A Resonant Tube Strength Static Stress Analysis and Modal Analysis

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Abstract. The structural performance of the resonator tube, as a core component of the thermoacoustic system, has a direct impact on the system performance. The study of the structural strength, stability, and dynamic characteristics of the resonator tube helps to comprehensively assess its performance and promote the development of thermoacoustic technology. In this paper, an asymptotic resonator tube made of stainless steel is analyzed under static stress and modal conditions. Static stress analysis identifies stress concentration areas, providing critical insights to avoid fatigue failure and improve structural longevity. Modal analysis reveals vibration modes and intrinsic frequencies closely aligned with operational requirements, highlighting potential resonance risks. These findings underscore the importance of synergistic structural-acoustic design and lay a foundation for future experimental validation and multiphysics coupling analysis, ultimately advancing thermoacoustic system efficiency and reliability.

Keywords: Asymptotically expanded resonant tube, Static stress analysis, Modal analysis.

1. Introduction

With the increasingly serious energy and environmental problems, thermoacoustic technology as a new energy conversion and utilization technology has attracted more and more attention, and there have been a series of advances based on this technology in recent years in the fields of natural gas power generation, industrial thermoacoustic generator, liquid nitrogen temperature zone refrigeration device and so on [1]. Thermoacoustic refrigeration is a new refrigeration technology in the 21st century. Compared with the traditional compression refrigeration system, thermoacoustic refrigeration machine has superior characteristics [2]. Thermally-driven thermoacoustic refrigerators can be applied in fields such as waste heat cooling and solar cooling, and are of great significance for energy conservation and emission reduction. Therefore, they have received extensive attention both at home and abroad [3]. The resonant tube is one of the important components of the thermoacoustic heat engine, which maintains the required frequency of the system, stores acoustic energy for the system, and the variation of its geometrical parameters changes the amplitude and phase of the oscillating pressure, which has a significant impact on the performance of the thermoacoustic system [4]. Replacing the equal-section tube with a variable-section resonant tube can substantially increase the pressure amplitude inside the tube and suppress the phenomena such as excitation and high harmonics [5].

In recent years, scholars have made some progress in the numerical simulation and experimental study of resonant tubes.

The acoustic flow field inside the exponential tube, as a kind of variable cross-section tube, has also received wide attention. Ning Fangli [6] et al. used the finite volume method to construct a one-dimensional model capable of calculating the nonlinear standing waves inside an axisymmetric resonant tube of arbitrary shape, and the results show that there is a close relationship between the physical properties of the nonlinear standing waves inside the resonant tube and the shape of the resonant tube.Feng [7] et al. used Fluent software to simulate and analytically study the nonlinear acoustic field characteristics inside the exponentially thermoacoustic resonant tubes with 11 different shapes to obtain the relationship between the intrinsic frequency and the theoretically calculated harmonic frequency of the exponential resonant tube. Luo [8] et al. compared the exponential tube case for piston drive and loudspeaker drive using the Galyokin method. Hou [9] et al. established a one-dimensional nonlinear acoustic field model that can be used to solve the sound propagation in a tube of variable cross-section and solved it, and the results show that the exponential tube has a high acoustic radiation efficiency and suggests that the exponential tube be used in the engineering development.

This paper combines the static stress-strength analysis of resonant tubes with multi-order modal frequency identification to provide a theoretical basis for the structural-acoustic synergistic design of thermoacoustic chillers. Moreover, the static stress analysis of the resonant tube reveals the stress concentration area, which can avoid fatigue failure due to cyclic loading and prolong the device life. At the same time, this paper quantifies the intrinsic frequency of the resonant tube to provide a reference basis for experimental research and practical engineering use. When the excitation frequency is consistent with the modal vibration frequency of the gas column, the kinetic energy of the excitation can be maximally converted into the displacement potential energy of the gas microcluster, which realizes a large-scale vibration of the gas column and improves the refrigeration efficiency of thermoacoustic refrigeration machine.

2. Theoretical basis for numerical simulation of thermoacoustics

In this study, the structural dynamics and acoustic coupling analysis of the resonant tube of a thermoacoustic chiller is carried out based on ANSYS Mechanical, and the key parameters of the structural stress distribution, intrinsic frequency, and acoustic-vibration coupling characteristics are obtained by discretizing and solving the elasticity mechanics control equations (e.g., Navier's equations) and acoustic fluctuation equations by the finite element method. The process is based on the physical modeling of thermoacoustic dynamics theory and linear acoustic theory, combined with Mechanical's modal analysis module and harmonic response analysis function, which realizes the quantitative characterization of the synergistic effect of the mechanical vibration of the resonant tube and the excitation of the acoustic field, and provides a numerical validation means for the study of the structural-acoustic coupling mechanism of the thermoacoustic system. The subsequent theoretical derivation will be based on the following core equations

2.1. The thermodynamic basis of thermoacoustics

The foundation of thermodynamics is governed by the First and Second Laws of Thermodynamics. Heat can be transferred between objects or converted into mechanical energy and other forms, while the total energy within a closed system remains conserved during such processes. Specifically, the First Law of Thermodynamics states that the change in a system's internal energy equals the heat absorbed by the system plus the work done on it. This principle encapsulates the conservation of energy, ensuring that energy cannot be created or destroyed, only transformed or transferred[10]:

$$dU = \delta Q - \delta W \tag{1}$$

The Clausius formulation of the second law of thermodynamics states that heat cannot be transferred spontaneously from a low-temperature object to a high-temperature object, and that the entropy of an isolated system never decreases in the course of nature. That is:

$$dS \geq \delta Q / T \tag{2}$$

2.2. The Acoustic Fundamentals of Thermoacoustics

For the acoustic flow field in a resonant tube, based on the small-amplitude assumption and rigid boundary conditions, the first-order acoustic governing equations for key physical parameters can be derived by linearizing the Navier-Stokes equations and energy equation[11]:

$$\text{Sound pressure equation: } p = p_m + Re[p_1(x)e^{iwt}] \tag{3}$$

$$\text{Velocity equation: } u = Re[u_1(x, y, z)e^{iwt}] \tag{4}$$

Temperature equation: $T = T_m + Re[T_1(x,y,z)e^{iwt}]$ (5)

Density equation: $\rho = \rho_m + Re[\rho_1(x,y,z)e^{iwt}]$ (6)

Displacement equation: $s = s_m + Re[s_1(x,y,z)e^{iwt}]$ (7)

2.3. The Fundamentals of Fluid Mechanics in Thermoacoustics

The basic hydrodynamic equations in thermoacoustics consist of three main governing equations: the continuity equation (conservation of mass), the Navier-Stokes equations (conservation of momentum), and the equation of conservation of energy.

Continuity equation: $\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0$ (8)

Momentum equation: $\rho[\frac{\partial v}{\partial t} + (v \cdot \nabla)v] = -\nabla P + \mu \nabla^2 v$ (9)

Energy equation: $\frac{\partial}{\partial t}(\rho\epsilon + \frac{1}{2}\rho|v|^2) = -\nabla \cdot \left[-k\nabla T - v\sigma' + (\rho h + \frac{1}{2}\rho|v|^2)v\right]$ (10)

Combine the three equations to obtain the entropy equation:

$$\rho T(\frac{\partial S}{\partial t} + v \cdot \nabla S) = \nabla \cdot k\nabla T - k\frac{|\nabla T|^2}{T} + \rho T S_{gen}, S_{gen} \geq 0 \quad (11)$$

3. Static Stress Analysis

The schematic structure of 1/4 wavelength thermoacoustic heat engine is shown, the cavity geometry is shown in Figure-1 and 4000 Hz is used as the design frequency.

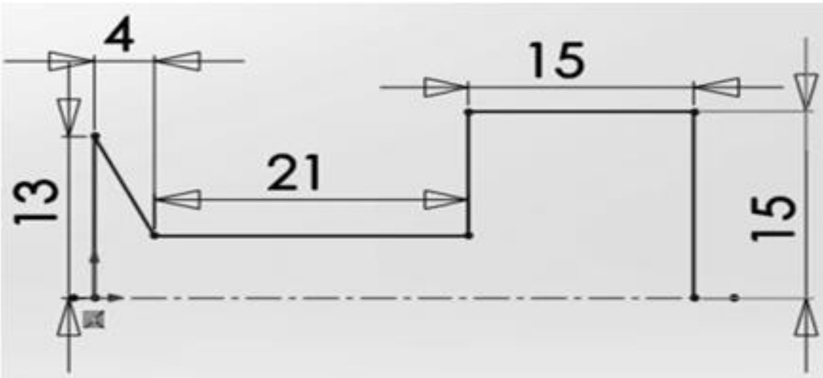

Figure 1. schematic structure

Static stress analysis is an important means to assess the structural strength and stability of the resonator tube. In this paper, the finite element analysis method is used to analyze the force on the resonator tube using ANSYS software, and its constraint point is at the end connection at the inlet, and the force direction is on the Y-axis, with the size of 500N, and the effect of the action of the effect is the surface effect. In the simulation process, stainless steel is used as the material of the resonator tube, whose modulus of elasticity is 200 GPa, Poisson's ratio is 0.28, and the density of the material is 7850 kg/m³.

Simulation results show that under normal operating conditions, the maximum stress concentration area of the resonator tube is located in the connection of the inlet end, and the maximum stress value is 22.537 Mpa.This result is known by the strength theoretical formula, the resonator tube material is Q235A, and its permissible stress is [σ]=115 Mpa. From the strength theoretical formula, it can be seen that $\sigma_{max} \leq [\sigma]$[12], the resonator tube suffers from the maximum stress is much smaller than the material's permissible stress. The maximum stress on the resonator tube is much smaller than the allowable stress of the material. In addition, the actual pressure on the surface of the inner wall of the resonator is not the same, and the maximum pressure is taken for the convenience of calculation, and the resulting stress is much smaller than the allowable stress, which indicates that the strength of the resonator is in line with the requirements, As shown in Figure 2.

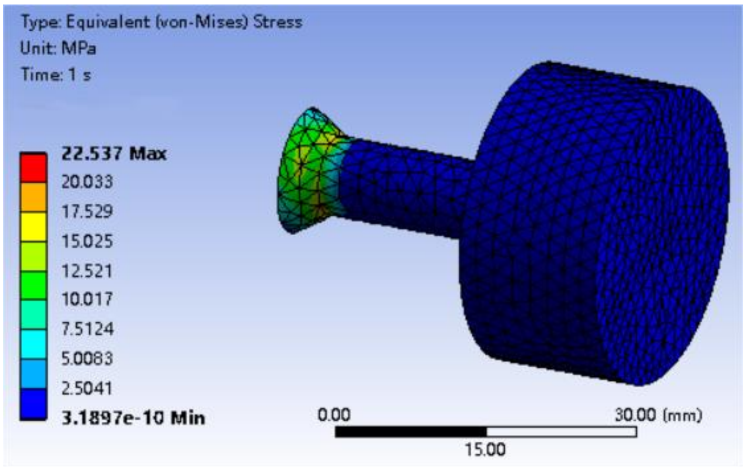

Figure 2. Static Stress Analysis

4. Modal Analysis

Modal analysis is an important method to study the dynamic characteristics of the resonant tube, which can reveal its intrinsic frequency and vibration mode characteristics. In this paper, the first six orders of the intrinsic frequency of the resonant tube are extracted by modal analysis, and its vibration mode is analyzed in detail. The simulation results are shown in Table 1 .

Table 1 Comparison of Values in different intrinsic frequency

Comparison	Power
First order	2370.2
Second order	4247.6
Third order	4249.6
Fourth order	13389
Fifth order	13400
Sixth order	15543

Resonant tube of the first and second-order vibration pattern is mainly manifested as axially symmetric vibration; the third-order vibration pattern for radial changes; the fourth and fifth-order vibration pattern for shaking changes. The second and third order resonance frequencies are 4247.6Hz and 4249.6Hz respectively, which are close to the acoustic drive frequency of 4000Hz, resulting in resonance phenomenon. (Modal simulation model are shown in Figure-3 to Figure-8)

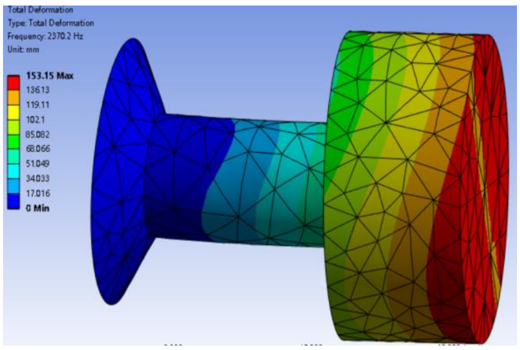

Figure 3. First order

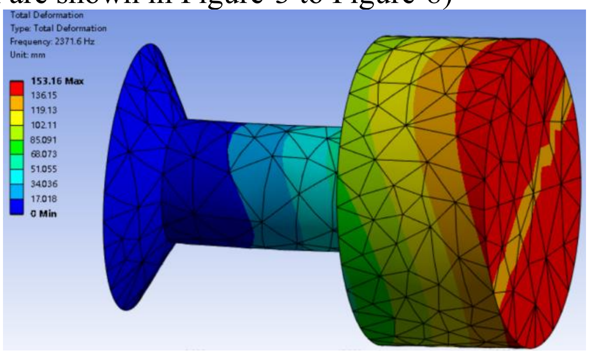

Figure 4. Second order

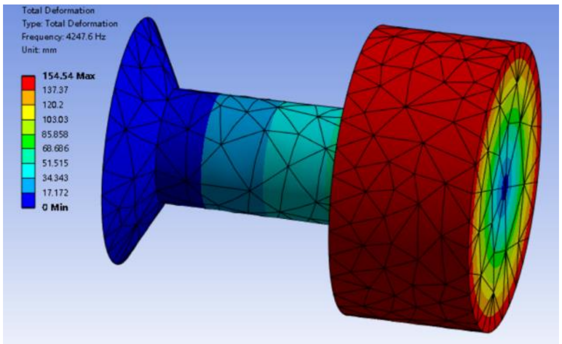

Figure 5. Third order

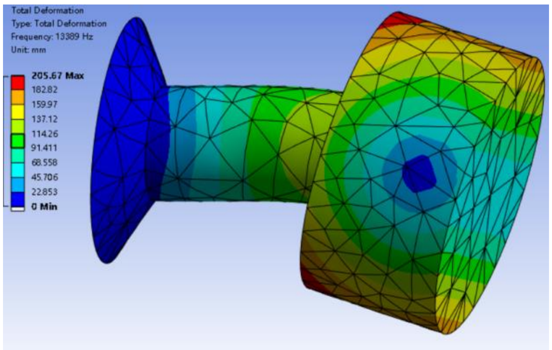

Figure 6. Fourth order

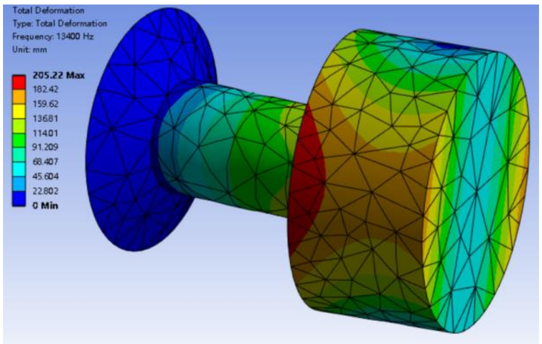

Figure 7. Fifth order

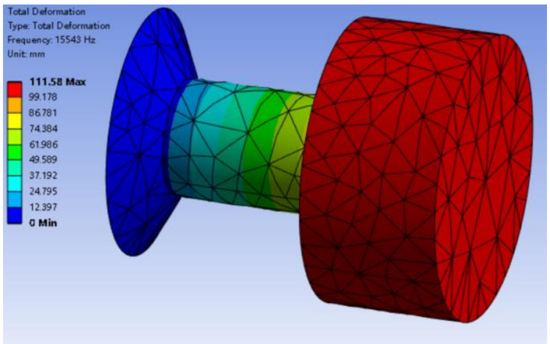

Figure 8. Sixth order

5. Conclusions

In this paper, the mechanical properties of a resonant tube are thoroughly investigated through static stress analysis and modal analysis. The results of static stress analysis show that the maximum stress concentration area of the resonant tube is located in the connection between the wall and the end, and the tapering resonant tube designed in this paper meets the stress requirements. Optimizing the end structure design is the key to improve its service life.

Meanwhile, the modal analysis reveals the vibration mode characteristics of the resonator tube and proposes several order corresponding frequencies, which correspond to the overall vibration, local bending vibration and higher order local vibration modes, respectively. The second and third order resonant frequencies obtained from the modal analysis are close to the intrinsic frequency of 4000 Hz, which is easy to produce resonance phenomenon.

Future research can further combine experimental verification and multiphysics field coupling analysis to more comprehensively evaluate the performance of the resonant tube and promote the development of thermoacoustic technology.

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