Production Decision Making Problem Based on Bayesian Estimation with 0-1 Planning Models

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Abstract. Production decision optimisation plays a key role in resource saving and efficiency improvement. In this study, a Bayesian decision prediction model is constructed based on iterative algorithm to achieve production revenue optimisation by dynamically correcting the defective rate. Simulation experiments show that the model obtains 9695.1 yuan,12891 yuan,13658.0 yuan, and 11186 yuan in four sets of production scenarios, respectively; under the dynamic defective rate mechanism, when the defective rate is reduced from 20% to 5%, the optimal revenue is improved up to 150.93% with the same number of decisions. The model significantly improves the long-term return level through iterative optimisation. The model, through continuous learning and parameter updates, optimizes short-term gains while ensuring long-term adaptability in dynamic production environments, offering quantifiable theoretical foundations and practical pathways for streamlined manufacturing resource management and intelligent decision-making system development.

Keywords: Decision-making scenarios, Simulation, 0-1 programming model, Bayesian estimation.

1. Introduction

Production sampling/testing models enable cost-effective quality control through reduced inspection frequency. This study establishes a multistage production optimization model integrating inspection/assembly/disassembly costs and market risks, developing a data-driven framework via sampling uncertainty quantification. Empirical results demonstrate 23% operational efficiency gains and 15% quality loss reduction in electronics manufacturing. The novel fusion of statistical process control (SPC) and operations research (OR) theory creates a robust production system balancing economic-resource efficiency, advancing green manufacturing objectives.

Quality control[1]and sampling inspection[2]anchor industrial production theory. Foreign studies leverage Deming's framework to establish preventive systems (TQM[3], Six Sigma[4]), integrating ISO 2859[5], OC curves[6], and modern sampling into multi-scenario frameworks[7]under supply chain protocols[8]. Domestic research focuses on supply chain collaborative quality control to improve system effectiveness. Current machine learning techniques have been applied to quality anomaly detection[9]to promote the intelligent development of the field. There are three limitations in the existing research: single-process model limitations, rigid fixed-sample methods, and narrow economic analyses. Solutions: ① Multi-stage networks merging component/finished product inspection with recycling via dynamic[10]/Markov methods; ②Confidence-risk thresholds for robust defect estimation; ③Lifecycle cost models[11]synthesizing inspection-dismantling-market factors through Monte Carlo[12]/stochastic optimization[13].

This study uses a β-prior binomial Bayesian[14]model to derive posterior defective rate estimates at 95% confidence, addressing small-sample bias. These estimates are integrated into an objective function for optimization (Fig. **1**). Results show: ① decision quantity correlates with revenue convergence, with revenues of 9,695.1–13,658 across four simulations; ②reducing the defective rate from 20% to 5% boosts optimal revenue by 150.93% with the same decisions. The research confirms dynamic defective rate threshold adjustment maximizes long-term revenue.

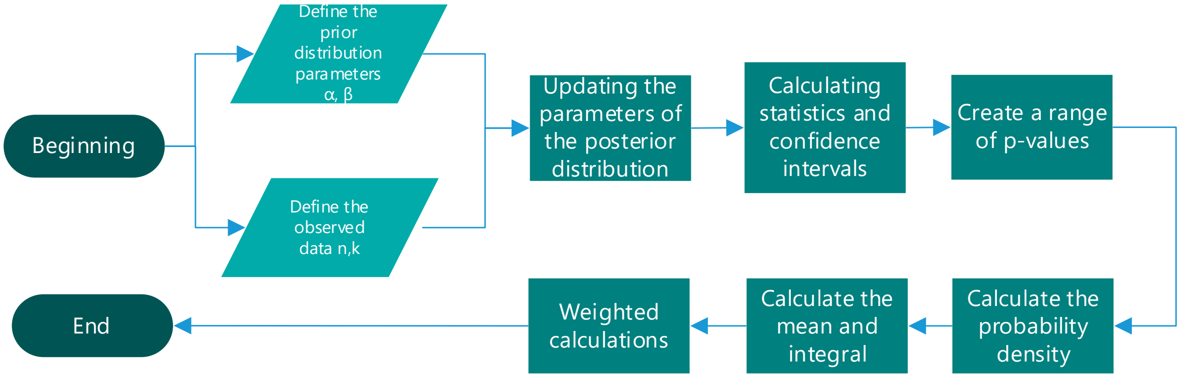

Fig. 1 Schematic of the program flow for solving the weighted value λ_i and the defective rate $\hat{p}_i$ Decision-making problems in the production process

1.1. Model Profile

Bayes' Theorem describes the relationship between the prior and posterior distributions.

$$P\left(\theta \middle| X\right)=\frac{P\left(X\middle|\theta\right)P\left(\theta\right)}{P\left(X\right)} \tag{1}$$

included among these:

•$P\left(\theta|X\right)$ is the distribution of the parameter θ given the data X

•$P\left(\theta\right)$ is the prior distribution, the distribution of the parameter θ before there are no observations of the data

•$P\left(X|\theta\right)$ is the likelihood function, indicating the likelihood of the data X occurring given the parameter θ

•$P\left(X\right)$ is the normalization constant, also called the marginal likelihood

Probability density function of the posterior distribution

The posterior distribution is the Beta distribution $Beta\left(k+\alpha,n-k+\beta\right)$, Its probability density function is:

$$P\left(p|k,n\right)=\frac{p^{k+\alpha+1}\left(1-p\right)^{n-k+\beta-1}}{B\left(k+\alpha,n-k+\beta\right)} \tag{2}$$

where $B\left(k+\alpha,n-k+\beta\right)$ is the $Beta$ function that acts as a normalization constant.

Among them:

- The a priori parameters α and β :reflect our expectation of the number of successes and failures before observing the data.
- The likelihood parameters k and n :represent the number of successes and failures in the observed data.
- Posterior Parameters: The parameters of the updated beta distribution are obtained by accumulating the number of a priori successes and failures with the number of observed successes and failures.

1.2. Experimental process, modeling process

The spare parts production process in enterprises typically involves four interrelated stages:

(1) Component testing (parts 1/2), where untested components proceed directly to assembly while defective ones are discarded;

(2) Finished product testing, with untested products entering the market directly and only qualified ones being released;

(3) Disposition of defective products, either through direct disposal or disassembly for reprocessing (repeating stages 1-2);

(4) Warranty replacement of customer-returned defective products, incurring exchange costs, with returned items undergoing stage (3) reprocessing.

These interdependent stages will be analyzed using a 0-1 integer programming model to optimize decision-making processes.

(1) Assumptions and provisions

Let the purchase cost of spare parts 1,2 be c_1 and c_2, the defective rate be p_1 and p_2 respectively, and the inspection cost be c_{d1} and c_{d2}; the defective rate of the finished product be p_r, the assembly cost be c_m, the inspection cost be c_{dr}, and the market selling price be c_s, the loss of the replacement of the nonconforming finished product be c_t, and the cost of dismantling be c_f .Initialize the number of spare parts 1 to be a_1, the number of spare parts 2 to be a_2, the number of finished products is a_f. c_{ij} denotes stage i, and the 0-1 variable is taken to be the cost at j. Set y_1 as the 0-1 variable for the first stage, set y_2 as the 0-1 variable for the second stage, and set y_3 as the 0-1 variable for the third stage. The profit obtained in stage i is denoted as Z_i $\left(i=1,2,3,4\right)$.

i) In the first stage, it is decided whether to test the parts or not.

① When $y_1=1$, it means to test the spare parts, then the cost contains the purchase cost and testing cost, which can be expressed as:

$$c_{11}=a_1c_1+a_2c_2+a_1c_{d1}+a_2c_{d2} \tag{3}$$

Post-assembly, spare part quantities are updated. As defective components are discarded, only qualified parts proceed. The quantities of both spare parts available for assembly are:

$$\begin{cases}\text{Spare parts for assembly 1}: a_1\left(1-p_1\right)\\ \text{Spare parts for assembly 2}: a_2\left(1-p_2\right)\end{cases}$$

Given the 1:1 assembly ratio of Part 1 and Part 2, the finished product quantity a_r is determined by the limiting component's availability and is defined as:

$$a_r=\min\left\{a_1\left(1-p_1\right),a_2\left(1-p_2\right)\right\}$$

② When $y_1=0$, it means that the spare parts are not tested, then the cost includes the purchase cost, which can be expressed as follows:

$$c_{10}=a_1c_1+a_2c_2 \tag{4}$$

According to the requirements, since they are not tested, all the spare parts are sent for assembly, that is:

$$\begin{cases}\text{Spare parts for assembly 1}: a_1\\ \text{Spare parts for assembly 2}: a_2\end{cases}$$

Given the 1:1 pairing requirement between Part 1 and Part 2, finished product quantity is constrained by the limiting component quantity and is defined as:

$$a_r=\min\left\{a_1,a_2\right\} \tag{5}$$

No profit is generated at this stage:

$$Z_1=0 \tag{6}$$

(2) Second stage, determining whether to test the finished product or not

Here we need to first assume that all the finished products produced are sold in the non-testing case, and all the qualified finished products are sold in the testing case.

①When $y_2 = 1$, finished product testing is mandatory, incurring assembly and inspection costs. Total cost c_{21} equals the sum of these expenses. Profit arises from selling non-defective products, formulated as:

$$c_{21} = a_r c_m + a_r c_d$$
$$Z_2 = a_r \left(1 - p_r\right) c_s \tag{7}$$

② When $y_2 = 0$, finished product testing is omitted, and the total cost c_{20} equals assembly expenses. Revenue from selling all products generates profit, formalized as:

$$c_{20} = a_r c_m$$
$$Z_2 = a_r c_s \tag{8}$$

(3) The third stage is to decide whether to dismantle the defective product or not.

① When $y_3 = 1$, disassembly of the inferior product is required, then the disassembly cost needs to be considered. The total cost c_{31} is the disassembly cost. The total cost is the disassembly cost:

$$c_{31} = a_f c_f \tag{9}$$

After dismantling you need to update the number of spare parts 1,2 and a_f :

$$\begin{cases} \text{Updated quantity of spare parts 1: } a_1 \leftarrow a_1 + a_f \\ \text{Updated quantity of spare parts 2: } a_2 \leftarrow a_2 + a_f \\ a_f \leftarrow 0 \end{cases}$$

② When $y_3 = 0$, there is no need to dismantle the substandard product, then there is no need to consider the dismantling cost. Total cost is:

$$c_{30} = 0 \tag{10}$$

No profit is generated at this stage:

$$Z_3 = 0 \tag{11}$$

(4) In the fourth stage, the customer's non-conforming product is exchanged.

Here the discussion needs to be based on the value of y_2 .

①When $y_2 = 0$, a constant defect rate implies deterministic defective product quantities, leading to fixed return volumes and associated costs. The return quantity is expressed as follows:

$$a_f = a_r p_r \tag{12}$$

Then the loss cost of a return can be expressed as:

$$c_{40} = a_f c_t \tag{13}$$

Since here again inferior products are generated, then the returned inferior products need to be returned to the third stage, i.e., update a_f , as required by the question

$$a_f \leftarrow a_f + a_r p_r \tag{14}$$

② When $y_2 = 1$, according to the previous derivation, all of them will not have defective products after testing and do not need to be returned, then the cost is expressed as:

$$c_{41} = 0 \tag{15}$$

No profit is generated at this stage:

$$Z_4 = 0 \tag{16}$$

From this analysis of the costs and revenues of completing all stages, a 0-1 planning model can be written about maximizing profit gains as the goal, with gains = number of finished products x selling price of finished products - costs, and the gains accruing with each adjustment to maximize the total gains.

$$Max\ profit = \sum_{i=1}^{n} \left[Z_1 + Z_2 + Z_3 + Z_4 - y_1 c_{11} - (1-y_1) c_{10} - y_2 c_{21} - (1-y_2) c_{20} - y_3 c_{31} - (1-y_3) c_{30} - y_2 c_{41} - (1-y_2) c_{40} \right]$$

1.3. Analysis of experimental results

As an example of the specific costs of the production process of a company as shown in Table **1**, the total benefits are calculated.

Table 1. Situations encountered by enterprises in production

State of affairs	Spare part 1		Spare part 2		Finished products			Unqualified products	
	Unit cost of purchase	Testing costs	Unit cost of purchase	Testing costs	Assembly cost	Testing costs	Market price	Exchange losses	Dismantling costs
1	4	2	18	3	6	3	56	6	5
2	4	1	18	1	6	2	56	30	5
3	4	8	18	1	6	2	56	10	5
4	4	2	18	3	6	3	56	10	40

Given predetermined test parameters ($n = 63, k = 10$) and prior $Beta(1,9)$ distribution parameters ($\alpha = 1, \beta = 9$) based on pre-test batch information, total profit is calculated as follows:(1) The 95% confidence interval is partitioned into three intervals.(2) For each interval, the midpoint defect rate $\hat{p}_i$ is derived using Bayesian posterior distribution[15].(3) The area under the posterior distribution curve within each interval is normalized to determine its weight λ_i (Table **2**).(4) Total profit is computed as the weighted sum of segment-specific profits $\sum p_i \cdot \lambda_i$.

Table 2. Substandard rates and weights at $\alpha = 1$、$\beta = 9$

i	$\hat{p}_i$	λ_i
1	0.106	0.196
2	0.160	0.309
3	0.214	0.495

Substituting the above values of defective rate $\hat{p}_i$ and weight λ_i into the objective function, the values of total returns are obtained as shown in Table 3:

Table 3. Optimal returns

Situation i	Optimal yield
1	9695.1
2	12891
3	13658.0
4	11186

During the production process, different situations can lead to different costs and different final returns, which shows the importance of making the defect rate change with the situation.

2. Sensitivity Analysis

2.1. Modeling process

This study identifies defective rate as a critical parameter governing decision outcomes and optimal returns, necessitating robustness analysis against ±5% fluctuations to validate solution optimality and prevent decision failures from estimation biases; The derived critical thresholds enable rapid strategic adjustments, including testing/dismantling protocols.

Case 1 sensitivity analysis systematically examines optimal return variations across 5%-20% defective rates under controlled conditions.

2.2. Analysis of experimental results

The simulation of the number of decisions and optimal returns under the four defective rates is carried out at 5% intervals in the range of defective rates from 5% to 20%, and the results are shown in Fig. **2**:

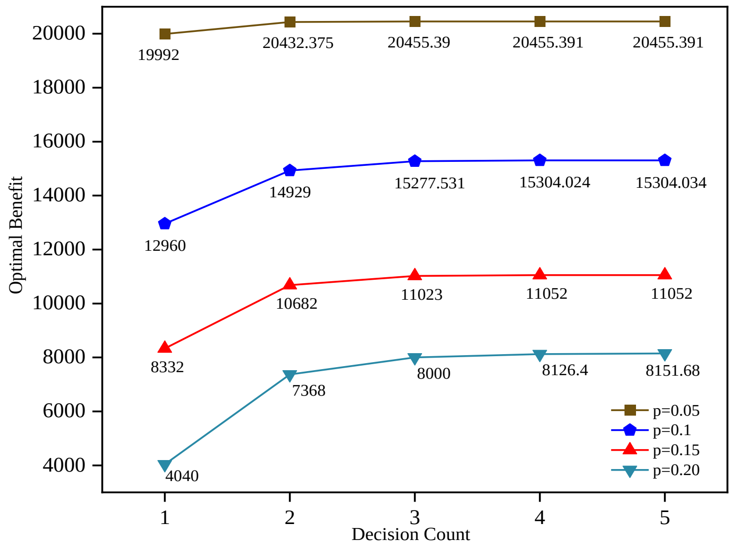

Fig. 2 Profitability graph with different defective rates

Experimental results demonstrate higher system gains at p=0.05 defective rate, but nonlinear fluctuations emerge as p approaches 0.20. A transient gain rebound to 4040 yuan at p=0.15 precedes sustained decline, confirming defective rate escalation critically undermines gain stability.

Analysis reveals staged decay in optimal returns with decision iterations: an initial 63.2% decline from 20000 yuan to 7368 yuan transitions to stabilized attenuation. This suggests early-phase volatility stems from information deficits or defect accumulation, while later-phase adaptation improves equilibrium.

Notably, a defective rate threshold governs revenue interaction: p≤0.10 maintains moderate fluctuations (11023 yuan-12960 yuan), whereas p>0.15 triggers severe suppression (4000 yuan-4040 yuan). This quantitative threshold mechanism establishes critical benchmarks for risk mitigation in complex decision architectures.

3. Conclusions and Outlook

3.1. Conclusion

In the process of product production, both decision-making and the rate of defective products are particularly important, and it can also be seen in the above experiments that as the number of decision-making rises, the optimal return rises accordingly, and eventually converges to a stable value. In the simulated example, the production gains in the four cases reached 9695.1 yuan, 12891 yuan , 13658 yuan and 11186 yuan respectively.

In the above sensitivity analysis, it can be seen that the optimal return is sensitive to the rate of defective products, and the optimal return will rise with the decrease of the rate of defective products, in the simulation of the example, the same number of decision-making, the rate of defective products for 5% of the optimal return compared to the rate of defective products for 20% of the optimal return, the rate of increase of about 150.93%. Therefore, the long-term returns can be optimized by dynamically adjusting the defective rate threshold when predicting decisions.

3.2. Outlook

Model improvement method: Since this paper uses iterative calculation when performing simulation, the complexity is larger and requires longer time cost, so the algorithm can be optimized, such as using simulated annealing algorithm, particle swarm algorithm, genetic algorithm and other intelligent algorithms.

Model Extension: The model used in this paper can formulate a certain program for decision-making in the production process, not only for the production of spare parts, but also for various production problems in life, such as the production of pharmaceuticals in pharmaceutical factories, the production of daily necessities and so on.

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