdots & \vdots \\ q_{H-1} & 1 \end{bmatrix} \times \begin{bmatrix} -a \\ u \end{bmatrix} \tag{13}$$

After a and u are solved with the Least Squares Method, the authors can attain the following model:

$$q_{i+1}' = \left(q_1 - \frac{u}{a}\right) e^{-ai} + \frac{u}{a} \tag{14}$$

q_{i+1}' stands for the calculated fitted value of the original value q_{i+1}. Conspicuously, when $i + 1 > H$, q_{i+1}' is a prediction given by the Gray Forecast Model.

Since series q is not a rearrangement by sorting the H different F_i from the same alternative or SDE in time order, so series $q' = \{q_1', q_2', q_3', q_4', \ldots, q_H', q_{H+1}', q_{H+2}', q_{H+3}', \ldots\ldots\}$ is not the final prediction. Therefore, the authors deduced series Q' with:

$$\begin{cases} Q' = \{Q_1', Q_2', Q_3', Q_4', \ldots, Q_H', Q_{H+1}', Q_{H+2}', Q_{H+3}', \ldots\ldots\} \\ Q_i' = q_i' - q_{i-1}' \\ i \in Z^+ \end{cases} \tag{15}$$

The series Q' is the final prediction given by the Gray Forecast Model.

The data used for predictions and thus the predictions are given in Table 2:

Table 2. Grey prediction partial results

Sport	Q_1	Q_2	Q_3	Q_4	Q_6'	Q_7'
Aquatics	0.046	0.0434	0.0414	0.0445	0.044813	0.0454
Archery	0.017	0.0187	0.0179	0.0181	0.017345	0.01706
Athletics	0.0418	0.0368	0.0345	0.0363	0.03511	0.034861
Badminton	0.0217	0.0216	0.0218	0.0241	0.026612	0.028157
Baseball and Softball	0.0184	0.0187	0.0214	0.0201	0.022195	0.022957
Basketball	0.0254	0.0278	0.0291	0.0266	0.026124	0.025579
(40 more)	(40 more)	(40 more)	(40 more)	(40 more)	(40 more)	(40 more)

In table 2, Q_1, Q_2, Q_3, Q_4 are average closeness coefficients F_i for years 2009~2012, 2013~2016, 2017~2020, 2021~2024 respectively. Q_6' is the predicted closeness coefficient for 2032 Olympics in Brisbane while Q_7' is the predicted closeness coefficient for Olympics in 2036. Q_5' is not in the table because 2028 data is known and thus prediction for 2028 is not required.

Since either having closeness coefficients $F_i \geq 0.015$ regardless of time or having increasing closeness coefficients over time would allow an SDE to be considered as a candidate for inclusion in the Olympic Summer Games, so most SDEs included in both 2024 and 2028 are predicted to be included in both 2032 and 2036.

Now the authors will identify SDEs that could be new additions or reintroductions for the 2032 Olympics in Brisbane. SDEs that are included in 2028 are not seen as new additions or reintroductions. The authors ranked the predicted closeness coefficients $Q_6{'}$ and $Q_7{'}$ of the other choices and thus determined that the SDEs for new additions or reintroductions. Boxing, Polo and Roque are determined for reintroductions, as their predicted closeness coefficients $Q_6{'}$ and $Q_7{'}$ are both greater than 0.23, which all are overwhelmingly high compared to other choices available.

3.3. Sensitivity Analysis

The authors conducted a sensitivity analysis to address the robustness of the overall model by calculate $Q_5{'}$, the predicted closeness coefficient for 2028, to obtain a prediction and thus compare it to the actual 2028 data provided by HiMCM_Olympic_Data.xlsx.[18]

Table 3 provides the results, where ‘y’ stands for inclusion and ‘n’ stands for exclusion:

Table 3. partial sensitivity check

Sport	$Q_5{'}$	Gray Forecast Model prediction	2028 actual data
Aquatics	0.044234	y	y
Archery	0.017636	y	y
Athletics	0.03536	y	y
Badminton	0.025153	y	y
Baseball and Softball	0.021458	y	y
Basketball	0.02668	y	y
(40 more)	(40 more)	(40 more)	(40 more)

The robustness of the overall prediction is acceptable as the accuracy reached 95.65%.

4. Conclusion

The article provides an evaluation model for sports programs by employing AHP and TOPSIS together to assess sports based on six IOC criteria, then uses the results obtained by TOPSIS as input data to preform predictions through Gray Forecast Model. The model augments these conclusions by enabling a data-driven evaluation of sports. It takes into account both the current popularity and levels of participation in each sport, as well as their ability to grow and develop in accordance with Olympic Values. With that information and past data, this study can model future trends to be able to optimally decide on sports that best suit the Olympic Games, and thus providing technical support for the IOC with higher levels of accuracy and better scientific basis.

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