Optimization Design of the Coiling Dragon Dance Movement

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Abstract. The Coiling Dragon is an important traditional folk cultural activity, and studying the movement of the coiling dragon dance team can enhance its visual appeal. This paper establishes a mathematical model for the coiling dragon based on isometric spiral motion, aiming to improve the scientific design of such traditional activities through mathematical models and optimization methods. The model analyzes the position and speed of each segment of the coiling dragon through geometric relationships and recursive formulas. It determines that at 300 seconds, the specific positions and velocities of all handles, among which the coordinates of the dragon head are (4.425059, 2.324049), the dragon tail's coordinates are (1.784408, 9.301190), and the speed of the dragon tail is 0.999910 m/s, with a slight velocity loss observed across successive segments. Additionally, the paper establishes a single-objective optimization model for the optimal pitch, solving it using a genetic algorithm, with the optimal pitch being 0.4386 m and the error 0.009780. This optimal configuration enables the coiling dragon to maneuver into the turning space through the most compact motion path, achieving a feasible and visually optimal turnaround. The proposed model and methodology effectively describe the coiling dragon's motion and provide scientifically grounded guidance for selecting motion parameters in coiling dragon performances.

Keywords: Isometric spiral motion, Genetic Algorithm, Coiling Dragon Dance Activity.

1. Introduction

The Bench Dragon Dance is a highly representative performance in Chinese traditional folk culture, especially in festivals in regions such as Fujian and Zhejiang. It consists of multiple "bench" segments connected together,also called the bench dragon, and requires collaboration from many performers to simulate the dragon's winding and leaping posture. However, the fluidity and visual appeal of the Bench Dragon Dance heavily depend on the rationality of the path planning. Traditional empirical designs can lead to issues such as insufficient space for turning and rigid movements, which urgently require scientific design through mathematical models and optimization methods. In recent years, the rapid development of computational kinematics and swarm intelligence has provided new ideas for quantitatively analyzing traditional folk activities. This paper establishes a bench dragon model based on isometric spiral motion to study this traditional cultural activity.

Currently, many scholars have conducted related research on isometric spiral motion. For example, Peng Zhang and others, based on the three-dimensional movement of biological snakes, studied the spiral climbing motion of snake-like robots on cylindrical surfaces and established a multi-objective optimization function for the robot's joint modules to optimize design parameters[1]. Yihan Xu and others proposed a dual-module fusion path planning algorithm to address the potential collision issues among multiple robots in limited space. They studied the spiral motion of UAVs and conducted simulation experiments[2]. Zihao Wang and others constructed a system based on OPA and used an equidistant spiral search algorithm to retrieve motion targets, improving search efficiency, and obtained an RMSE of around 10 for target image reconstruction with a sampling rate of 1[3]. Jhon Jasper D. Apan and others used computational fluid dynamics (CFD) to determine how the anastomosis angle affects the spiral flow-induced transplant design's hemodynamics, improving bypass graft performance and patency, with a 30° anastomosis angle producing the most favorable flow conditions[4]. Dengbiao Liu and others, addressing the dynamics of pipe-climbing robots in space, used an improved Lagrange-Newton-Euler method to establish multiple dynamic models, including the spiral motion of particle robots around the pipe's axis[5]. Guoqiang Dun and others improved the structure of spiral fertilizer discharge mechanisms to enhance filling rate and uniformity, optimizing the structure through EDEM simulation and response surface experiments[6]. Liu Shaobo and others developed a broadband magnetic field sensor based on an Archimedean spiral structure, significantly increasing the sensor's sensitivity and reducing amplitude errors by 0.2%[7]. Yan Zhao and others designed a chiral metamaterial absorber based on an Archimedean spiral structure, offering large-angle incidence stability and 360° polarization insensitivity[8]. Teguh Prasetyo and others studied the effect of the tilt angle of an Archimedean spiral pump driven by a Savonius Helical Windmill on water flow, finding that the fastest water-filling speed occurred at a 48º tilt angle, filling 1000 ml of water[9]. Dai Yuanjun and others explored the relationship between the structure and aerodynamic performance of an Archimedean Spiral Wind Turbine (ASWT) and proposed an improved ASWT that enhanced aerodynamic performance by 16.88%[10].

While there is abundant literature on isometric spiral motion, very few papers have applied this to traditional folk activities like the Bench Dragon Dance. This paper, based on isometric spiral motion, establishes a mathematical model for the bench dragon movement and uses a genetic algorithm to solve for the optimal pitch, thus enabling the scientific design of the Bench Dragon Dance performance.

2. Methods

2.1. Determining the Position and Speed of the Bench Dragon

Let the bench dragon consist of *n* segments, where the first segment is the dragon head, the middle segments are the body, and the last segment is the dragon tail. The dance team moves along a spiral with a pitch of *p* and an initial radius of *r* in a clockwise direction. A rectangular coordinate system is established with the center of the spiral as the origin. This spiral is also known as an Archimedean spiral and can be represented by the following polar coordinate equation:

$$\rho(\theta) = \rho_0 + \frac{p}{2\pi} \cdot \theta \tag{1}$$

At the initial moment, the angle of the dragon head front handle is denoted as θ_0, and its movement velocity is denoted as v_0. The position of the front handle at time t can be expressed as:

$$\begin{cases} x_0(t) = [\rho_0 + \frac{p}{2\pi}(\theta_0 - \frac{2\pi v_0 t}{p}] \cdot \cos(\theta_0 - \frac{2\pi v_0 t}{p}) \\ y_0(t) = [\rho_0 + \frac{p}{2\pi}(\theta_0 - \frac{2\pi v_0 t}{p}] \cdot \sin(\theta_0 - \frac{2\pi v_0 t}{p}) \end{cases} \tag{2}$$

Let the distance between the front and rear handles of the dragon head be L_a, and the distance between the front and rear handles of the dragon body and dragon tail be L_b. The front and rear handles of each dragon body segment and tail are connected by handles, so:

$$\theta_i(t) = \theta_0(t) + \frac{2\pi L_a}{p} + (i-2)\frac{2\pi L_b}{p} \tag{3}$$

At time, when $i = 1, 2, \cdots, n-2$, $\theta_i(t)$ denotes the angle of the front handle of the i-th dragon body segment, and when $i = n-1$ and $i = n$, $\theta_i(t)$ denote the angles of the front and rear handles of the dragon tail at time t.

Therefore, the coordinates of each dragon body segment and the dragon tail can be expressed as:

$$\begin{cases} x_i(t) = [\rho_0 + \dfrac{p}{2\pi}\theta_i(t)] \cdot \cos\theta_i(t) \\ y_i(t) = [\rho_0 + \dfrac{p}{2\pi}\theta_i(t)] \cdot \sin\theta_i(t) \end{cases} \tag{4}$$

The velocity of the front handle of the dragon body segment is represented by the rate of change of its position at each moment. Since the positions of the front handles of the dragon body segments and the front and rear handles of the tail can be determined at each time, the velocity at any given moment can be calculated by examining the position change between adjacent moments. Let $v_i(t)$ denote the velocity of the front handle of the i-th dragon body segment at time t, then:

$$v_i(t) = \frac{\sqrt{(x_i(t+\Delta t) - x_i(t))^2 + (y_i(t+\Delta t) - y_i(t))^2}}{\Delta t} \tag{5}$$

where Δt represents an infinitesimal time interval.

2.2. Determination of the Minimum Pitch

This paper need to determine the minimum pitch that allows the dragon head's front handle to spiral into the turning boundary along the corresponding spiral line. Let the turning space be a circle centered at the origin with a radius r. If (x_{cir}, y_{cir}) is a point on this circle, the slope of the tangent line at this point can be calculated as:

$$k_{rcir} = -\frac{x_{cir}}{y_{cir}} \tag{6}$$

For the spiraling line, when the angle corresponding to a point on the spiral line is θ, the derivative can be calculated using the chain rule:

$$k_{spi} = \frac{\dfrac{p}{2\pi}\sin\theta + (\rho_0 + \dfrac{p}{2\pi}\theta)\cos\theta}{\dfrac{p}{2\pi}\cos\theta - (\rho_0 + \dfrac{p}{2\pi}\theta)\sin\theta} \tag{7}$$

The rotational angle corresponding to the intersection point between the turning space and the spiral satisfies:

$$\theta_{\tan} = \frac{2\pi(r - \rho_0)}{p} \tag{8}$$

Thus, the coordinates of this point are:

$$\begin{cases} x_{\tan} = r\cos\dfrac{2\pi(r-\rho_0)}{p} \\ y_{\tan} = r\sin\dfrac{2\pi(r-\rho_0)}{p} \end{cases} \tag{9}$$

By substituting equation (8) into equation (7) and equation (9) into equation (6), the tangent slopes of the spiral and the circle at the intersection point, denoted as k_{spi} and k_{rcir}, can be determined. These two slopes must be equal, so the paper have:

$$\frac{\dfrac{p}{2\pi}\sin\dfrac{2\pi(r-\rho_0)}{p} + r\cos\dfrac{2\pi(r-\rho_0)}{p}}{\dfrac{p}{2\pi}\cos\dfrac{2\pi(r-\rho_0)}{p} + r\sin\dfrac{2\pi(r-\rho_0)}{p}} = -\cot\frac{2\pi(r-\rho_0)}{p} \tag{10}$$

To determine the minimum pitch that allows the dragon head's front handle to spiral into the given turning space boundary, this paper set the objective function as:

$$\min\ p \tag{11}$$

In addition to the condition in equation (10), the pitch cannot be too small, or it will be difficult to perform the Dragon Dance activity. Also, the pitch should not be too large, as it would limit the number of turns that can be made within a limited area, reducing the visual appeal. Therefore, the following constraints are applied:

$$p_0 \le p \le p_1 \tag{12}$$

where p_0 and p_1 are the minimum and maximum pitch values, respectively.

Thus, this paper obtain the single-objective optimization model to determine the minimum pitch:

$$\min\ p$$
$$s.t.\begin{cases} \dfrac{\frac{p}{2\pi}\sin\frac{2\pi(r-\rho_0)}{p}+r\cos\frac{2\pi(r-\rho_0)}{p}}{\frac{p}{2\pi}\cos\frac{2\pi(r-\rho_0)}{p}+r\sin\frac{2\pi(r-\rho_0)}{p}}=-\cot\dfrac{2\pi(r-\rho_0)}{p} \\ p_0 \le p \le p_1 \end{cases} \tag{13}$$

2.3. Genetic Algorithm

The Genetic Algorithm (GA) is an optimization method based on natural selection and genetic principles. It is a global search algorithm that can effectively handle complex nonlinear problems. GA simulates the biological evolution process in nature (such as selection, crossover, and mutation) to find the optimal solution from a population of candidate solutions.

Each candidate solution is represented as an "individual," usually encoded to represent the individual's characteristics. A group of individuals forms a "population," and the population size is usually kept fixed. The fitness function is used to assess the quality of each individual. The higher the fitness function value, the greater the probability that the individual will survive and reproduce.

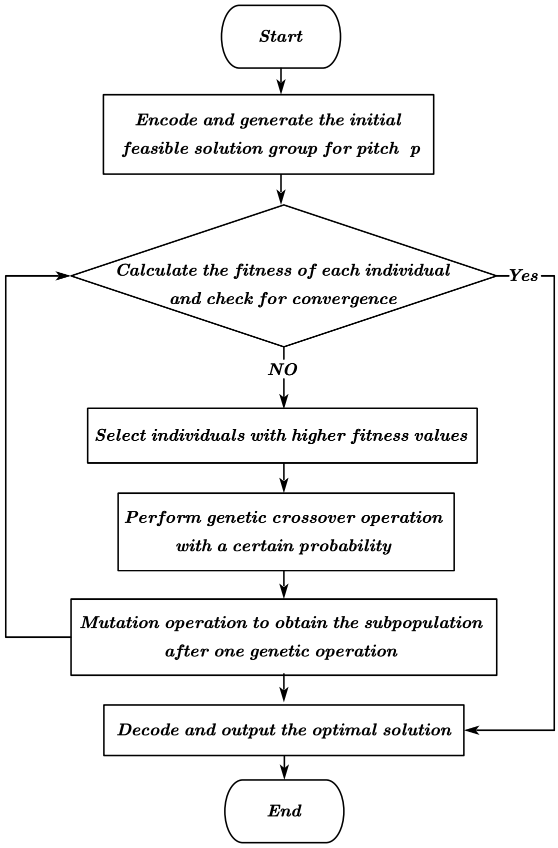

Figure 1. Genetic Algorithm Solution for the Minimum Pitch

Let D be the dimension of the optimization parameters, and N be the population size. Each individual x_i is represented as:

$$x_i = [x_{i1}, x_{i2}, \cdots, x_{iD}], i = 1, 2, \cdots, N \tag{14}$$

The fitness function $f(x_i)$ is defined by the error function $E(x_i)$:

The selection x_i probability of individual p_i is:

$$f(x_i) = \frac{1}{1 + E(x_i)} \tag{15}$$

The selection x_i probability of individual p_i is:

$$p_i = \frac{f(x_i)}{\sum_{k=1}^{N} f(x_k)} \tag{16}$$

The cumulative probability $q_i = \sum_{k=1}^{i} p_k$ is used to generate the mating pool through random sampling. For selected parent individuals x_p and x_q, arithmetic crossover is applied to generate offspring y_p and y_q:

$$y_p = \alpha x_p + (1-\alpha) x_q, y_q = (1-\alpha) x_p + \alpha x_q \tag{17}$$

where $\alpha \in [0,1]$ is the crossover coefficient. For individual x_i, each parameter has a probability P_m of undergoing mutation:

$$x'_{ij} = x_{ij} + \sigma \cdot N \ (0,1) \tag{18}$$

where σ is the mutation step size, and $N\ (0,1)$ is the standard normal distribution. The iteration stops when the maximum number of iterations is reached or when the change rate of the optimal fitness is less than the threshold ε. The process of using the genetic algorithm to solve the minimum pitch is shown in Figure 1.

3. Experimental

3.1. Model Solution

The initial position of the dragon head is at (8.8, 0) on the 16th loop of the spiral. The following parameters are substituted into the model: the length of the dragon head board $L_a = 3.41m$, the length of the dragon body board $L_b = 2.2m$, the pitch $p = 0.55m$, the speed of the dragon head $p = 0.55m$, and $\theta_0 = 32\pi(rad)$. The solution is obtained through MATLAB programming. The motion trajectory of the bench dragon is shown in Figure 2.

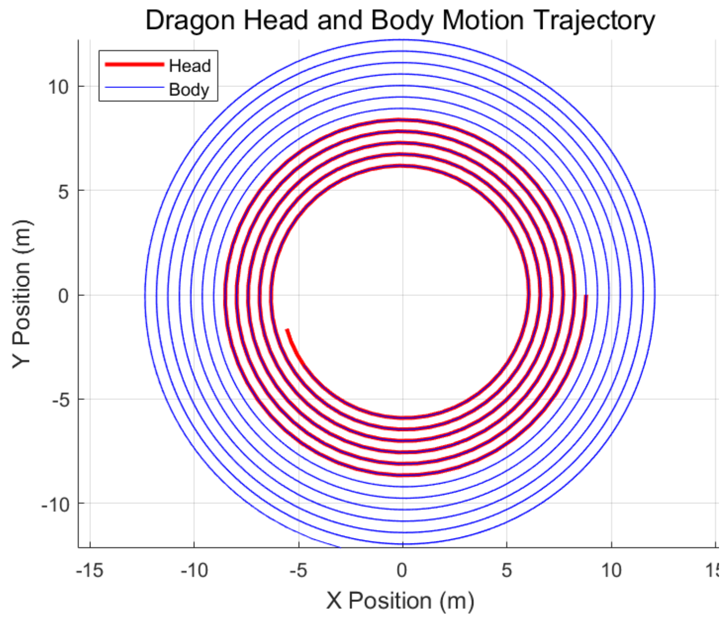

Figure 2. The Motion Trajectories of the Dragon Head and Entire Dragon Body

In this paper, the final moment, 300 seconds, is chosen to determine the specific positions and velocities of the front handle of the dragon head, as well as the front handles of the 1st, 51st, 101st, 151st, and 201st sections of the dragon body, and the rear handles of the dragon's tail, as shown in Table 1.

Table 1. Specific Position and Velocity of the Particular Handle at Final Time

	Dragon Head	1st	51st	101st	151st	201st	Dragon Tail
x(m)	4.425059	2.459624	-6.304733	-6.219842	7.044951	-7.470060	1.784408
y(m)	2.324049	4.400114	0.464300	3.928487	4.372673	-5.266860	9.301190
v(m/s)	1.000000	0.999985	0.999970	0.999955	0.999940	0.999925	0.999910

The optimal pitch is solved using the following algorithm steps: First, determine the initial parameter values, set the initial radius ρ_0, the radius of the circular region r, the range of the pitch, and the initial pitch value p. Next, the MATLAB ga function is used to uniformly and randomly initialize the individuals in the population, where each individual corresponds to a pitch value p. A specified number of n chromosomes are randomly generated as the initial population. Based on the error calculation method, the fitness of each individual p is computed; the smaller the error, the greater the fitness. Then, it is checked whether the convergence condition has been met. If not, selection is performed, where individuals with higher fitness are chosen for crossover operations, with the selection probability for each individual being P_i.Next, crossover operations are performed: two distinct individuals p are randomly selected from the population and, with a certain probability, their genes are exchanged to produce two new individuals p. This process is repeated $n/2$ times, generating a new population made up of the newly formed individuals. Following this, mutation operations are performed: a random individual p is selected from the population and, with a certain probability, undergoes chromosomal mutation to generate a new population. This new population is treated as a subpopulation that has undergone one genetic operation. At this point, convergence is checked again. The steps are repeated until the convergence condition is met. The encoding is then converted into the actual value p, and the optimal individual value p, along with its corresponding fitness and error, is output.

To minimize the uncertainty of the genetic algorithm's results as much as possible, running the algorithm multiple times can ensure that the results are stable. The optimal solution obtained in each run is recorded in Table 2.

Table 2. Genetic Algorithm Results for Solving the Minimum Pitch

Serial Number	Pitch	Error	Number of Iterations
1	0.4000	0.014147	51
2	0.4615	0.000630	67
3	0.4386	0.009780	65
4	0.4389	0.017360	69
5	0.4185	0.011248	67

Since the objective function requires the pitch to be as small as possible, in cases where the errors are similar, the value with the smallest pitch, $p_{\min} = 0.4386$, is selected, with an error of 0.009780.

3.2. Experimental Result Analysis

As shown in Figure 3, 0.4386 meters is selected as the optimal pitch. At this pitch, it is the smallest, and during the actual dragon dance, more turns can be made within the same area, resulting in a better visual effect. The turns are quicker and smoother, and the overall movement of the dragon is more compact and aesthetically pleasing. Although the error for this pitch is slightly larger than that of 0.4615 meters, it is still within the 0.001 range, which is not significant enough to affect the turning space during the actual dragon dance.

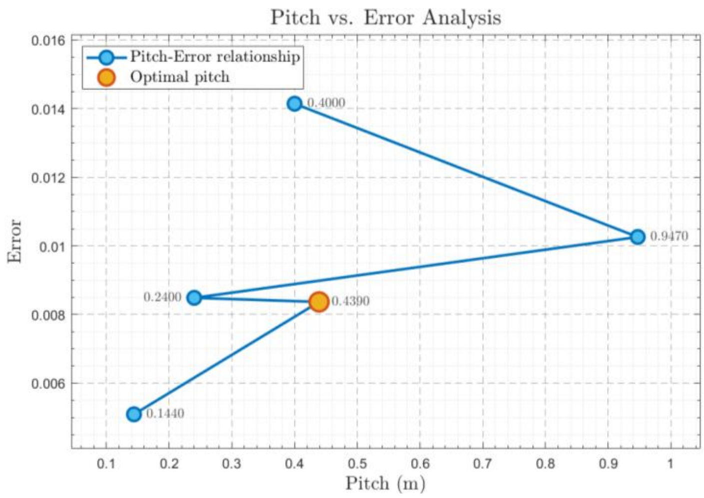

Figure 3. The Relationship Between Pitch and Error

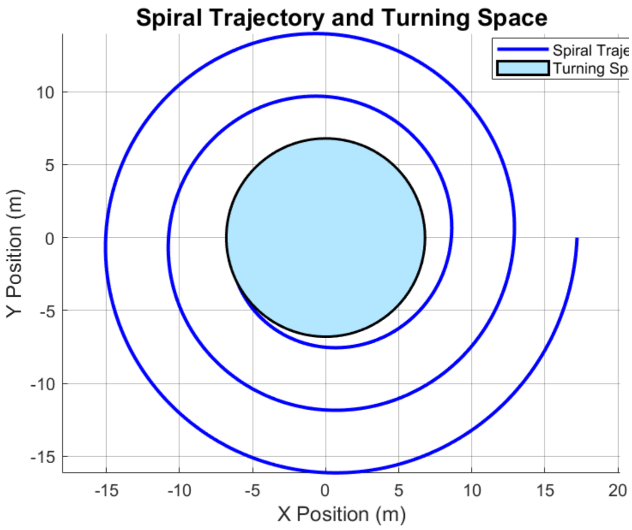

Figure 4. Tangent Point Between the Dragon Dance Trajectory and the Turning Space Boundary

As shown in Figure 4, when 0.4386 meters is selected as the pitch, the dark blue spiral trajectory represents the dragon's path, and the light blue area indicates the turning space with a diameter of 9 meters. At this point, the dragon's head coordinates are (-3.54, -2.38). During the actual dragon dance, the bench follows the most compact motion path toward the turning space, and the trajectory circles gradually reduce in size as they approach the boundary of the turning space. Eventually, the path enters the turning space, completing a feasible and highly aesthetic turn.

3.3. Sensitivity Analysis

By changing the radius r of the turning space circular area while keeping other parameters fixed, the effect of different radii on the optimal pitch is observed, as shown in the following figure:

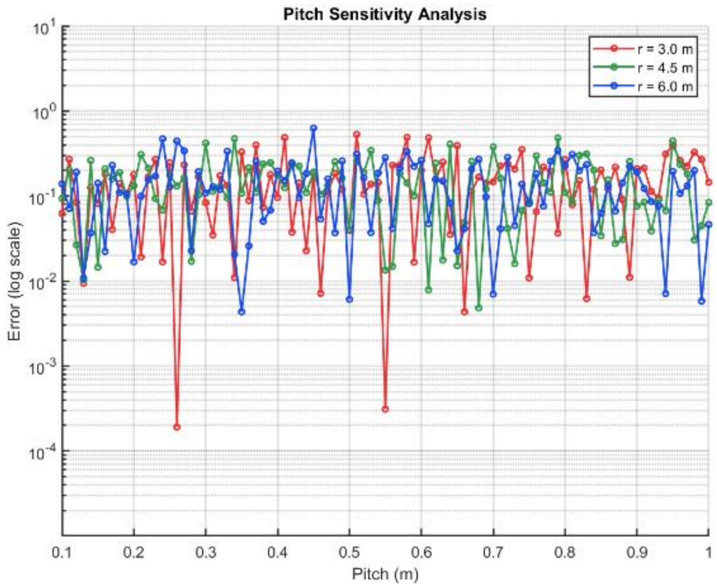

Figure 5. Sensitivity Analysis of Pitch and Error

As shown in Figure 5, the minimum error and corresponding pitch values (top five) for turning space radii of 3 meters, 4.5 meters, and 6 meters are presented. When the radius is 3 meters, the optimal pitch is 0.387 meters with an error of 0.003826. For a 4.5-meter radius, the optimal pitch is 0.439 meters with an error of 0.008370. When the radius is 6 meters, the optimal pitch is 0.774 meters with an error of 0.003826.

By analyzing the overall trend, it is evident that as the turning space radius increases, the optimal pitch also increases. This indicates that larger turning spaces, typically found in larger areas, allow for a larger pitch to maintain a smaller error.

In summary, as the radius of the turning space increases, the running pitch of the bench dragon team can be appropriately increased to maintain a smaller error. However, for smaller turning space radii, a smaller pitch should be chosen to enhance visual appeal.

4. Conclusions

This paper first establishes the bench dragon motion model, which determines the equidistant spiral equation of the path based on the initial position of the dragon head. It then defines the expression for the dragon head's position over time based on its traveling speed. Using the distance between the current position of the dragon head and the bench handles, the iterative recursive expressions for the remaining handles are determined. The velocity of each handle is calculated by taking the derivative of the displacement with respect to time. The paper provides the specific position and velocity of the dragon head's front handle and the front handles of the 1st, 51st, 101st, 151st, and 201st sections of the dragon body, as well as the rear handles of the dragon's tail at the 300s mark.

Based on this, the tangent slope expressions for a point on the spiral and a point on the circle are derived using the chain rule. By solving the system of equations, the rotational angle corresponding to the intersection point of the spiral and the circle is calculated and transformed into the rectangular coordinate system. This leads to the tangent constraint condition and establishes the single-objective optimization model for determining the minimum pitch. Genetic algorithm is then used to solve the model. As a result, five minimum pitch values are obtained, and 0.4386m is chosen as the minimum pitch with an error of 0.009780.

Additionally, the tangent point coordinates for the dragon head's entry into the turning space are given as (-3.54, -2.38). A sensitivity analysis of the turning space radius is conducted, revealing that as the radius of the turning space increases, the spiral pitch for the bench dragon team can be slightly increased to maintain a smaller error. However, when the radius of the turning space is smaller, a smaller pitch should be chosen to enhance the visual appeal.

This paper addresses the motion problem of the bench dragon. Based on mathematical derivation and using knowledge from plane geometry, linear algebra, and kinematics, the position and velocity determination model for each section of the bench dragon is established. The results are analyzed, and sensitivity analysis is conducted, ensuring a rigorous and comprehensive approach. Genetic algorithm is ultimately chosen for solving the model, as it helps overcome the obstacles often encountered in traditional search and optimization algorithms and is more likely to find the global optimal solution. However, the model assumes that the entry and exit paths are equidistant spirals, which may be too simplistic and idealized to fully reflect the complex path situations in real life. Additionally, the model only considers the two-dimensional motion of the dragon team, and does not capture the potential vertical movement of the dragon team on the field, especially the more flexible and free motion of traditional dragon dance. Therefore, this model can be extended to a three-dimensional spiral path.

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