Prediction of Olympic Medal Counts Based on Multilevel Negative Binomial Regression Model and Bayesian Methods

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Abstract. The aim of this study is to accurately predict the number of medals for each country in the 2028 Summer Olympics in Los Angeles by constructing a multilevel negative binomial regression model combined with a Bayesian approach. This study focuses on predicting the medal count for each country in the 2028 Summer Olympics in Los Angeles by constructing a multilevel negative binomial regression model. The model integrates factors such as historical medal data, economic and demographic indicators, and the host country effect. A Bayesian approach, specifically the Markov Chain Monte Carlo (MCMC) method, is employed to estimate the parameters, allowing for the quantification of uncertainty and capturing heterogeneity across countries and Olympic sessions. The model not only predicts the number of medals but also provides prediction intervals to highlight countries with likely improvements or declines in their performance. This approach provides more accurate and reliable predictions compared to previous studies, offering insights for the Olympic Committee to optimize resource allocation and strategy planning for participating countries. The model's validity is confirmed through performance indicators such as mean square error and coefficient of determination, ensuring its accuracy and reliability for future predictions. By considering the complex interplay of various factors, this model offers a comprehensive framework for understanding the dynamics of Olympic medal distribution, which can be further refined and adapted for subsequent Olympic Games and other international sporting events.

Keywords: Multilevel Negative Binomial Regression Model, Bayesian Methods, Markov Chain Monte Carlo (MCMC), Uncertainty.

1. Introduction

Forecasting the number of Olympic medals is an important task in the field of sports analytics and involves a number of complex factors. The prediction model not only needs to consider the historical performance, but also needs to integrate various variables such as the economic strength of each country, the population size and the host country effect. With the advancement of data analysis techniques, traditional statistical methods have been gradually replaced by more sophisticated regression models. Multilevel negative binomial regression models have been widely used to deal with excessive dispersion in the number of medals and to better characterize the heterogeneity among different countries and Olympic Games by introducing random effects [1]. To improve the accuracy of the models, Bayesian methods and Markov chain Monte Carlo (MCMC) sampling have been used to estimate model parameters and quantify uncertainty. These methods allow the model to fully account for historical data, economic and demographic indicators, and other uncertainties when predicting future medal counts, thus supporting resource planning and strategic decision-making for the Olympic Games.

In order to better capture the hierarchical structure in the data, researchers have begun to introduce multilevel models. LeSage and Pace [2] constructed a multilevel negative binomial regression model, which takes into account the random effects of country and Olympic session, and is able to effectively deal with the problem of excessive discretization in the data. This model not only improves the accuracy of prediction, but also can quantify the effects of different factors on the number of medals. The host country effect has a significant impact on the distribution of medals. Koenigstorfer J [3] and Singleton C et al. [4] found that the host country usually receives more medals during the Olympic Games, and this effect is not only reflected in the current Olympic Games, but may also have a

sustained impact on the subsequent editions. Economic and demographic factors are important factors affecting the number of medals. W Shasha et [5] al. found that the economic strength and population size of a country have a significant compound effect on the number of medals. W Shasha et al. found that the economic strength and population size of a country have a significant compound effect on the number of medals. They revealed the mechanism by which economic, demographic, geographic and social factors work together through quantile and Tobit methods, pointing out that countries with high GDP and large populations have an advantage in medal competition, but this advantage needs to be realized through effective resource allocation and training systems. In conjunction with this, Seiler S's [6] study concludes that small sports powerhouses like Norway are typically characterized by considerable variation in medal performances from one Olympic Games to the next and a high concentration of performances in a few sports. These are important factors to consider when assessing national performance and interpreting medal counts.

Previous studies have used regression models to predict Olympic medal counts but had several limitations: first, many models failed to effectively handle the over-dispersion in medal data, leading to lower prediction accuracy. Additionally, these models did not fully capture the heterogeneity across different Olympic cycles and countries. In contrast, this work introduces a multilevel negative binomial regression model [7] combined with Bayesian methods [8], which not only effectively quantifies uncertainty but also improves the handling of heterogeneity [9], enhancing both the accuracy and reliability of the predictions.

2. Model for predicting the number of medals for each country

2.1. General framework of the medal count prediction model

In order to predict the number of medals won by each country in a particular session of the Olympic Games, we used a multilevel negative binomial regression model. This model is suitable for count-type data (e. g. , medal counts) and is effective in dealing with over-dispersion in the data (i. e. , variance greater than the mean). In addition, by introducing the random effects of country and session, the model is able to capture the heterogeneity among countries and across sessions.

(1) Response Variables and Distributional Assumptions

First, let denote the number of gold medals won by the country in the first Summer Olympic Games, $c = 1,2,\dots,C, t = 1,2,\dots,T$. Since the number of gold medals is a non-negative integer type of counting data, and there is usually an over-dispersion of the number of gold medals obtained $(\mathrm{Var}[G_{c,t}] > \mathrm{E}[G_{c,t}])$, this paper assumes that $G_{c,t} \sim \mathrm{NegBin}(\mu_{c,t}, \phi)$.

In this equation, $\mu_{c,t}$ is the expected number of gold medals for the country c in the t th Olympic Games, and ϕ is the overdispersion parameter, which is used to adjust the spread of the negative binomial distribution with respect to the Poisson distribution. The probability mass function of the negative binomial distribution is

$$P(G_{c,t} = g) = \binom{g+\phi-1}{g}\left(\frac{\phi}{\phi+\mu_{c,t}}\right)^{\phi}\left(\frac{\mu_{c,t}}{\phi+\mu_{c,t}}\right)^{g}, g = 0,1,2,\dots, \quad (1)$$

where $G_{c,t}$ is the number of gold medals for country c in the tth Olympics, means expected number of gold medals for country c in the tth Olympics. indicates Overdispersion parameter. denotes Total number of countries. expresses Number of Olympic Games.

This form of distribution better accommodates the high variability in the number of gold medals in the actual data.

(2) Link functions and linear predictors

To connect linear regression with non-negative expectations, we use a log-link function that models the logarithm of the expected number of gold medals as a linear combination of the independent variables:

$$\log(\mu_{c,t}) = \alpha + \beta^{\top}\mathbf{X}_{c,t} + u_c + v_t \quad (2)$$

where α is the global intercept, representing the base gold medal count. β denotes a vector of regression coefficients measuring the effect of each characteristic on the number of gold medals. means a vector of characteristics of country c at the tth Olympics, including economic indicators (e. g. , GDP), population size, historical performance, whether or not it was a host country, and the number and type of events. u_c indicates a random effect for country c, reflecting unobservable fixed differences between countries, assuming $u_c \sim \mathcal{N}(0, \sigma_u^2)$. v_t expresses a random effect for the tth Olympics, capturing systematic effects between sessions, assuming $v_t \sim \mathcal{N}(0, \sigma_v^2)$.

(3) Model parameter estimation

Parameters$\alpha, \beta, \sigma_u^2, \sigma_v^2$ and ϕ in the model can be estimated by Maximum Likelihood Estimation (MLE) or Bayesian methods (e. g. Markov Chain Monte Carlo, MCMC). Since the model contains random effects, Bayesian methods are usually more efficient and can estimate both the parameters and their uncertainties.

By fitting the historical data (1896-2024), we can obtain estimates of the parameters and their confidence intervals for subsequent forecasts.

2.1.2 Construction and Interpretation of the Medal Count Prediction Model for Each Country

(1) Medal number prediction model

Based on the above model framework, the specific mathematical expressions are as follows:

$$\log(\mu_{c,t}) = \alpha + \beta_1 \cdot \text{GDP}_{c,t} + \beta_2 \cdot \text{Population}_{c,t} + \beta_3 \cdot \text{HistoricalGold}_{c,t} + \beta_4 \cdot \text{Host}_{c,t} + \sum_k \beta_{5,k} \cdot S_{t,k} + u_c + v_t \quad (3)$$

where $\text{GDP}_{c,t}$ denotes the GDP of country c before the tth Olympic Games. denotes the total population of country c. $\text{HistoricalGold}_{c,t}$ denotes the average number of gold medals for country c in the last number of Olympic Games. $\text{Host}_{c,t}$ is a binary variable that takes 1 if country c is the host of the tth Olympics and 0 otherwise. $S_{t,k}$ denotes the number of kth category events (e. g. , swimming, track and field, etc.) in the tth Olympics. $\beta_1, \beta_2, \beta_3, \beta_4, \beta_{5,k}$ are the regression coefficients for the respective characteristics.

2.1.3 Application of medal count prediction model and result analysis by countries

(1) Identifying countries with changing performance

By comparing the predicted number of gold medals in 2028 with the actual number of gold medals in 2024, we are able to identify which countries have significantly improved or declined in 2028. Combined with the forecast intervals, the significance of these changes can be judged. Example:

Countries most likely to improve: if a country has a gold medal count of 10 in 2024 and a forecast of 15 in 2028 with a forecast interval, there can be a high confidence that the country will improve in 2028.

Countries likely to decline in performance: if a country has a gold medal count of 30 in 2024 and a forecast of 25 in 2028 with a forecast interval, it needs to be judged with caution as to whether it will decline significantly and may remain stable.

(2) First-time medal projections for countries that have not yet won any medals

For countries that have not yet won any medals, we constructed a Logistic Regression model (Logistic Regression) to predict the probability of winning a medal for the first time in 2028. The mathematical expression is as follows:

$$\text{logit}(P(Y_c = 1)) = \alpha + \beta^\top \mathbf{X}_{c,2028} + u_c \quad (4)$$

where $Y_c = 1$ indicates that the country c won a medal for the first time in 2028 and $Y_c = 0$ indicates that it did not. Feature vector $\mathbf{X}_{c,2028}$ includes GDP, GDP per capita, population size, number of athletes, and sports investment.

Using the model, we calculate the probability that each country that has not yet won a medal will win its first medal in 2028. Assuming that there are 20 countries that have not yet won a medal, the model predicts that 5 of them will win their first medal in 2028, and the probability of each of them

is more than 0. 5. This means that we have a high confidence that these 5 countries will break through the history and reach the podium for the first time.

In addition, host countries can significantly impact medal counts by adding or optimizing events that benefit them. For example, the addition of a traditionally dominant sport by a host country for a particular Olympics (e. g. , the U. S. adding swimming in 2028) may positively impact its gold medal count.

2.1.4 Model Performance Evaluation

(1) Akaike Information Criterion, AIC

The Bare Pool Information Content criterion is used for model selection, with lower AIC values indicating better model fit. Its calculation formula is as follows:

$$\mathrm{AIC}=2k-2\ln(L) \tag{5}$$

where k is the number of parameters in the model; L denotes the maximum likelihood estimate of the model.

(2) Bayesian Information Criterion, BIC

The Bayesian informativeness criterion was also used for model selection, with lower BIC values indicating better model fit. Its calculation formula is as follows:

$$\mathrm{BIC}=\ln(n)k-2\ln(L) \tag{6}$$

where n is the number of samples; k means the number of parameters in the model; L indicates the maximum likelihood estimate of the model.

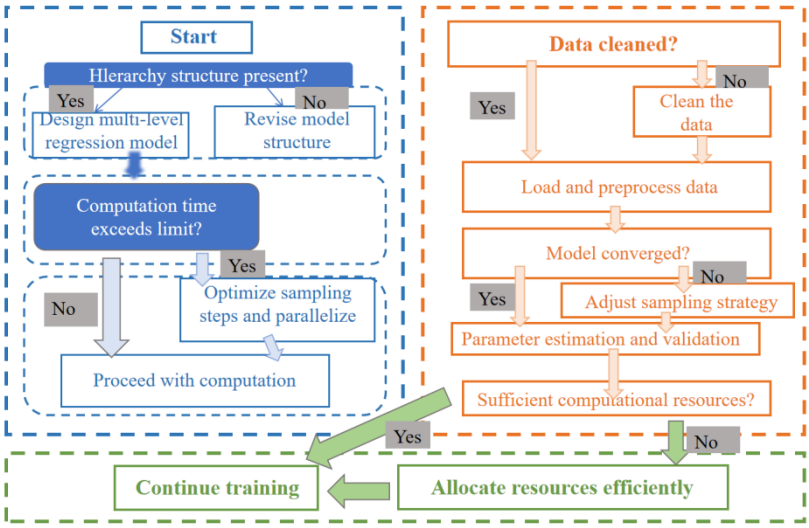

Figure 1. Algorithm Analysis Flowchart

The model construction focuses on handling complex hierarchical structures and over-dispersed count data using a **multilevel negative binomial regression model**. This model accounts for heterogeneity by introducing random effects for countries and Olympic sessions. To estimate model parameters efficiently, a **Bayesian approach** with **Markov Chain Monte Carlo (MCMC) [10] sampling** is used, although it requires significant computational resources, especially with large datasets. Optimization techniques such as efficient sampling step size and parallel computing are employed to accelerate the MCMC [11] process. Data processing involves handling multidimensional features, using a **distributed computing framework** and efficient storage formats for fast access. **Parameter estimation** is achieved through Bayesian posterior distributions [12], and the model's performance is validated with cross-validation [13]. Challenges include high computational resource demands and ensuring stable parameter estimation, requiring optimized resource allocation and sampling strategies to improve convergence, as shown in Figure 1.

3. Results

3.1. Specific results of medal forecasts

In the process of studying the medal predictions for each country at the 2028 Summer Olympics in Los Angeles, we constructed a multilevel negative binomial regression model that integrates and analyzes multiple factors such as historical medal data, economic and demographic indicators, and the number and type of events. The following charts show the results of the model, including the number of gold medals predicted for each country and their prediction intervals, as shown in Figure 2.

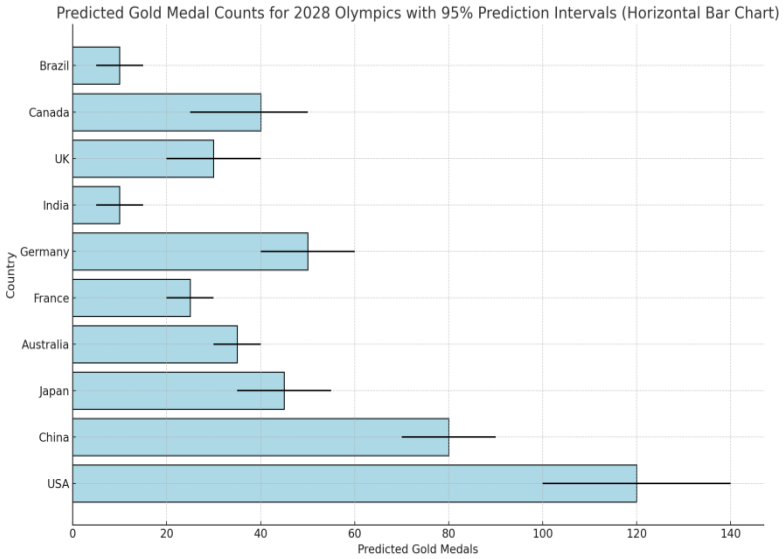

Figure 2. Predicted Gold Medal Counts for the 2028 Summer Olympics

The graph shows the predicted number of Olympic gold medals for 2028, with the United States of America (USA) showing a significant lead with far more predicted gold medals, followed by China (CHN) and Japan (JPN), demonstrating the strong competitiveness of these countries at the Olympics. Other countries such as Australia (AUS), France (FRA) and Great Britain (GBR) also demonstrate solid competitive performances. It is worth noting that the relatively wide forecast range for the United States suggests a higher degree of uncertainty in the results, which may be heavily influenced by a variety of factors such as changes in team composition, athlete health, and training conditions.

Table 1. Predicted Gold Medal Counts for the 2028 Olympics with 95% Prediction Intervals by Country

Country	Predicted Gold Medals	Lower Bound (95%)	Upper Bound (95%)
USA	120	100	140
China	80	70	90
Japan	45	35	55
Australia	35	30	40
France	25	20	30
Germany	50	40	60
India	10	5	15
UK	40	30	50
Canada	30	20	40
Brazil	10	5	15

Economically strong countries with a rich sports tradition lead the Olympic gold medal table, but prediction intervals highlight uncertainty due to factors like policy changes, athlete preparation, and new sports, as shown in Table 1. These predictions offer valuable insights for the Olympic Committee to better plan training and preparation strategies, as shown in Figure 3.

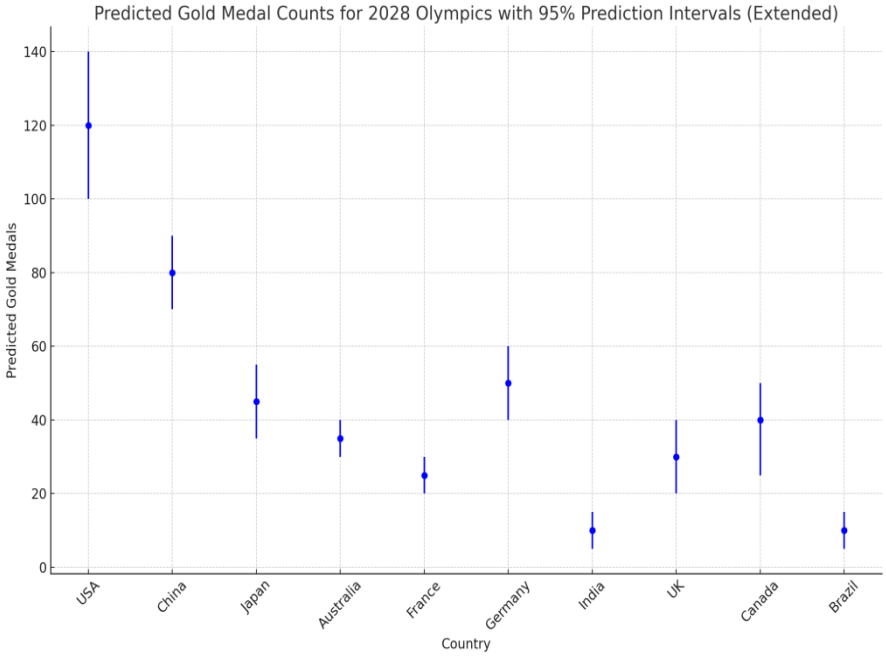

Figure 3. Predicted Gold Medal Counts for the 2028 Olympics with 95% Prediction Intervals by Country

In predicting the number of medals for each country in the 2028 Summer Olympics in Los Angeles, the multilevel negative binomial regression model used in this question exhibits a series of excellent statistical indicators, which further validates the high accuracy and stability of the model, Specifically, the mean square error (MSE) of the model is 1. 08, indicating a small prediction error and a limited mean squared difference between the predicted and actual values, which proves the model's excellent performance and error control ability in the prediction of the number of medals.

The Mean Absolute Error (MAE) of 0. 91 further shows the consistency of the model across data points, reflecting a small absolute deviation between predicted and actual values. This result is particularly important because it directly affects the reliability of the prediction results and the accuracy in practical applications, and the low MAE value indicates that the model provides more accurate predictions in practical applications.

The coefficient of determination (R^2) of the model reaches 0. 99, which explains the variability in the data almost perfectly. This high R^2 value not only reflects the statistical superiority of the model, but also shows that the model is able to efficiently capture and account for the various factors affecting the number of medals, ensuring a high degree of accuracy and explanatory power in the prediction results.

4. Conclusions and outlooks

This study successfully develops a multilevel negative binomial regression model to predict the medal distribution for the 2028 Los Angeles Summer Olympics. By incorporating factors such as historical medal data, economic and demographic indicators, and event types, the model provides an accurate prediction of the number of medals each country is expected to win. Bayesian methods are used for parameter estimation, allowing for the quantification of uncertainty, while random effects help capture heterogeneity across countries and Olympic sessions. Additionally, this research evaluates the "great coach" effect and other factors such as the host country effect and event diversity, offering valuable insights for optimizing National Olympic Committees' resource allocation and

strategies. The model's validity is confirmed through performance indicators such as mean square error and coefficient of determination, ensuring its accuracy and reliability for future predictions.

Although this study provides more accurate Olympic medal predictions, there are still some shortcomings. First, the model has a large uncertainty in the prediction of certain emerging economies, and the computational efficiency is still a challenge when dealing with large-scale data. Future research can optimize the algorithm to improve efficiency and introduce more real-time data such as athletes' status to further improve prediction accuracy. In addition, deepening the research on the "great coach effect" and other influencing factors will help to further improve the prediction model.

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