Research On Nonlinear Mathematical Models in Power Load Distribution and Their Engineering Applications

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Abstract. This paper focuses on the field of power load distribution, and devotes itself to the study of nonlinear mathematical model and its application in power engineering. By systematically analyzing the factors that affect the distribution of power load, such as user behavior, characteristics of electrical equipment, time and environment, a nonlinear mathematical model with strong pertinence is constructed. In the solution and verification, the model is trained by iterative method and the actual monitoring data. The results show that the model shows high accuracy in forecasting power load distribution, with an average absolute error (MAE) of 23.5kW and an average absolute percentage error (MAPE) of 3.15%. In the application of power engineering, this model helps to rationally arrange power supply and power grid in power system planning, realize the optimal utilization of energy in power dispatching, and ensure that the equipment adapts to the load demand when selecting power equipment. To sum up, the model is of great significance to improve the operation efficiency and reliability of power system.

Keywords: Power load distribution, Nonlinear mathematical model, Engineering application, Power system planning, Power dispatching.

1. Introduction

With the rapid development of economy and the continuous progress of society, as a key energy source in modern society, the demand of electricity shows a trend of continuous growth [1]. In this context, ensuring the stable and efficient operation of the power system has become an important issue facing the power industry [2]. The study of power load distribution plays a vital role in the optimal operation of power system, which is related to the rational allocation of power resources, the effective utilization of power equipment and the improvement of power supply quality [3].

In the research field of power load distribution, nonlinear mathematical models have gradually emerged, showing unique advantages. The traditional linear model has some limitations in describing the complex and changeable characteristics of power load, and it is difficult to accurately describe the real relationship between load and various influencing factors [4]. The nonlinear mathematical model can reflect the actual situation of power load distribution more accurately by virtue of its powerful description ability of complex phenomena, and provide a more reliable basis for the relevant decision-making of power engineering [5].

Judging from the research status, many scholars have made some achievements in the nonlinear mathematical model of power load distribution. Part of the research is devoted to the theoretical construction of the model. Through in-depth analysis of power load characteristics, a variety of novel nonlinear model frameworks are proposed. Other studies focus on the application of the model in practical engineering, and verify the effectiveness of the model in power system planning and dispatching [6]. However, there are still some shortcomings in the existing research. In view of this, this paper aims to deeply study the nonlinear mathematical model of power load distribution and its engineering application. Through systematic analysis of various factors affecting power load distribution, a concise and practical nonlinear mathematical model is constructed, and its specific application in power system planning, power dispatching and other engineering fields is discussed in detail.

2. Power load distribution related theoretical basis

Electric load refers to all kinds of equipment or components that consume electric energy in the power system. From the point of view of load nature, it can be divided into resistive load, inductive load and capacitive load. Resistive loads, such as ordinary incandescent bulbs and electric furnaces, have the same phase of current and voltage, and electric energy is mainly converted into heat energy [7]. Inductive loads, such as motors, transformers, etc., the current lags behind the voltage, and the magnetic field is established to consume reactive power when working. There are relatively few capacitive loads, which are common in some compensation circuits, and the current leads the voltage. According to the use classification, there are lighting load, power load, electric heating load, etc. Different types of loads have significant differences in power consumption characteristics and power requirements, which has an important impact on the distribution of power loads.

Power system is a huge and complex organic whole, which is mainly composed of five links: power generation, transmission, substation, distribution and electricity consumption. The power generation link converts other forms of energy into electric energy through various power plants, such as thermal power plants, hydraulic power plants and wind power plants [8]. The power transmission link transmits the electric energy from the power plant to the load center by means of high-voltage transmission lines. Transformer is used in the substation to increase or decrease the voltage level according to different requirements. In the power distribution link, electric energy is distributed to all users through medium and low voltage distribution lines and distribution equipment. The electricity consumption link means that all kinds of electric loads consume electric energy. In the process of operation, all links cooperate closely and follow Kirchhoff's law and other basic electrical principles to maintain the balance and stable transmission of power.

A nonlinear mathematical model is a mathematical expression that describes the nonlinear relationship between variables. Compared with the linear model, its variable relationship is more complicated and cannot be simply represented by straight lines or planes. When building a nonlinear mathematical model, it is necessary to choose appropriate nonlinear function forms according to the characteristics of practical problems, such as polynomial function, exponential function and trigonometric function [9]. The commonly used modeling methods are regression analysis, which fits the relationship between variables through a large number of data; There is also a mechanism analysis method to determine the model structure based on basic principles such as physics and chemistry. The solution of nonlinear model is usually more complicated than that of linear model, and iterative method and numerical solution are often needed.

3. Construction of nonlinear mathematical model in power load distribution

3.1. Analysis of factors affecting power load distribution

The distribution of power load is influenced by many complex factors. First of all, there are significant differences in user behavior. For example, residential users use electricity in the morning and evening for lighting and household appliances. Commercial users rely on all kinds of electrical equipment for office and business during business hours, and their electricity consumption characteristics are quite different from those of residential users [10]. Secondly, the characteristics of electrical equipment are key, and the power, running time and starting mode of different equipment are different. For example, when a large industrial motor is started, the impact current is large, which has obvious influence on the local power grid load. Furthermore, the time factor cannot be ignored. Seasonal changes, working days and rest days will all lead to changes in electricity consumption patterns, and the demand for cooling in summer and heating in winter will make the seasonal electricity consumption difference prominent. Environmental factors, such as temperature and humidity, will also affect the load, and the demand for electricity in high temperature space-time will increase.

3.2. Assumption and establishment of nonlinear mathematical model

Based on the analysis of the above factors, the following assumptions are made:

Ignore the impact of short-term sudden extreme weather and other small probability events on the load; It is assumed that all kinds of users' electricity consumption behaviors are relatively stable in the statistical period. A nonlinear mathematical model is constructed with time t, user type u and equipment power p as main variables. Considering the coupling effect of user behavior and time factors, a function $f(t,u)$ that changes with time and user type is introduced, which comprehensively reflects the power consumption tendency of different users at different times. At the same time, the influence of the power of electrical equipment on the load is not simply linear superposition, and the following nonlinear relationship is constructed:

$$L(t,u) = \sum_{i=1}^{n} p_i \cdot f(t,u) \cdot g(p_i) + \epsilon \tag{1}$$

Where $L(t,u)$ represents the load of u users at t time; p_i is the power of the i-th electrical equipment, and $g(p_i)$ is a nonlinear correction function about the equipment power, which is used to describe the nonlinear influence of the equipment power on the overall load. ϵ is a random error term, which represents a small influence factor that has not been considered. Further refine $g(p_i)$:

$$g(p_i) = 1 + \alpha \cdot p_i^2 + \beta \cdot p_i^3 \tag{2}$$

Among them, α and β are coefficients obtained by fitting the actual data, which reflect the differences in the influence of equipment in different power sections on the load.

3.3. Solution and verification of the model

The model is solved by iterative method, starting from the initial value and gradually approaching the optimal solution satisfying the equation. The model training is carried out by using the investigation data of regional power load influencing factors. Firstly, the initial guess values of α and β are determined, and the predicted load value $L_{pred}(t,u)$ is calculated by substituting it into the model, and then the coefficient is adjusted by minimizing the mean square error (MSE) between the predicted value and the actual load value L_{actual}:

$$MSE = \frac{1}{N}\sum_{t=1}^{T}\sum_{u=1}^{U}(L_{pred}(t,u) - L_{actual}(t,u))^2 \tag{3}$$

Where $N = T \cdot U$, T is the number of time samples, and U is the number of user types. By iteratively updating α and β, MSE converges to the minimum.

In the model verification stage, the data is divided into training set and test set, and the model performance is tested on the test set after the optimal coefficient is obtained by using the training set. If the error between the predicted load and the actual load is within an acceptable range, such as MAE is less than a certain threshold, it shows that the model has high accuracy and reliability and can describe the distribution of power load well.

4. Model testing and analysis

After completing the construction and solution of the nonlinear mathematical model of power load distribution, the model is fully verified by experiments to evaluate its accuracy and reliability. The experimental data comes from the long-term monitoring of power load in a representative area, covering the power consumption of different types of users (residents, businesses and industries) in different time periods (working days, weekends and different seasons).

In order to visually show the prediction effect of the model, the predicted load value is compared with the actual load value, and the results are shown in Table 1:

Table 1: Comparison between Actual Load and Predicted Load by Nonlinear Mathematical Model

Date	Time Slot	User Type	Actual Load (kW)	Predicted Load (kW)	Error (%)
Monday	19-21 o'clock	Residential	350	342	2.29
Monday	9-11 o'clock	Commercial	520	508	2.31
Friday	14-16 o'clock	Industrial	1200	1165	2.92
...	...	...	...	...	...

The load data of different periods and different user types in a week are selected in the table. As can be seen from Table 1, in terms of residential electricity consumption, the predicted value of the model is close to the actual value. At 7-9 pm on Monday, the actual load of residential electricity consumption was 350kW, and the predicted load was 342kW, with an error of only 2.29%. At 9-11 am on weekdays, the actual load of commercial users is 520kW, and the predicted value is 508kW, with an error of 2.31%. Due to the complexity of electrical equipment and production process, it is relatively difficult for industrial users to predict, but the overall error is also within the acceptable range. For example, at 2-4 pm on Friday, the actual load is 1200kW and the predicted load is 1165kW, with an error of 2.92%.

In order to further quantify the accuracy of the model, MAE, root mean square error (RMSE) and MAPE are calculated, and the results are shown in Table 2:

Table 2: Accuracy Evaluation Indicators of Nonlinear Mathematical Model

Evaluation Indicator	Value
MAE (kW)	23.5
RMSE (kW)	30.2
MAPE (%)	3.15

MAE reflects the average absolute value of the error between the predicted value and the actual value, and the MAE of this model is 23.5kW, indicating that the absolute value of the average prediction error is relatively small. RMSE considers the sum of squares of errors, and is more sensitive to larger errors, with a value of 30.2kW, which shows that the model has better control over extreme errors. MAPE shows the error in percentage form, and the MAPE of this model is 3.15%, which means that the average deviation between the predicted value and the actual value is about 3.15%, which fully proves that the model has high accuracy in forecasting the power load distribution.

5. Application of nonlinear mathematical model in electric power engineering

In power system planning, the nonlinear mathematical model can predict the future power load in different regions. By considering regional development planning, population growth and other factors, the model predicts the load change. For example, when planning the power supply facilities in the new development area, the capacity of new substations, the number and trend of transmission lines are determined according to the load scale and distribution predicted by the model. In this way, the power system planning can be synchronized with the regional development demand, the shortage or excess of facilities can be avoided, and the stability and economy of the long-term operation of the power system can be improved.

In power dispatching, this model monitors the load distribution in real time. With the analysis of real-time data, the dispatching center can master the load dynamics of each region. At the peak of power consumption, according to the load information provided by the model, the power generation resources are reasonably allocated, and the power consumption in key areas is given priority. At the same time, different power sources are coordinated to generate power, reducing power generation cost and energy consumption, ensuring stable power supply and improving the operating efficiency of the power system.

The nonlinear mathematical model is also used in power equipment management. It evaluates the operation status of equipment according to the load characteristics, such as analyzing the long-term load data of transformers and predicting potential faults. According to this, the operation and maintenance personnel make targeted maintenance plans to deal with hidden dangers in advance, prolong the service life of equipment and reduce power failure. Moreover, when the equipment is upgraded, the model helps to select the new equipment to ensure that the equipment performance meets the load demand and improve the overall reliability of the power system.

6. Conclusions

In this paper, the nonlinear mathematical model of power load distribution and its engineering application are deeply studied. On the theoretical level, the basic theory of power load distribution is systematically expounded, covering the concept classification of power load, the structural principle of power system and the basic theory of nonlinear mathematical model, which lays a solid foundation for subsequent research.

In the aspect of model construction, the multivariate factors affecting the distribution of power load are comprehensively analyzed, and reasonable assumptions are made accordingly, and a nonlinear mathematical model is successfully established. The model is solved by iterative method and trained and verified by actual data. The results show that the model has high accuracy and stability, and can accurately describe the distribution law of power load. In the field of engineering application, the model plays a significant role in power system planning, dispatching and equipment selection. It provides scientific prediction for power system planning and optimizes power grid layout; Help power dispatching realize real-time decision-making and improve energy utilization efficiency; Guide the selection of power equipment, ensure the matching between equipment and load, and improve the reliability of equipment operation.

Future research can further expand the model, incorporate more complex factors and enhance its universality in different scenarios. At the same time, with the intelligent development of power system, the integration of models and emerging technologies can be explored to provide more powerful support for the development of power industry.

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