Optimal Weighted Combination Forecasting Based on Holt-Winters and SARIMA Intervention Models

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Abstract. Excessive emissions of SO_2 will lead to the formation of acid rain. Accurate prediction of the content of SO_2 in the atmosphere can provide reasonable suggestions for the government's policy formulation and residents' travel. Each prediction model has its drawbacks. For complex problems such as weather prediction, using a single prediction model often leads to a relatively large error. Optimal weighted prediction is a commonly used model combination prediction method, which usually yields better prediction results. In order to accurately predict the content of SO_2 in the atmosphere of Urumqi City, the SARIMA intervention model, the Holt-Winters additive model, and the optimal weighted combination model composed when the weight coefficients of the two are 0.6 and 0.4 respectively were used to predict the content of SO_2 in the atmosphere of Urumqi City from April 2024 to March 2025. And the fitting situations and prediction accuracies of the three models were compared. As a result, the MAE values of these three models are 0.800, 0.758 and 0.715 respectively, the $RMSE$ values are 1.008, 0.991 and 0.945 respectively, and the $MAPE$ values are 0.154, 0.153 and 0.141 respectively. The prediction effect of the combined model is the best. Finally, the content of SO_2 in the atmosphere of Urumqi City is predicted.

Keywords: SO_2 concentration, Intervention model, Holt-Winters additive model, Prediction accuracy, Weighted combination model.

1. Introduction

With the rapid development of the economy, both the government and enterprises are attaching increasing importance to the research and development as well as the application of SO_2 pollution reduction and control technologies. Urumqi City, as the largest city in Xinjiang, is rich in coal resources. The burning of coal has produced a large amount of SO_2 gas, which has had a great impact on the lives of local residents. In recent years, with the introduction and implementation of environmental protection policies, many heavy industry enterprises have increased their investment in environmental protection, resulting in a significant reduction in the content of SO_2 in the atmosphere. However, the detrimental effects of SO_2 remain a significant concern. Therefore, it is of great significance to accurately predict the content of SO_2 in the atmosphere in the future. In terms of predicting the content of atmospheric pollutants, many scholars have done a lot of research. Liu Mao and others used the ARIMA model and the GM (1,1) grey model to predict the atmospheric SO_2 concentration in Nanjing City respectively [1]. They concluded that the model better fits SO_2 concentration data in Nanjing, enabling short-term predictions. However, the prediction results of the models based solely on the past sequence of SO_2 concentration values were unstable, and it was necessary to incorporate the relevant influencing factors affecting SO_2 concentration values into the models to improve the prediction accuracy of the models. Jin Lishan and others used the BP neural network model and the support vector machine model to predict the concentrations of PM2.5 and PM10 in Hohhot City respectively [2]. Both models achieved good prediction effects, but no further modifications were made to the models. He Yuhan predicted the concentration of PM2.5 in the atmosphere of Shanghai City by using the STL-decomposition and LSTM-RF-combination model, and concluded that the prediction effect of the combination model was better than that of the single model [3]. Using a single prediction data is less stable and has a higher error. The optimal weighted combination prediction model can incorporate various advantages to improve the prediction accuracy. In order to predict the content of SO_2 in the atmosphere of Urumqi City more accurately, this paper first established the SARIMA model for prediction. Then, an intervention model was constructed by adding policy factors, and then the Holt-Winters three-parameter exponential additive model was constructed. Through the R software, the optimal combination of weighted coefficients was screened out based on the principle of the lowest mean absolute percentage error. In the test set, the values of MAE, $RMSE$ and $MAPE$ of the three models were compared, and it was concluded that the prediction effect of the combination model was better than that of the single model. Finally, the content of SO_2 in the atmosphere of Urumqi City from April 2024 to March 2025 was predicted according to the combination model.

2. Data sources and division

The monthly data of SO_2 content in the atmosphere of Urumqi from December 2013 to March 2024 were sourced from the China Air Quality Online Monitoring and Analysis Platform. A total of 124 sample data were collected, with a sampling interval of one month. The data from December 2013 to March 2023 were used as the training set, and the data from April 2023 to March 2024 were used as the test set.

3. Construction of the SARIMA model

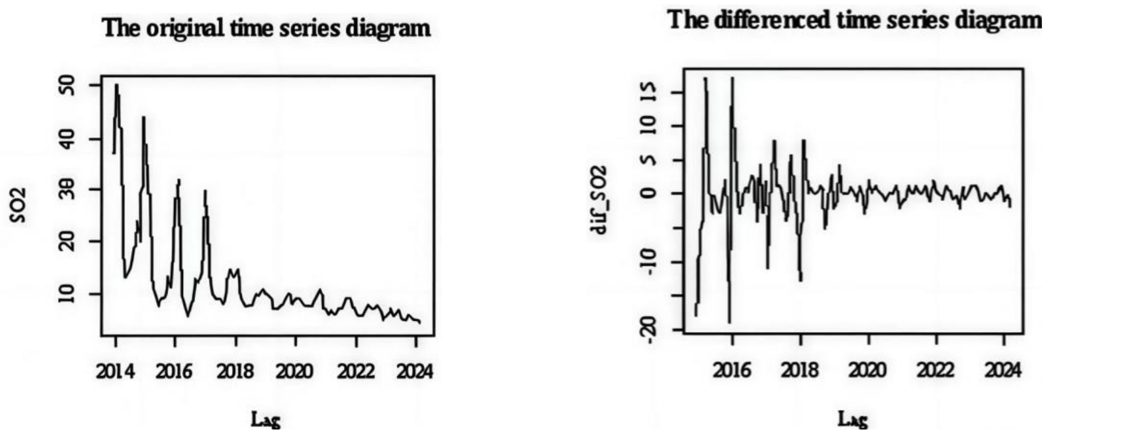
3.1. SARIMA model

3.1.1. Introduction of the SARIMA Model

The SARIMA model is a mathematical model that predicts future values by comprehensively analyzing the long-term trends, periodic changes and characteristics of random disturbances of a time series. Generally, the model is simply denoted as $SARIMA(p,d,q)(P,D,Q)_S$ [4]. Among them, p,d and q represent the order of autoregression, the number of differences and the order of moving average respectively, P and Q represent the order of seasonal autoregression, the number of seasonal differences and the order of moving average respectively, and S represents the seasonal cycle and cycle length [5]. When a non-stationary time series shows an obvious seasonal pattern, the $SARIMA(p,d,q)(P,D,Q)_S$ model has a relatively high prediction accuracy.

3.1.2. Data processing

It can be seen from the original SO_2 time series diagram (Figure 1) that the sequence has obvious seasonal and trend characteristics. After Performing first-order and 12-step differences on the original sequence., it is found through observing the time series diagram after the differences that the cycle and trend are basically eliminated. The " *adf.test()* " function is used to test the differenced data, and since $P < 0.05$, it can be judged that the data after the differences has become a stationary sequence [6]. Then, the “ *Box.test()* ”function is used to conduct a white noise test on the differenced stationary data. Since $P < 0.05$, it can be judged that the sequence after the differences is a non-white noise sequence. Therefore, the differenced data can be used for modeling.



(a) SO_2 time sequence diagram (b) Differenced Time Sequence Diagram of SO_2

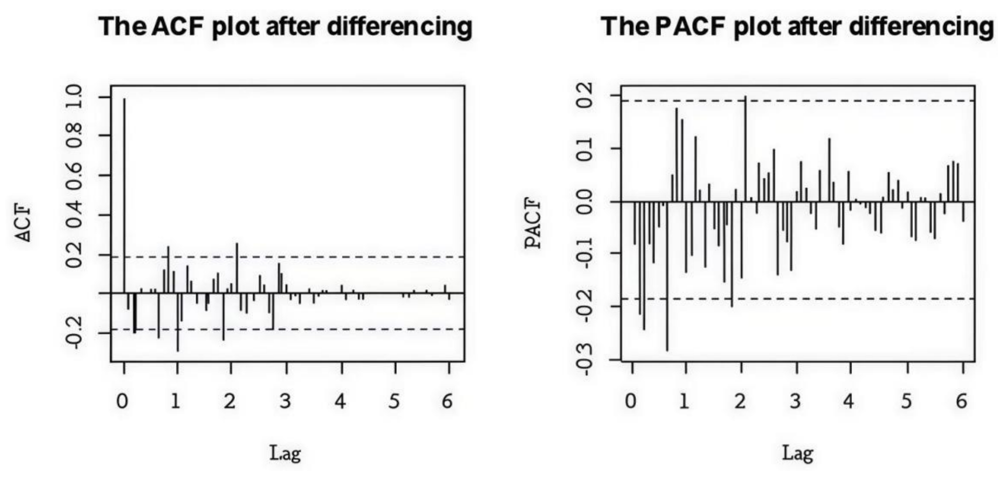
Figure 1. Time series plots of SO_2 before and after differencing

3.1.3. Model Selection and Order Determination

The seasonality of the original data is obvious, so the $SARIMA(p,d,q)(P,D,Q)_S$ model is used for modeling. Since the first-order 12-step difference processing has been carried out on the original data, $d=1, D=1$, and $S=12$. Through the ACF and PACF graphs after the difference, it can be judged that when the autocorrelation graph is truncated at the third order, the partial autocorrelation graph has a trailing effect, and when the partial autocorrelation graph is truncated at the third order, the autocorrelation graph has a trailing effect. By observing the characteristics of the autocorrelation and partial autocorrelation coefficients with the cycle length as the unit, the autocorrelation and partial autocorrelation coefficients at lags 12 and 24 are both significantly greater than 0, and the autocorrelation and partial autocorrelation coefficients at lag 36 are both less than twice the standard deviation [7]. Therefore, the autocorrelation and partial autocorrelation coefficients can be regarded as being truncated at the cycle of 2 respectively, with the other one having a trailing effect. Thus, the fitted multiplicative models are respectively.

$$\begin{gathered} SARIMA(3,1,0)(2,1,0)_{12} \\ SARIMA(3,1,0)(0,1,2)_{12} \\ SARIMA(0,1,3)(2,1,0)_{12} \\ SARIMA(0,1,3)(0,1,2)_{12} \end{gathered} \tag{1}$$

After model and parameter tests, it was found that only the model and parameters of $SARIMA(0,1,3)(0,1,2)_{12}$ passed the tests.

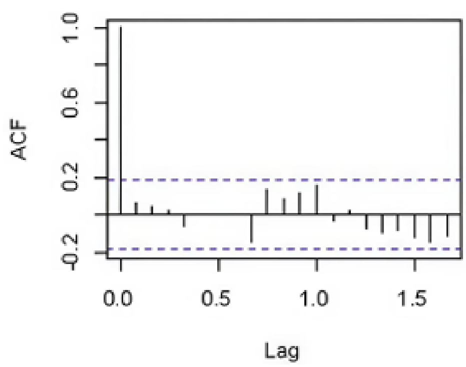


(a) ACF Diagram of SO_2 (b) PACF Diagram of SO_2

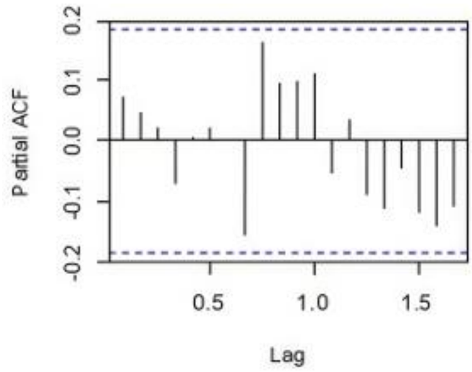
Figure 2. ACF and PACF plots after differencing.

Table 1. Parameter Test Table of the $SARIMA(0,1,3)(0,1,2)_{12}$ Model.

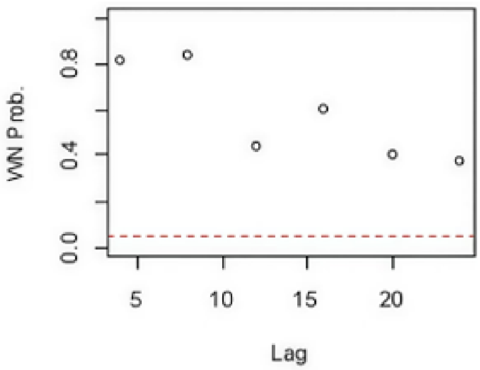
$SARIMA(0,1,3)(0,1,2)_{12}$	$MA(1)$	$MA(2)$	$MA(3)$	$SMA(1)$	$SMA(2)$	
	–0.4621	–0.2049	–0.1648	–0.4337	0.3117	
SE	0.1153	0.1137	0.1011	0.1096	0.1459	pass



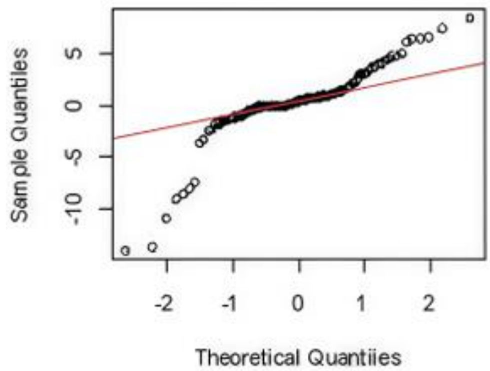
(a) ACF Diagram of the Residual Sequence



(b) PACF Diagram of the Residual Sequence



(c) White Noise Test Diagram of the Residual Sequence



(d) QQ Plot of the Residual Sequence

Figure 3. Residual Diagnostic Plot of the $SARIMA(0,1,3)(0,1,2)_{12}$ Model.

3.1.4. Model performance testing and error evaluation

To visually evaluate the prediction effect of this model, first draw a comparison chart of the model's prediction effect. The overall fitting effect is quite good. By calculating the mean squared error, the mean absolute error and the mean relative percentage error, the prediction effect of this model is quantitatively measured. The calculation results are as follows:

$$MAE = 0.813, RMSE = 1.134, MAPE = 0.163 \tag{2}$$

The mean relative percentage error of this model is 16.3%. Since it is within the allowable error range, this model can be chosen to predict the future SO_2 content in the atmosphere of Urumqi.

The main reason for the prediction errors generated by the SARIMA model is that the government introduced relevant policies to control SO_2 emissions before 2017, which had a great impact on the SO_2 content in the atmosphere. The SARIMA model couldn't accurately capture such changes [8]. When making predictions in the future, these external factors can be taken into account to improve the prediction accuracy of the model.

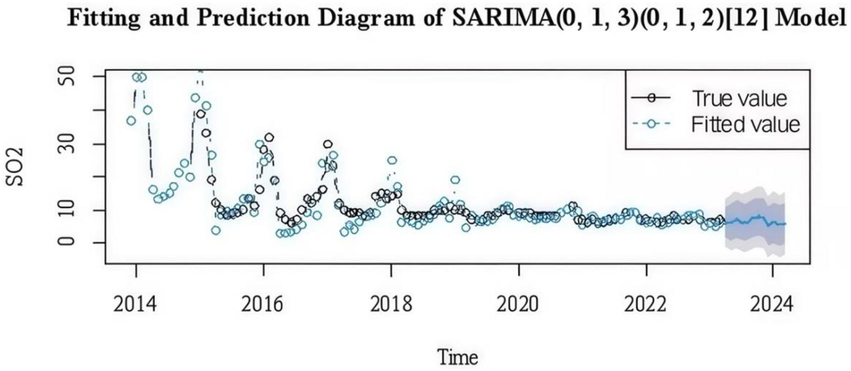


Figure 4. Fitting and Prediction Effect Diagram of the $SARIMA(0,1,3)(0,1,2)_{12}$ Model.

3.2. Intervention model

3.2.1. Introduction of the intervention Model

Time series are often affected by special events and situations. Such external events are called interventions. Evaluating the impact of external events on the series is known as intervention analysis. The intervention analysis model adds external variables on the basis of the original time model. In addition to being influenced by itself, the occurrence of future events is also affected by external events [9]. For the prediction of things like weather and stocks, the influence of external factors often needs to be considered. For the prediction of such events, the intervention model can often achieve better prediction results. The two most common types of intervention variables are step intervention and impulse intervention, and their variables are usually set as follows:

$$x_t = \begin{cases} 0, \text{Before the occurrence of the intervention event}(x < T) \\ 1, \text{After the occurrence of the intervention event}(x \geq T) \end{cases} \tag{3}$$

$$x_t = \begin{cases} 0, \text{When the intervention event occurs}(x = T) \\ 1, \text{Other moments}(x \neq T) \end{cases} \tag{4}$$

The $SARIMA(0,1,3)(0,1,2)_{12}$ model has been established earlier in this paper to predict the SO_2 content in the atmosphere of Urumqi. It can be seen from Figure 5 that the SO_2 content in the atmosphere of Urumqi decreased rapidly after April 2017. In order to further improve the prediction accuracy of the model, as can be seen from Figure 5, the SO_2 content in the atmosphere decreased rapidly after April 2017, which was related to the relevant environmental protection policies issued by the local government. Therefore, policy factors were added:

$$x_t = \begin{cases} 0, t < \text{April 2017} \\ 1, t \geq \text{April 2017} \end{cases} \tag{5}$$

3.2.2. Establishment of the Intervention Model

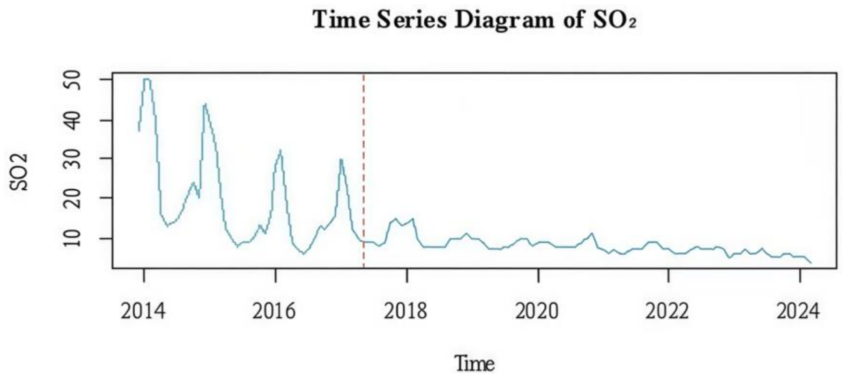


Figure 5. Time Series Diagram of SO_2 Content in the Atmosphere of Urumqi.

In order to examine whether there is an obvious correlation between the SO_2 content in the atmosphere of Urumqi and the intervention variable, the cross-correlation diagram (Figure 6) of the NO_2 sequence and the intervention variable is used to study the intervention mechanism of the intervention variable on the sequence. Figure 6 shows that the correlation is the strongest when the intervention variable has a 0-order lag. It can be assumed that the intervention variable only has a horizontal impact and there is no delay. The reason why the correlation is the strongest at the 0-order lag is that various environmental protection policies in Urumqi had already been implemented as early as 2016, and the effect of the policies became significant around April 2017. After performing a 12-step difference on the NO_2 content sequence in Urumqi, the stationarity and white noise tests are carried out, and then the intervention variable is introduced. A regression model is established based on the data of the NO_2 content sequence after the difference.

$$\nabla_{12SO2} = \beta_0 + \beta_1 x_t + \varepsilon_t \tag{6}$$

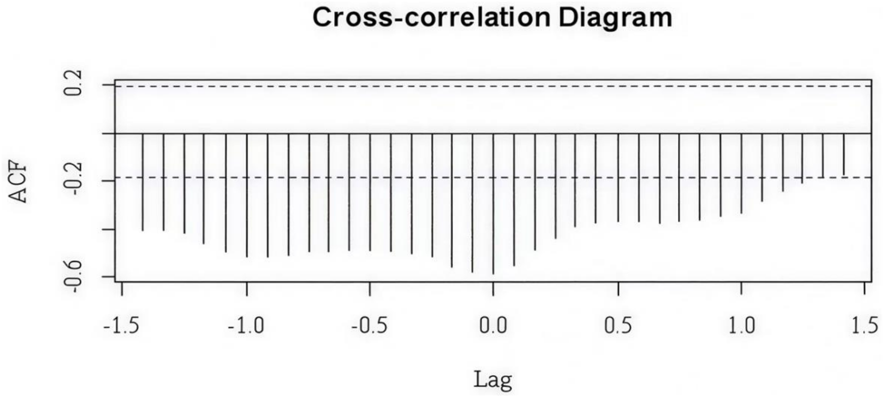
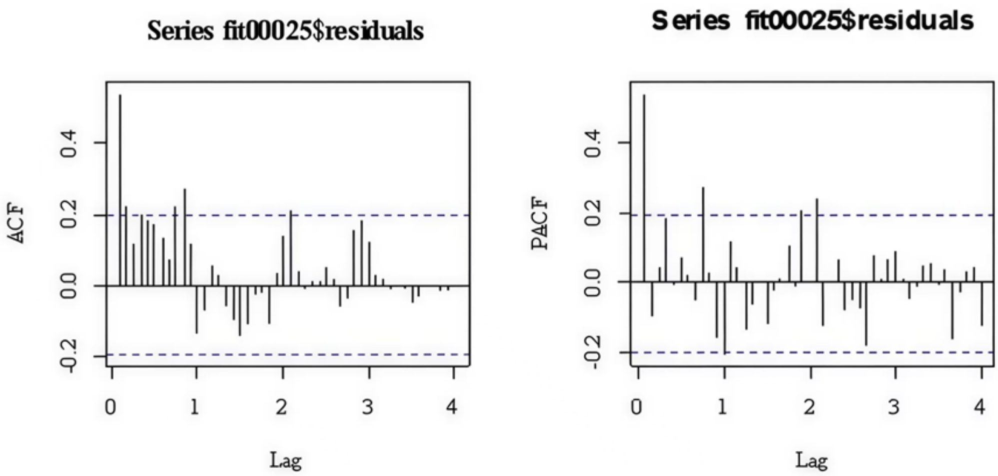


Figure 6. Cross-correlation Diagram of the SO_2 Sequence and the Intervention Variable.

By examining the characteristics of the autocorrelation coefficients and partial autocorrelation coefficients of the residual sequence (Figure 7), it can be considered that the autocorrelation graph is truncated at the 2 order and the partial autocorrelation graph is trailing, or it can also be thought that the autocorrelation graph is trailing while the partial autocorrelation graph is truncated at the 1 order. The seasonal partial autocorrelation coefficients and autocorrelation coefficients are both significantly non-zero at lags of 12 and 24, and 0 at a lag of 36. Both can be regarded as either truncated or trailing respectively, and a total of 9 combinations of model parameters are obtained. After model and parameter tests, only the $SARIMA(0,1,3)(0,1,2)_{12}$ intervention model passes the test, and the model test table (Table 2) is obtained. It can be seen from Table 3 that the MAE (Mean Absolute Error), $RMSE$ (Root Mean Square Error) and $MAPE$ (Mean Absolute Percentage Error) of the model with policy factors added are all smaller than those of the original model. By using the $SARIMA(0,1,3)(0,1,2)_{12}$ intervention model, better prediction accuracy can be achieved. Construction of the intervention model.

This study focuses on the impact of policy interventions on the changes in SO_2concentrations. Although attempts were made to incorporate other potential intervention factors (such as seasonality or meteorological conditions), the statistical results show that the effects of these variables are not significant compared to policy interventions. Therefore, policy factors are the most important external factors leading to changes in the SO_2 content in the atmosphere of Urumqi, and the impacts of other factors are not significant. Thus, only the policy factor as an external factor can be used to construct the SARIMA intervention model for explanation.



(a) ACF Diagram of the Regression Residua Sequence

(b) ACF Diagram of the Regression Residua Sequence

Figure 7. Autocorrelation Diagram and Partial Autocorrelation Diagram of the Regression Residual Sequence.

Table 2. Parameter Test Table of the $SARIMA(0,1,3)(0,1,2)_{12}$ Intervention Model.

$SARIMA(0,0,2)(2,1,0)_{12}$ Intervention Model	$MA(1)$	$MA(2)$	$SAR1(1)$	$SAR2(2)$	X_2xun	
	0.4580	0.2165	–1.0546	–0.4645	2.2606	pass
SE	0.1143	0.1028	0.1134	0.1113	1.6138	

Table 3. Comparison Table of Prediction Accuracy of the Two Models.

	MAE	$RMSE$	$MAPE$
$SARIMA(0,1,3)(0,1,2)_{12}$	0.831	1.134	0.163
$SARIMA(0,0,2)(2,1,0)_{12}$ Intervention Model	0.800	1.008	0.154

3.3. Holt-Winters Model

3.3.1. Introduction of the Holt-Winters Model

The Holt-Winters three - parameter exponential smoothing model is mainly used for time - series forecasting and contains three parameters: level, trend, and seasonality. The three-parameter exponential model extends the simple and two-parameter exponential smoothing models by incorporating seasonal components. The simple exponential smoothing model only takes into account the level factor of the data. The Holt two - parameter exponential smoothing model takes into account the level and trend factors, and the three - parameter exponential smoothing model also takes into account the seasonal factor. It can be seen from Figure 8 that the SO_2 sequence has an obvious downward trend and periodicity. Therefore, the three - parameter exponential smoothing model is considered for modeling [10]. The model is divided into two types: the additive model and the multiplicative model. It can be seen from Figure 5 that the seasonal fluctuation amplitude of the sequence is relatively stable and does not depend on the level of the time - series. Therefore, the additive model is used. Based on the optimal fitting principle of the R language, the three parameters are obtained as follows:

$$alpha = 0.1606555 beta = 0.07625786, gamma = 1 \tag{7}$$

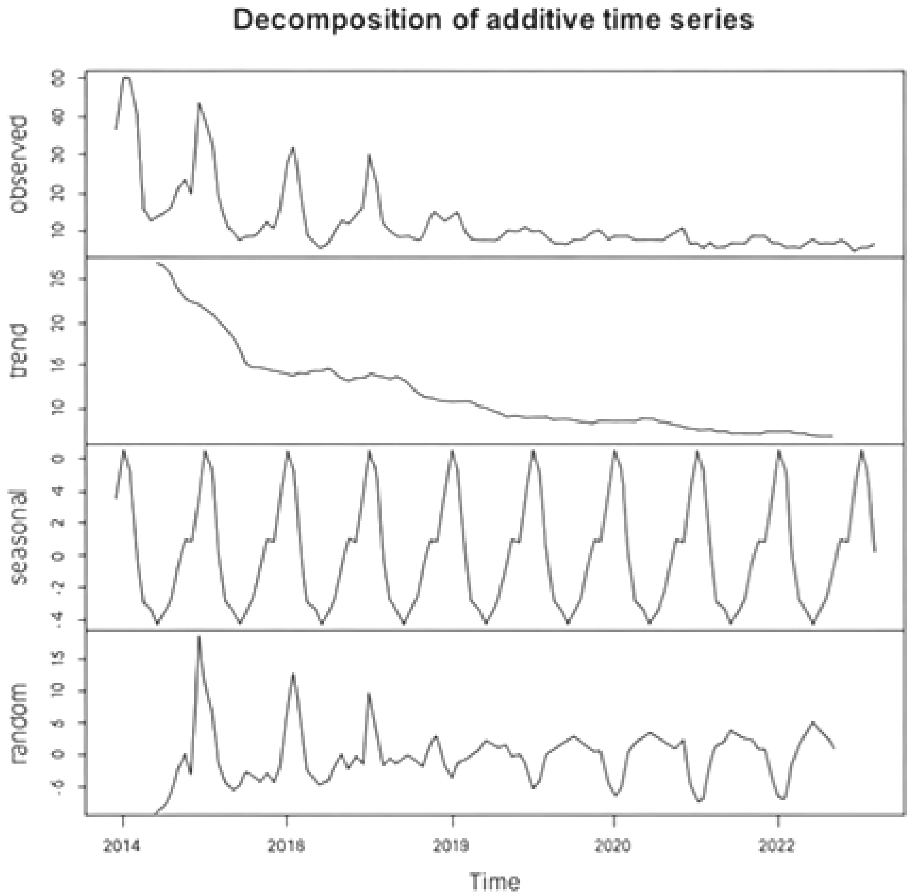


Figure 8. Decomposition Diagram of SO_2 Time Series Data.

3.3.2. Model performance testing and adjustment

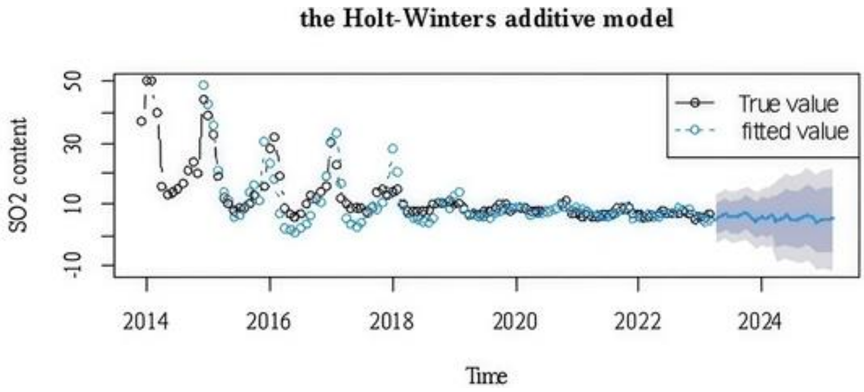


Figure 9. Comparison Diagram of the Prediction Effect of the Holt-Winters Additive Model.

It can be seen from Figure 9 that there is a slight deviation between the true value and the fitted value, and the overall fitting degree is good. Next, a quantitative measurement of the model's prediction effect is carried out, and the results are as follows:

$$MSE = 0.758, MAE = 0.991, MAPE = 0.153 \tag{8}$$

The mean absolute percentage error is 15.3%. The prediction effect using this model is relatively good. The reason for the error is that the three parameters are generated by R language based on the principle of optimal fitting, which has limitations. Therefore, in order to find a more appropriate combination of parameters, the loop traversal method in R language (based on the principle of minimizing the mean square error) is used to search for the optimal combination of parameters. In this way, the error can be reduced and the prediction accuracy of the model can be improved.

3.4. Optimal weighted combined forecasting model

3.4.1. Model Introduction and Selection

The time series prediction combined model is a kind of model that combines two or more single time series prediction methods [11]. Common combination methods include optimal weighted average, concatenation, stacking and so on. Among them, the weighted average method is the most straightforward combination approach [12]. The weights of each model are determined according to principles such as minimizing the mean square error or minimizing the mean absolute percentage error, and the sum of the weights is 1. Then, the prediction results of each model are multiplied by their respective weights respectively, and finally added together to obtain the final prediction result. A single model is easily affected by outliers in the data and the limitations of the model itself. Usually, each model has different applicable conditions. The combined model can make up for the deficiencies of a single model, combine the advantages of each model and improve the prediction accuracy of the model [13].

The advantages of the SARIMA intervention model are that it can fully take into account the influence of external variables and can provide relatively stable prediction results. However, compared with some simple linear models, its interpretability is poorer and the parameter estimation is more complex. The Holt-Winters model has the advantages of being able to effectively capture the seasonality and trend of time series, not requiring complex operations, and having strong interpretability. Its disadvantages lie in the limitations of its assumptions and the limitations in long-term prediction. The combined model of these two can superimpose the advantages of both, thereby improving the prediction accuracy of the model. Based on the principle of minimizing the mean absolute percentage error, this paper conducts a weighted combination of the $SARIMA(0,1,3)(0,1,2)_{12}$ intervention model and the Holt-Winters additive model. Through traversal with a step size of 0.1 for weights using the R language, the optimal weight combination of the $SARIMA(0,1,3)(0,1,2)_{12}$ intervention model and the Holt-Winters additive model is obtained as 0.4 and 0.6. Then, the fitting effect diagram of the predicted values and the true values in the test set (Figure 10) is obtained. It can be seen that in the test set, the true values and the predicted values of the combined model are closer, and the prediction effect is better. It can also be seen from Figure 11 that in the test set, the predicted values of the combined model are closer to the true values. Through the calculation of the combined model, we obtain:

$$MSE = 0.751, MAE = 0.945, MAPE = 0.141 \tag{9}$$

The MAE, $RMSE$ and $MAPE$ of the combined model are all smaller than those of the single models, which quantitatively measures that the combined model has higher prediction accuracy.

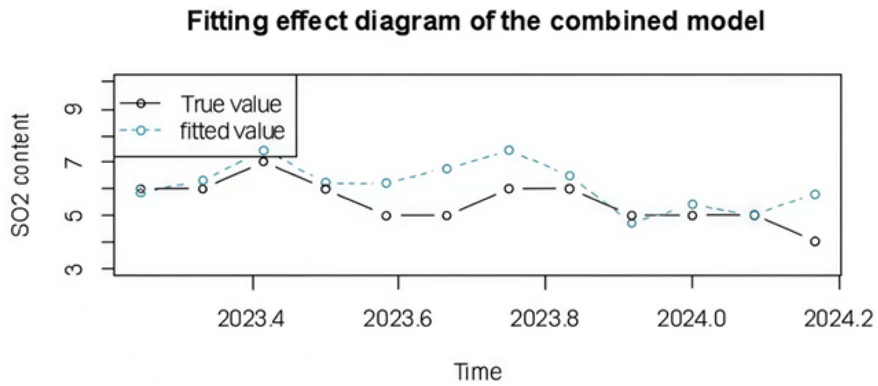


Figure 10. Fitting Effect Diagram of the Optimal Weighted Combined Model

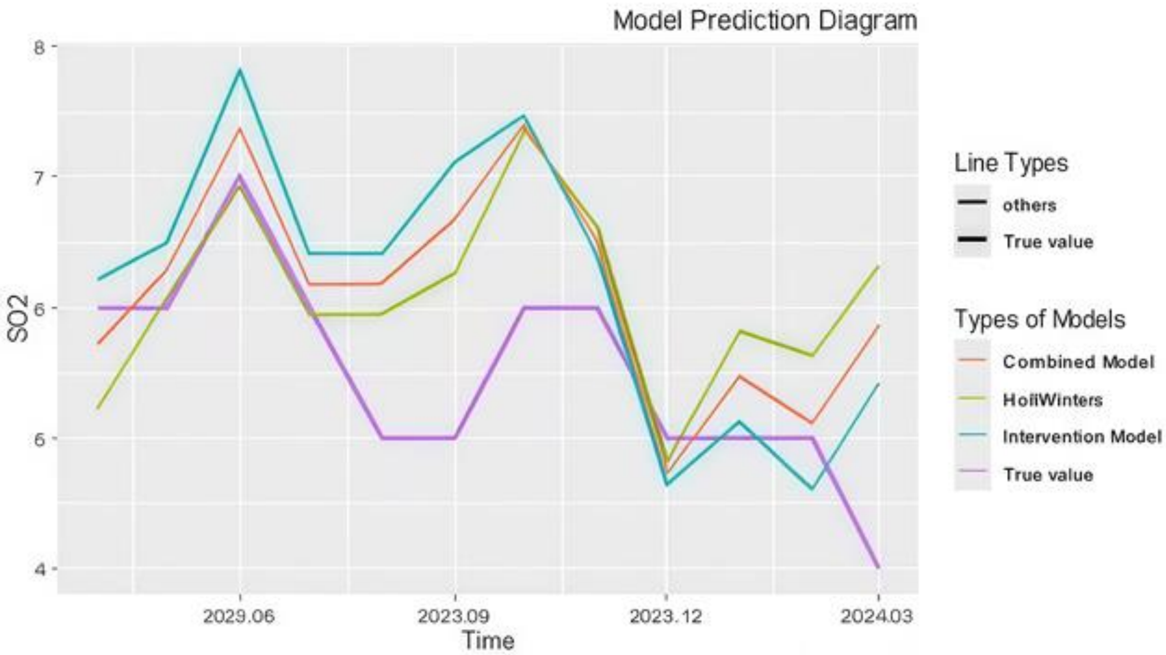


Figure 11. Fitting Effect Comparison Chart of Three Models.

4. Model prediction

The $SARIMA(0,1,3)(0,1,2)_{12}$ intervention model and the Holt-Winters additive model were used respectively to predict the content of SO_2 in the atmosphere of Urumqi from April 2024 to March 2025. And the prediction results of the combined model were obtained through weighting. Then, the prediction effect diagram of the three models (Figure 12) was obtained. It can be seen that the content of SO_2 in the atmosphere of Urumqi in the next year will be within the range of $3\text{-}8ug/m^3$, all remaining within the normal level and tending to be stable.

It can be concluded that the content of SO_2 in the atmosphere of Urumqi will always remain at the normal level in the next year, and the air quality has been effectively improved. This is related to the efforts of the government and the people. It is hoped that in the future, the local government will continue to implement environmentally friendly strategies, organize environmental protection knowledge lectures, social activities and other activities, so that residents can have a deeper understanding of the causes, hazards and prevention methods of air pollution, thereby enhancing residents' environmental protection awareness and sense of responsibility.

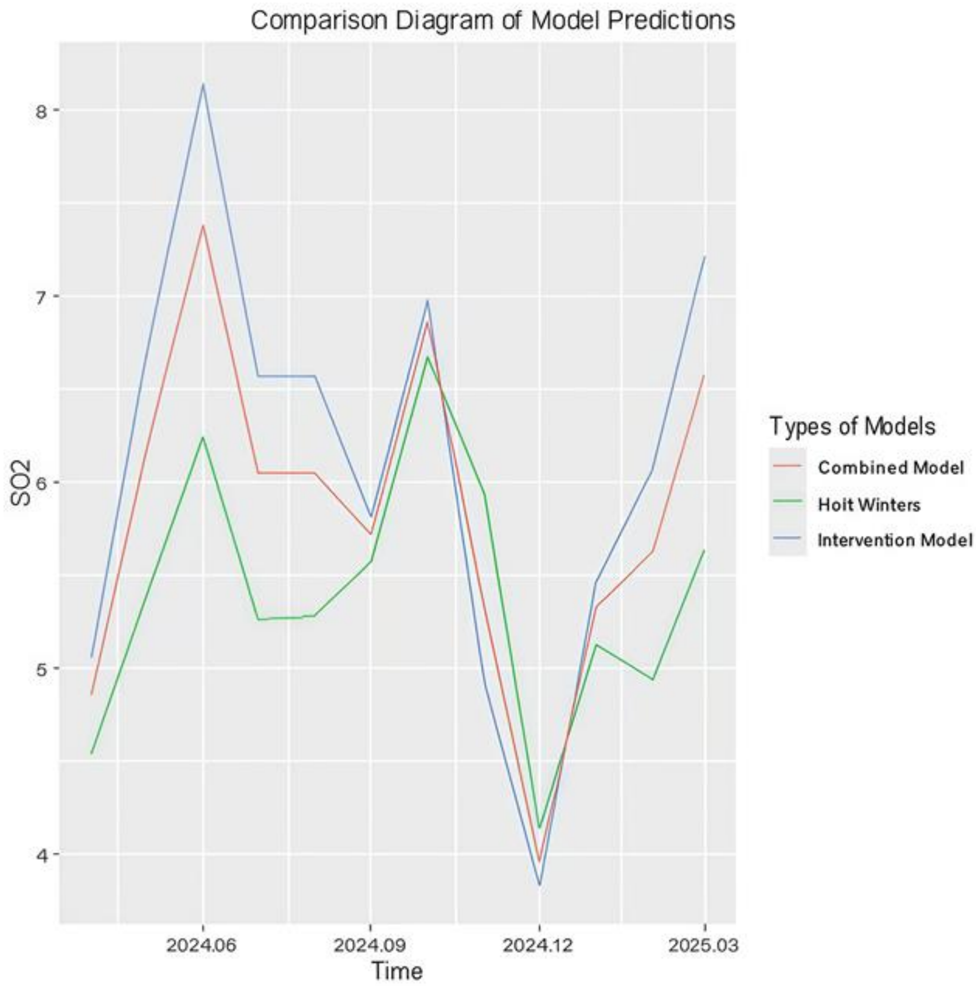


Figure 12. Prediction Effect Diagrams of Three Models

5. Conclusion

In predicting the content of air pollutants, the optimal weighted combination model outperforms single models in terms of prediction accuracy and stability. Therefore, it can be adopted when forecasting the air quality of a region. Regarding combination models, the current major challenges lie in determining which models to combine and what combination methods to use. An increasing number of researchers have delved deeper into these two issues. It is believed that the application of combination models will become increasingly widespread in the future.

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References

[1] Liu Mao, Wang Qingqing, Ding Zhen. Time - series Prediction and Analysis of Atmospheric SO_2 Concentration in Nanjing Based on ARIMA Model [J/OL]. Journal of Environment and Health, 2025, 1 - 4.

[2] Jin Lishan, Liu Fang. Research on the Changing Trend of Air Quality in Hohhot from 2019 to 2022 [J]. Journal of Environment and Health, 2024, 41 (08): 698 - 701.

[3] He Yuhan. Prediction of PM2.5 Concentration Based on STL - Decomposition and LSTM - RF Combined Model [J]. Changjiang Information & Communications, 2024, 37 (10): 38 - 42.

[4] Ye Chenfeng, Xu Ganggang, Gao Mengjie, et al. Application of Intervention Analysis and Seasonal SARIMA Model in the Incidence of Schistosomiasis in Hubei Province [J/OL]. Journal of Ningxia University (Natural Science Edition), 1 - 6 [retrieved January 15, 2025].

[5] Zhu Jiang, Meng Yang, Chen Yongjun, et al. Prediction of the 2024 Varicella Epidemic Trend in Baoshan District of Shanghai Based on the SARIMA Model [J]. Infectious Disease Information, 2024, 37 (06): 541 - 545.

[6] Jiang Yilin, Wang Jian, Liang Duoduo, et al. Analysis and Prediction Research on the Pollution Characteristics of Atmospheric Particulate Matter in Alar City Based on the SARIMA Model [J]. Arid Environmental Monitoring, 2024, 38(04): 179 - 185.

[7] Li Yu, Liu Minxia, Guan Chengxuan, et al. Pollution Characteristics and Prediction of Near - surface PM2.5 and O_3 in the Yangtze River Delta Region [J/OL]. Acta Scientiae Circumstantiae, 1 - 12 [retrieved January 15, 2025].

[8] Hu Xuejun. Research on Prediction of Pump Inspection Rate for Pumping Wells Based on SARIMA [J]. Chemical Engineering & Equipment, 2024, (09): 110 - 112, 152.

[9] Zhang Haodan, Gui Guoping, Guo Feng, et al. Impact of COVID - 19 Pandemic on Hand, Foot and Mouth Disease Based on Intervention Analysis Model [J]. Chinese Journal of Disease Control & Prevention, 2023, 27(03): 339 - 344.

[10] Ouyang Di, Cui Jia. A Study on the Temporal and Spatial Dynamics of Air Pollutants in Harbin City under the Carbon Neutrality Strategy — Based on the Holt - Winters Time Series Model [J]. Environmental Protection Science, 2023, 49 (02): 112 - 119.

[11] Zhou Xiaoye, Cheng Jiahui, Dai Sicong. Optimal Weighted Combined Prediction of Express Delivery Volume Based on Holt - Winters and PSO - SVR [J]. Mathematics in Practice and Theory, 2023, 53 (07): 51 - 60.

[12] Huang Shenglong, Yuan Hongjun, Hu Lingyun. Combined Prediction Analysis Based on Genetic Algorithm for Weight Optimization [J]. Journal of Shanxi Datong University (Natural Science Edition), 2024, 40 (06): 48 - 53.

[13] Li Hongbing, Tang Chenghao, Liu Ke, et al. Research on the Application of the Optimal Combination Model in Predicting the Number of Earthquake Casualties [J]. Modern Occupational Safety, 2024, (12): 19 - 26.