Research on the Spiral Motion Model of Bench Dragon Based on Archimedean Spiral

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Abstracts; The aim of this study is to accurately describe the trajectory and dynamics characteristics in traditional dragon dance performances through mathematical modeling, so as to provide a theoretical basis for the science, standardization and teaching of the art of dragon dance. The spiral motion of the bench dragon dance can be simulated and analysed by the Archimedean solenoid model, which provides a theoretical and practical basis for the study of its trajectory and velocity. Here, this paper is based on the known pitch of the screw line using Archimedes screw line to find out the screw line equation in polar coordinates, according to the handle centre straight line distance constant conditions, to establish a differential equation, and then the fourth-order Lung Kuta algorithm to solve the next hand position and then use recursive ideas to get all the bench position. And then according to the different moments of the position of the head, get different moments under the position of the bench, according to the position information using the numerical difference method to approximate the speed of each moment. Finally, a series of bench position and speed information is obtained. The 60s, the faucet is at (5.799m,-5.771m), the speed is 1m/s. This paper provides theoretical guidance for the study of the motion analysis of disc-type mechanical parts.Qualitative analysis and quantitative calculations were also performed.

Keywords: Bench Dragons, Archimedean Solenoids, Fourth-order Dragons Gekuta, Numerical Difference Method.

1. Introduction

As an important part of traditional Chinese folk culture, the Bench Dragon Dance is not only a highly ornamental form of festival performance but also carries rich historical and cultural values[1]. In traditional festivals and folklore events, dragon dance teams demonstrate complex and varied movement choreography and team coordination by following a spiral path, and their rhythmic performances often attract many spectators[2]. However, despite the long history and deep cultural heritage of dragon dance activities, previous academic studies have largely failed to develop rigorous mathematical models of trajectory and velocity characteristics. Existing methods usually rely on simplified linear approximations that fail to accurately capture the complex spatial relationships between dragon segments in spiral motion. These simplifications lead to significant discrepancies between theoretical models and actual performances, limiting their practical application in training and choreography. The Archimedean spiral, as a classical mathematical curve with linear expansion characteristics, has attracted much attention because of its wide application in engineering, physics and mathematics. The application of Archimedean solenoids to model the trajectory of the dragon dance team can accurately simulate the characteristics of its spiral motion and provide a scientific basis for the spiral motion of the dragon dance team[3]

In this study, based on Archimedes' theory of solenoids, the solenoidal equations were established based on the motion characteristics of the bench dragon. Subsequently, based on the geometric relationship between the centre of each bench handle, the differential equation equation was derived through the Euclidean distance calculation formula[4]. Combined with the recursive idea and the four-section dragon Gekuta algorithm, the position information of all the handles over time was gradually solved[5]. Next, based on the obtained position information over time, the velocity under each moment was approximated using the numerical difference method. Finally, the time-varying position and velocity information of the centre of each bench handle is obtained[6].This paper provides a comprehensive modeling approach based on differential equations and geometric analysis that not only accurately captures positional information over time, but also uses numerical differentiation to derive velocity characteristics. This integrated approach enables, for the first time, a quantitative analysis of the entire dragon dance system as a whole, providing valuable insights for performance optimization, training standardization, and preservation of this cultural art form.

2. Simulation of Spiral Motion of the Bench Dragon

2.1. Bench Dragon simulation idea

In this study, the data from www.mcm.edu.cn are used to simulate the trajectory of the "Bench Dragon" dance team along the Archimedean spiral path and to calculate the velocity and position information of the dragon head, the dragon body, and the centre of the front handle and the centre of the rear handle of the dragon tail from 0s to 300s. In the first step, the motion trajectory of the dragon dance team is an Archimedean spiral with a known pitch; therefore, the equations of polar coordinates are used to inscribe the trajectory of its spiral motion. Subsequently, we determine the expression for the linear velocity of the handle centre of each section of the bench when the dragon dance team performs the spiral motion, and substitute the velocity of the handle centre in front of the dragon's head into the expression for the linear velocity to determine the positional information of the dragon's head, and the straight-line distance of the adjacent handle centres is always constant, and by using the Euclidean distance calculation formula, the use of the recursive method, according to the position of the dragon head to determine the position information of the dragon body, the dragon tail, the establishment of the bench dragon position transfer model, call the solve function in MATLAB for the solution, so that you can determine the position information of the entire dragon dance team. Then, according to the linear velocity expression of the centre of each bench handle, the bench dragon velocity transformation model is established. Since the angle data θ is discrete, this study adopts the numerical difference method to approximate the velocity and solve the velocity information of the whole dragon dance team[7-9].

2.2. Preparation of data

The "Bench Dragon" consists of 223 sections of benches, the first section is the dragon's head, the last 1 section is the dragon's tail, and the rest of the benches are the dragon's body, with the centre of the rear handles of the former benches connected to the centre of the front handles of the latter benches to form a mortise-and-tenon joinery structure,as shown in Figure 1, Figure 2, where the widths of all benches are $30cm$, the lengths of the dragon's headboards $341cm$, the dragon's body and the dragon's tail boards $220cm$, and the distance of two holes in the benches to the nearest head is, and the diameter of the holes are $27.5cm$ and the diameter of the holes is $5.5cm$.

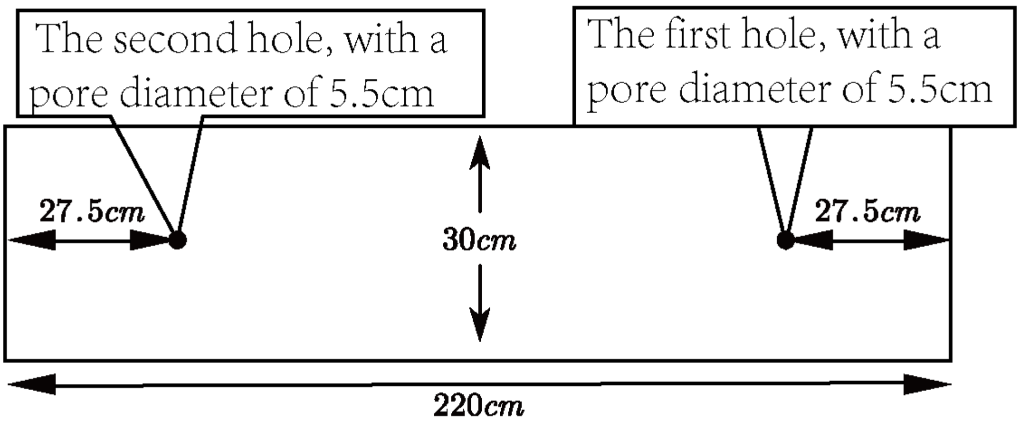

Figure 1. Schematic diagram of section I specifications

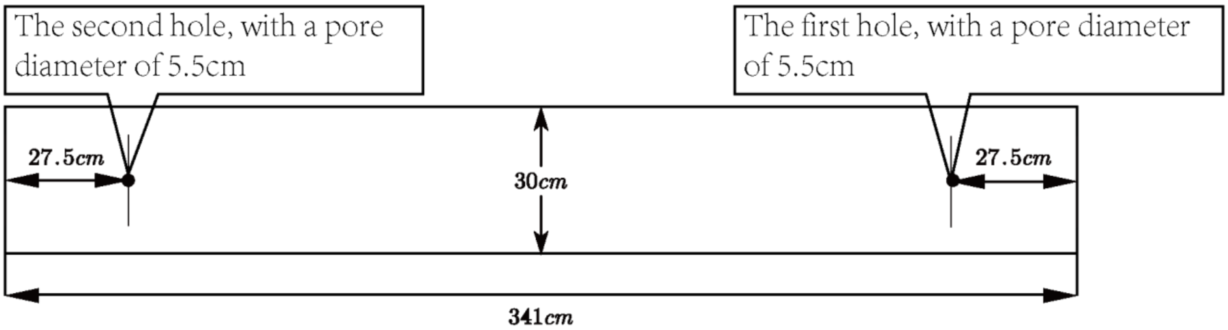

Figure 2. Schematic diagram of the remaining section specifications

2.3. Bench Dragon Position Transfer Model

Firstly, this study needs to determine the parametric equation of the solenoid firstly it is known that the dragon dance team moves clockwise along the equidistant solenoid (Archimedean solenoid) with the pitch $0.55m$ during the process of coiling in and the basic feature of the Archimedean solenoid is the linear relationship between the radial distance r and the polar angle θ :

$$r = a + b\theta \tag{1}$$

Where r is the radial distance from the pole to a point on the solenoid, θ is the polar angle in radians, a is the initial radius of the solenoid if starting from the origin, and $a = 0$, b are constants related to the pitch P .

The pitch P is the increase in radial distance r for each revolution of the screw thread (i.e., rotation of 2π radians), in this case $P = 0.55m$, so that the constant b associated with the pitch P can be solved for:

$$P = r(\theta + 2\pi) - r(\theta) = b(\theta + 2\pi) - b(\theta) = 2\pi b \tag{2}$$

Substituting the value of the pitch P solves for $b = \frac{0.55}{2\pi}$, and since the Archimedean spiral in this problem starts at the origin $a = 0$, it ultimately yields the polar equation for the Archimedean spiral:

$$r = \frac{0.55}{2\pi}\theta \tag{3}$$

It is known that the centres of the handles of the bench dragon are on the Archimedean solenoid and that the centre of the handles in front of the dragon's head maintains a rate of advancement along the solenoid $1m/s$ so that the position of the dragon's head can be determined before solving for the position of the whole dragon dance team by recursive thinking.

Firstly this paper constructs the polar coordinate system, the linear velocity v in the polar coordinate system is a combination of radial and angular components ([2]):

$$v = \sqrt{\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\theta}{dt}\right)^2} \tag{4}$$

Where $\frac{dr}{dt}$ denotes the radial component of linear velocity and $r\frac{d\theta}{dt}$ denotes the angular component of linear velocity[10, 11].

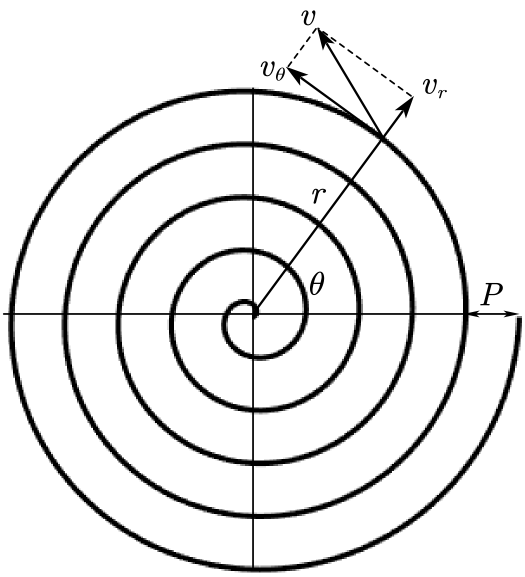

Figure 3. Schematic of velocity synthesis at any point on the Archimedean spiral

As shown in Fig. 3, since the centre of each handle of the bench dragon is always on the screw line r and θ always satisfies the polar coordinate equation of Archimedes' screw line, the relationship between the radial velocity $\frac{dr}{dt}$ and the angular velocity $\frac{d\theta}{dt}$ can be obtained by conducting the derivation of both sides of Eq. (3):

$$\frac{dr}{dt} = \frac{0.55}{2\pi}\frac{d\theta}{dt} \tag{5}$$

Substituting equation (5) into the equation for linear velocity (4) and collating gives:

$$v_1 = \sqrt{\left(\frac{0.55}{2\pi}\frac{d\theta_1}{dt}\right)^2 + \left(\frac{0.55}{2\pi}\theta_1\frac{d\theta_1}{dt}\right)^2} \tag{6}$$

Also because the linear velocity at the centre of the handle in front of the faucet is constant, i.e. $v_1 = 1m/s$, substituting into equation (6) and collating gives:

$$\frac{d\theta_1}{dt} = \frac{2\pi}{0.55\sqrt{1+\theta_1{}^2}} \tag{7}$$

By solving the above differential equation, the polar coordinates of the centre of the faucet's front handle at any given moment can be determined(θ_1, r_1).

The whole dragon dance team in the process of advancing, as shown in Fig. 1, the centre of the front handle of the dragon head and the centre of the front handle of the first section of the dragon body the straight line distance between the centre of the front handle of the dragon head and the centre of the first section of the dragon body$L = 2.86m$, the formula of the straight line distance between the two points under the polar coordinate is defined as

$$L = \sqrt{r_1{}^2 + r_2{}^2 - 2r_1r_2\cos(\theta_1 - \theta_2)} \tag{8}$$

Wherer_1 andr_2 are the radial distances between the two centre points,θ_1 andθ_2 are the polar angles between the two centre points, and$L$ represents the straight line distance between the two centre points.

And because the two centre points(θ_1, r_1)、(θ_2, r_2) are located on the Archimedean solenoids, the equation$r = \frac{0.55}{2\pi}\theta$ is satisfied and is obtained by association with equation (8):

$$\theta_1{}^2 + \theta_2{}^2 - 2 \cdot \theta_1 \cdot \theta_2 \cos(\theta_1 - \theta_2) = \frac{L^2}{b^2} \tag{9}$$

Where $L = 2.86m$ $b = \frac{0.55}{2\pi}$ since the polar position of the centre of the front handle of the dragon's head has been obtained (θ_1, r_1), the polar position of the centre of the front handle of the first section of the dragon's body can be solved by substituting into equation (9).

The above relationship can be determined in the same way to determine the determination of the remaining position of the dragon dance team, extended to the general, it is known that the entire dragon dance team is in the process of advancing, as shown in Figure 2, the former section of the dragon body and the latter section of the dragon body (dragon tail) of the front handle centre of the straight line distance is always constant $L = 1.65m$ $b = \frac{0.55}{2\pi}$ and the two centre points (θ_n, r_n)、(θ_{n+1}, r_{n+1}) are satisfied with the Archimedean solenoidal equations, and equation (9) associated with the collation of the obtained:

$$\theta_n{}^2 + \theta_{n+1}{}^2 - 2 \cdot \theta_n \cdot \theta_{n+1} \cos(\theta_n - \theta_{n+1}) = \frac{L^2}{b^2} (n = 2, 3, \cdots, 223) \tag{10}$$

Since the polar coordinate position information of the front handle centre of the first section of the dragon body is known, the polar coordinate position from the front handle centre of the second section of the dragon body to the rear handle centre of the dragon's tail can be obtained recursively through equation (10), and subsequently converted to Cartesian coordinates using the coordinate conversion formula from polar to Cartesian coordinates:

$$\begin{cases} x = r(\theta)\cos\theta \\ y = r(\theta)\sin\theta \end{cases} \tag{11}$$

The Cartesian position coordinates of the entire dragon dance team can be found.

The bench dragon position transfer model established above can be used to establish the bench dragon speed transformation model, and in this way, the position coordinates of the whole dragon dance team at any moment can be obtained by collating equation (6):

$$v_n = -b \frac{d\theta_n}{dt} \sqrt{1 + \theta_n{}^2} \tag{12}$$

By solving the differential equations using the numerical difference method, the velocity information of the whole dragon dance team at any moment can be obtained.

3. Results and discussion

Firstly, the nonlinear differential equation of θ_1 about t is solved by the numerical differential equation solver $ode45$ in MATLAB, so as to obtain the polar angle corresponding to the centre of the faucet front handle at different moments, and then, combined with the parametric equation of the Archimedean solenoid and the formula of the Euclidean distance determined in the positional transfer model, the polar position corresponding to the centre of the faucet front handle at every interval of $1s$ is determined at the location of (θ_i, r_i) starting from the initial moment until the end of the first $300s$. Since equation (10) is a more complex nonlinear equation and it is difficult to find an analytical solution, the numerical iteration method is used to gradually approximate the solution of this nonlinear equation. By calling the $fsolve$ function in MATLAB, the results obtained by the $fsolve$ function depend on the initial guessing value, and the selection of the initial guessing value has a large impact on the accuracy of the solution results, so the selection of the initial guessing value is very important, after a lot of testing, this paper selects $\theta_n + 0.01$ as the guessing solution for solving

θ_{n+1}, which can quickly approximate the solution and also avoid missing the exact solution with a large step size, and set the following constraints at the same time:

Firstly, the polar angle corresponding to the centre of the front handle of the preceding dragon body θ_n shall be less than the polar angle corresponding to the centre of the front handle of the following dragon body θ_{n+1} Secondly the difference between the corresponding polar diameters of the centres of the front handles of the adjacent benches Δr shall be less than half of the pitch P.

If the solution obtained θ_{n+1} deviates far from the actual solution, the results are made more accurate by adjusting the initial guess value appropriately, using $optimoptions$ to adjust the tolerance, iteration number and other parameters, and finally finding the position coordinates of the whole dragon dance team at the same moment.

The position information of the whole dragon dance team from 0s to 300s is further solved. In the previous section, the position information of the centre of the front handle of the dragon head is obtained from 0s to 300s. By taking the polar coordinates of the centre of the front handle of the dragon head at each moment as the initial value and importing them into the $fsolve$ function in MATLAB for subsequent operations, and so on, the position information of the whole dragon dance team from 0s to 300s can be obtained. The position information of the whole dragon dance team in the 300s is shown in Fig. 4, which is the final coordinate position conversion and dynamic drawing.

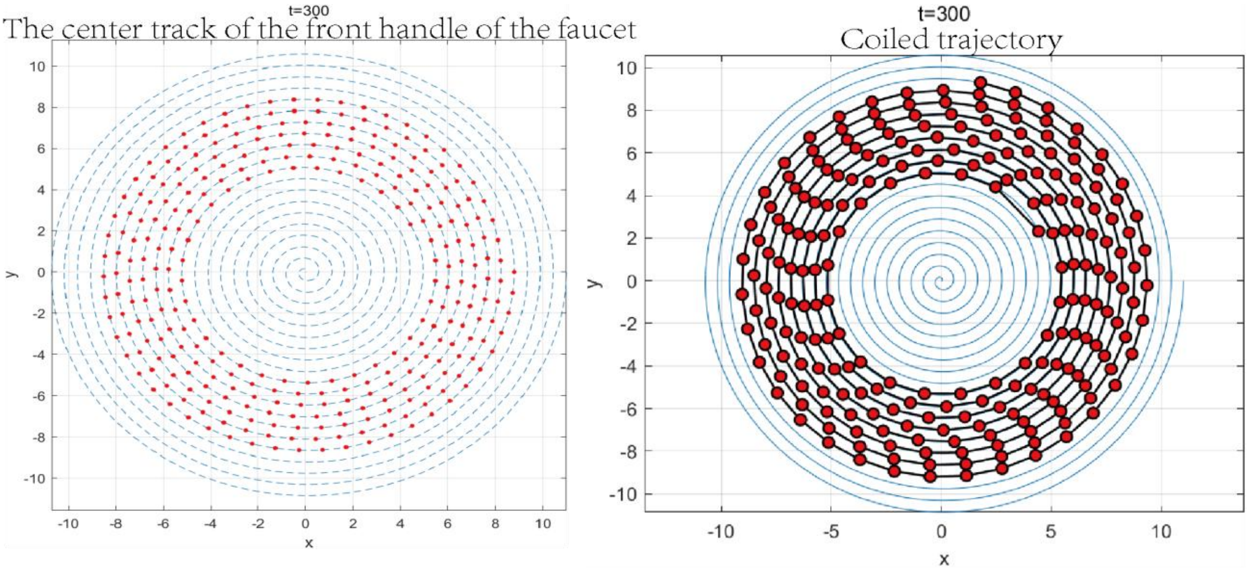

Figure 4. Coiled trajectory

4. Conclusions.

Firstly, the Archimedean solenoidal equation in polar coordinates is determined, and the Euclidean distance between the centres of the adjacent handles of the bench is fixed, then this paper establishes the differential equation for the transfer of the position of the dragon's head with constant linear velocity in polar coordinates according to the established solenoidal parametric equations, and solves it by the fourth-order dragon Gekuta algorithm, and then establishes a nonlinear equation on the position by the constant Euclidean distance between the dragon's head and the dragon's body, and uses the recursive idea, the solve function in Matlab is used to solve the position information of each hand under different moments of the dragon dance team, and the position information of each hand at each moment is obtained. At the same time, for the position information sampled under different time nodes, the numerical difference method is used to carry out the centre difference, discretize the angle θ, so as to approximate the rate of change of speed, and derive the speed information under different points of the handles. Finally, a series of positional velocity data of different positions of the dragon dance team with a time step of 1 is derived, and at the 120th s, the coordinates of the 51st section of the dragon body are (-5.54m, 6.38m), and the velocity is 0.99 m/s. The velocity accuracy of the solution in this paper is not high enough, but it can be improved in the future from the perspective of better-solving methods and models.

This paper provides a research idea and framework applied to the field related to kinematic analysis to solve the problems related to bench dragons based on Archimedean solenoids, proving the feasibility of the method.

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