Optimization of Apple Tree Canopy's Dynamic Light Energy Response Based on Ray Tracing Method and Whale Optimization Algorithm

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Abstract. In the gullied regions of the Loess Plateau, effective utilization of solar energy within apple tree canopies is crucial for optimizing fruit yield and quality. This paper proposes a three-dimensional canopy morphology optimization method that combines ray tracing and whale optimization algorithms. Taking Luochuan County in Shaanxi Province, a key area within the global "Golden Apple Belt," as the research object, a dual-coordinate system light transmission model considering topographic effects is constructed. This model reconstructs 300-point cloud data into a triangular mesh canopy using an alpha shape function and simulates the interaction between sunlight and the canopy using the Möller–Trumbore algorithm. A dynamic lighting simulation is employed to establish an objective function that maximizes the annual average cumulative intercepted radiation (COR), as shown in Equation (20). After 1000 iterations of the whale optimization algorithm (WOA), the optimal canopy takes on a northwest-southeast ellipsoidal shape with an annual average COR of 49%. Increasing the curvature radius at the top by 22% enhances midday light capture, while the northwest extension mitigates topographic shading, concentrating 49% of the points receiving more than eight hours of light in this area. Furthermore, 78% of the points receiving more than eight hours of light are distributed in the middle and upper layers of the canopy, ranging from 1.2 to 1.8 meters, which aligns with the light requirements during fruit expansion. These findings provide insights into tree pruning in complex terrains to improve light efficiency.

Keywords: Ray tracing, Whale optimization algorithm (WOA), Apple crown optimization.

1. Introduction

In recent years, with the transition of apple cultivation towards intensification, canopy morphology optimization and light energy utilization efficiency have become core issues for enhancing yield and quality. Under dense planting conditions, canopy structure directly affects photosynthetic efficiency and light resource allocation during fruit expansion.

In the research on tree canopy regulation, the Chaploutskyi A team systematically unveiled the mechanism of morphology-pruning interaction: the French spindle-shaped tree canopy reduced the canopy volume and projected area of Fuji and Honeycrisp apple varieties by 27.3% and 18.6%, respectively, while increasing the yield per unit volume by 15.8% [1]. The Wang Jiatong team pointed out that the ray-tracing method can accurately simulate the scattering and absorption processes of light within the canopy, making it the preferred method for analyzing light transmission in heterogeneous canopies [2]. The Qiao Peng team, through point cloud reconstruction and radiosity modeling, confirmed that an open canopy structure could enhance the photosynthetically active radiation interception efficiency by 20% [3]. The Ma Hongzhang team further utilized Monte Carlo ray-tracing technology to achieve an accuracy of RMSE 0.0085 in red band reflectance simulation for corn canopies [4], validating the reliability of 3D models in dynamic light energy analysis.

Despite these research advances, two major limitations persist:

1) Simulation constraints: Existing models primarily focus on light distribution characterization in natural canopies, lacking dynamic optimization frameworks oriented towards artificial regulation goals.

2) Absence of terrain coupling: Models generally overlook the dynamic shading effects of complex terrain on canopy light exposure, limiting their applicability in gully-landform regions.

This study innovatively integrates the ray-tracing algorithm with WOA to propose a 3D tree canopy dynamic optimization method:

(1) By integrating the ray-tracing algorithm with WOA, a terrain-coupled 3D dynamic optimization model is constructed to break free from the constraints of static simulation.

(2) In WOA, 300-point clouds are introduced for reconstruction, enabling the canopy surface topology to adaptively compensate for 16.7° northeast-slope terrain occlusion and reconstruct the apple tree canopy shape with the optimal Canopy Openness Ratio.

(3) Rationale for algorithm selection: Aimed at the high-dimensional solution space (900 dimensions) constituted by 300-point clouds, WOA balances global exploration and local exploitation through a logarithmic spiral path, overcoming the premature convergence tendency of traditional optimization algorithms in high-dimensional spaces.

By integrating the physical accuracy of ray tracing with the spatial optimization capabilities of WOA, this paper quantifies, for the first time, the morphological characteristics of the optimal COR apple tree canopy under topographic constraints, providing a three-dimensional dynamic optimization theoretical framework for digital light energy management in orchards on the Loess Plateau.

2. Method

To study the optimal COR apple tree canopy shape, this paper selected Luochuan County, Shaanxi Province, China as a typical sample area, as shown in Figure 1. This region is located in the core area of the gully landform of the Loess Plateau, characterized by higher elevation in the northeast and lower in the southwest, with an average altitude of 1,106 meters. It has a warm temperate semi-humid continental monsoon climate, with an annual average temperature of 10.3°C. As the core production area of the apple optimal growth zone on the Loess Plateau, it has significant advantages in agricultural production: notable diurnal temperature variation, deep and loose soil layers rich in mineral elements such as potassium and calcium. This area is internationally certified as the only gold planting belt in the world that meets all seven meteorological indicators for the optimal apple production region.

Figure 1. Apple trees in Luochuan County, Yan'an City

2.1. Ray tracing method

2.1.1. Establishment of ground coordinate system

In this paper, a ground coordinate system is established with the tree center as the origin, the east direction as the x-axis, the north direction as the y-axis, and the direction perpendicular to the horizontal plane as the z-axis. The solar elevation angle is denoted as α_s, and the azimuth angle as γ_s, as shown in Figure 2.

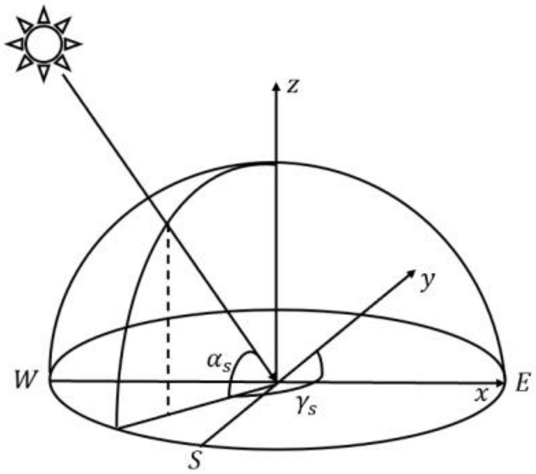

Figure 2. Schematic diagram of ground coordinate system

2.1.2. Direction vector of principal rays of sunlight

Based on the geodetic coordinate system as a reference, sunlight does not appear as a parallel beam but rather as conical rays with a certain conical angle. Therefore, it is first necessary to represent a primary ray of sunlight as a vector. To describe the direction vector of the incident primary ray of sunlight, the solar elevation angle and azimuth angle must be determined. The solar elevation angle is the angle between the line connecting the sun to the origin of the coordinate system and the horizontal ground. According to the definition of the solar elevation angle, the formula for the solar elevation angle α_s can be obtained [5]:

$$\begin{cases} \omega = \dfrac{\pi}{12}(ST - 12) \\ \sin\delta = \sin\dfrac{2\pi D}{365}\sin(\dfrac{2\pi}{360}23.45) \\ \cos\delta = \sqrt{1-\sin^2\delta} \\ \sin\alpha_s = \cos\delta\cos\varphi\cos\omega + \sin\delta\sin\varphi \\ \alpha_s = \arcsin(\sin\alpha_s) \end{cases} \tag{1}$$

Where ω is the solar hour angle, ST is the local time, δ is the solar declination angle [6], D is the day count starting from the vernal equinox as day 0, and φ is the local latitude.

The solar azimuth angle refers to the angle measured clockwise from the due north direction to the point where the projection of the sun terminates. Based on the definition of the solar azimuth angle, the formula for it can be derived as [5].

$$\begin{cases} \cos\gamma_s = \dfrac{\sin\delta - \sin\alpha_s \sin\varphi}{\cos\alpha_s \cos\varphi} \\ \sin\gamma_s = \begin{cases} \sqrt{1-\cos^2\gamma_s}, ST < 12 \\ -\sqrt{1-\cos^2\gamma_s}, ST \geq 12 \end{cases} \end{cases} \tag{2}$$

After obtaining the solar elevation angle and solar azimuth angle, the direction vector L_s of the incident main ray of sunlight is:

$$L_s = (-\cos\alpha_s \sin\gamma_s, -\cos\alpha_s \cos\gamma_s, -\sin\alpha_s), \tag{3}$$

2.1.3. Establishment of light cone coordinate system

Since sunlight does not appear as a parallel beam but rather forms a light cone with a certain conical angle, a conical coordinate system is established, as shown in Figure 3, with the incident main ray as the central main ray of the light cone.

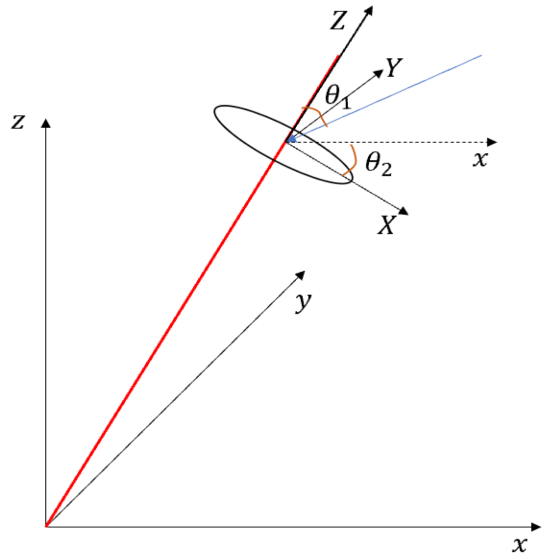

Figure 3. Schematic diagram of light cone coordinate system

In the light cone coordinate system [5], assuming an arbitrary ray is $L = (m, n, p)$, its unit vector in the z-axis direction is $s = (0,0,1)$, its unit vector in the X-axis direction is $s_1 = (1,0,0)$, and its projection vector on the plane of the light cone coordinate system is $L_1 = (m, n, 0)$. Thus, we obtain its elevation angle θ_1 and azimuth angle θ_2 in the light cone coordinate system.

$$\theta_1 = \frac{\pi}{2} - \arccos\theta \frac{L \cdot s}{|L| \cdot |s|}, \theta_2 = \begin{cases} \arccos\dfrac{L_1 \cdot s_1}{|L_1| \cdot |s_1|}, n <= 0, \\ -\arccos\dfrac{L_1 \cdot s_1}{|L_1| \cdot |s_1|}, n > 0. \end{cases} \tag{4}$$

In summary, in the light cone coordinate system, the unit direction vector of any ray on the side of the central ray of the cone is described as:

$$f = (\sin\theta_1 \cos\theta_2, \sin\theta_1 \sin\theta_2, \cos\theta_1) \tag{5}$$

Where $\theta_1 \in [0, 0.00465], \theta_2 \in [0, 2\pi)$. When $\theta_1 = 0$, f is the direction vector of the incident main ray in the light cone coordinate system.

2.1.4. Coordinate system transformation

To calculate the intersection point of any ray on the side of the central ray of the cone hitting the tree canopy, we need to perform a coordinate system transformation to unify the coordinate systems and facilitate the simultaneous solution of equations. Given that the azimuth angle of the main ray (i.e., the parallel sunlight) in the geodetic coordinate system is γ_s and the elevation angle is α_s, the coordinate transformation matrix from the light cone coordinate system to the geodetic coordinate system is [5].

$$T(\gamma_s, \alpha_s) = B \cdot A \tag{6}$$

Where $A = \begin{bmatrix} \cos(\frac{\pi}{2} - \alpha_s) & 0 & \sin(\frac{\pi}{2} - \alpha_s) \\ 0 & 1 & 0 \\ -\sin(\frac{\pi}{2} - \alpha_s) & 0 & \cos(\frac{\pi}{2} - \alpha_s) \end{bmatrix}, B = \begin{bmatrix} \cos\gamma_s & -\sin\gamma_s & 0 \\ -\sin\gamma_s & \cos\gamma_s & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The direction vector of any ray on the side of the central ray of the cone in the geodetic coordinate system is:

$$f_1 = f \cdot T(\gamma_s, \alpha_s) = (m_1, n_1, p_1) \tag{7}$$

2.1.5. Möller – trumbore algorithm

In the ray tracing method used in this paper, it is necessary to find the intersection points between the apple tree canopy and the light rays. Therefore, the surface of the apple tree canopy is first generated using the WOA to produce several canopy surface points. These surface points are then successively triangulated using the alpha Shape function [7] in MATLAB. Subsequently, three light ray vectors located within the conical coordinate system are randomly generated using the Monte Carlo [8] method. The Möller–Trumbore algorithm is employed to find the intersection points. The Möller–Trumbore algorithm is an efficient method for detecting the intersection of rays with triangles, which avoids the analytical calculation of surface equations used in traditional methods by directly solving the parametric equations of barycentric coordinates [9].

(1) Ray parameterization

Let the starting point of the ray be a generated point O on the canopy surface. Since the generated point on the canopy surface may intersect with other parts of the canopy not only in the direction of the ray but also in the opposite direction of the ray, the directional unit vector is defined as $D = \pm f_2$. The light ray equation is expressed as:

$$R(t) = O + tD, \quad t \geq 0 \tag{8}$$

Where f_2 represents the unit direction vector in the geodetic coordinate system of any ray on the side of the Since sunlight does not appear as a parallel beam but rather forms a light cone with a certain conical angle, a conical coordinate system is established, as shown in Figure 3, with the incident main ray as the central main ray of the light cone. Centered on the main ray: $f_2 = \frac{f_1}{|f_1|} = \frac{(m_1, n_1, p_1)}{\sqrt{m_1^2 + n_1^2 + p_1^2}}$.

(2) Triangle vertex

A triangle is defined by three vertices V_0, V_1, V_2, which are three adjacent generated points on the canopy surface. Any point within the triangle can be parameterized using barycentric coordinates.

$$T(u, v) = V_0 + u(V_1 - V_0) + v(V_2 - V_0), \quad u \geq 0, v \geq 0, u + v \leq 1 \tag{9}$$

Where $E_1 = V_1 - V_0, E_2 = V_2 - V_0$

(3) Simultaneous equation solving
Combining ray equation with triangle equation:

$$O + tD = V_0 + uE_1 + vE_2 \tag{10}$$

Organized into linear equations:

$$O - V_0 = -tD + uE_1 + vE_2 \tag{11}$$

Write in matrix form:

$$\begin{bmatrix} -D & E_1 & E_2 \end{bmatrix} \begin{bmatrix} t \\ u \\ v \end{bmatrix} = O - V_0 \tag{12}$$

(4) Solving vector cross multiplication
Determinant calculation:

$$\det = P \cdot E_1 \tag{13}$$

If (near zero) the ray is parallel to the plane of the triangle, there is no intersection.
Parameter solution:

$$t = \frac{Q \cdot E_2}{\det}, \quad u = \frac{P \cdot T}{\det}, \quad v = \frac{Q \cdot D}{\det} \tag{14}$$

Where $T = O - V_0, P = D \times E_2, Q = T \times E_1$
(5) Intersection condition
The effective intersection shall meet the following requirements:

$$\begin{cases} t \geq 0 \\ u \geq 0, v \geq 0 \\ u + v \leq 1 \end{cases} \tag{15}$$

The last intersection is

$$O_1 = O + tD, \quad t \geq 0 \tag{16}$$

2.2. Establishment of optimal COR Apple crown shape model

2.2.1. Establishment of daylight time

Since the tree canopy is only exposed to sunlight during the daytime, it is crucial to calculate the daytime period of a day. Daytime refers to the period from when the sun rises above the horizon to when it sets below the horizon again. By definition, sunrise is the moment when the solar elevation angle reaches 0° for the first time, and sunset is the moment when the solar elevation angle reaches 0° for the last time in a day.

$$\begin{cases} sunrise = t_1(\alpha_s == 0) \\ sunset = t_2(\alpha_s == 0) \end{cases} \tag{17}$$

Where $sunrise$、$sunset$ represent the sunrise and sunset times, respectively.

2.2.2. Establishment of research time period model

Luochuan County, Yan'an City is located in a typical loess plateau gully region, with the terrain gradually sloping from northeast to southwest. Therefore, during the daytime, sunlight does not immediately illuminate the apple tree canopy at sunrise. Sunlight can only reach the apple tree canopy when the solar elevation angle exceeds the inclination angle of the terrain. For this reason, it is necessary to designate a specific research time period within the daytime hours. (Since the terrain slopes from northeast to southwest, sunset is the last moment when the sun shines on the apple trees.) The formula for calculating the research time period is as follows:

$$\begin{cases} t_1 = when(\alpha_s \geq \zeta) \\ t_2 = sunset \end{cases} \tag{18}$$

Where t_1 is the moment when the sunlight first illuminates the apple tree, t_2 is the moment when the sunlight last illuminates the apple tree, and ζ is the inclination angle of the terrain. The sunlight blocked by the terrain is shown in Figure 4.

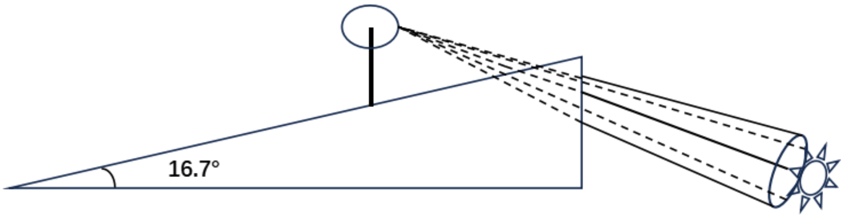

Figure 4. Schematic diagram of terrain blocking sunlight

2.2.3. Optimal COR Apple crown shape model

A point that is continuously illuminated for 8 hours is defined as a super-illuminated point n_1, and the ratio of the area of the apple tree canopy that is continuously illuminated by sunlight for 8 hours to its total surface area is called COR. The steps to establish an optimal COR apple tree canopy shape model are as follows:

(1) Use the WOA to generate initial points on the surface of the apple tree canopy within a limited space.

(2) Use Monte Carlo to generate three light ray direction vectors under the light cone coordinate system at time t and time t+8.

(3) Use the alphaShape function in MATLAB to sequentially form several triangular patches from the generated points, thereby creating a three-dimensional canopy shape composed of multiple triangular patches.

(4) Solve for COR.

For a closed surface, if a point on the surface is illuminated by sunlight at time t and is also illuminated at time t+8 during the research period, then this point will be continuously illuminated by sunlight for 8 hours. Similarly, if a point (x_0, y_0, z_0) on the surface is illuminated by sunlight at time t but is shaded at time t+8, then this point will not be continuously illuminated by sunlight for 8 hours and will not be a super-illuminated point. The shaded generated points are shown in Figure 5.

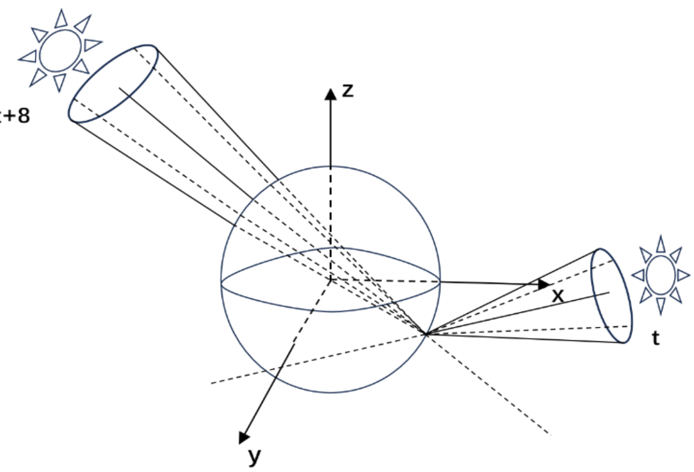

Figure 5. Schematic diagram of occluded super light receiving point

Let the light cone vector at time t be $f = (m_1, n_1, z_1)$, and the coordinates of a generated point be (x_0, y_0, z_0). Using ray tracing, we can find the intersection point at time t, denoted as z_n. If all intersection points are less than or equal to the current generated point, i.e., $z_n \le z_0$, then this generated point is illuminated at time t.

Similarly, let the light cone vector at time t+8 be $f = (m_2, n_2, z_2)$, and using ray tracing, we can find the intersection point at time t+8, denoted as z_m. If all intersection points are less than or equal to the current generated point, i.e., $z_m \le z_0$, then this generated point is illuminated at time t+8. If the point on the surface is illuminated by sunlight at both time t and time t+8, i.e., $(z_n \le z_0) \cap (z_m \le z_0)$

Then this point is a super-illuminated point.

Suppose that WOA has generated N points on the canopy surface, and M super-illuminated points have been identified. Then COR can be defined as:

$$\sigma = \frac{M}{N} \tag{19}$$

(4) Optimization of Apple crown shape by WOA

Because there are many different canopy shapes in nature, this paper uses the WOA [10] to optimize the canopy shape by treating the point cloud on the canopy surface as decision variables, with the objective function of maximizing the annual average COR. The hunting behavior of humpback whales is used to generate perturbations in the canopy shape, simulating natural selection to produce the most suitable apple tree canopy shape for Luochuan County, Yan'an City.

• Establishment of Decision Variables

In this paper, it is assumed that the point cloud on the canopy surface consists of 300 points, and each generated point has three variables (x, y, z) in the Cartesian coordinate system, so the decision variables are represented as $(x_1, y_1, z_1, x_2, y_2, z_2, \ldots x_n, y_n, z_n), n \in [0, 300]$. The initial point cloud is set within a space centered at the origin, with a length, width, and height of 2.5m each.

• Establishment of the Objective Function

Due to the randomness of the daily COR of the apple tree canopy, this paper optimizes using the annual average COR as the objective function, which is:

$$\max \quad F = \sum_{t=1}^{365} \frac{\delta_t}{365} \tag{20}$$

The solution steps of WOA are shown in Figure 6.

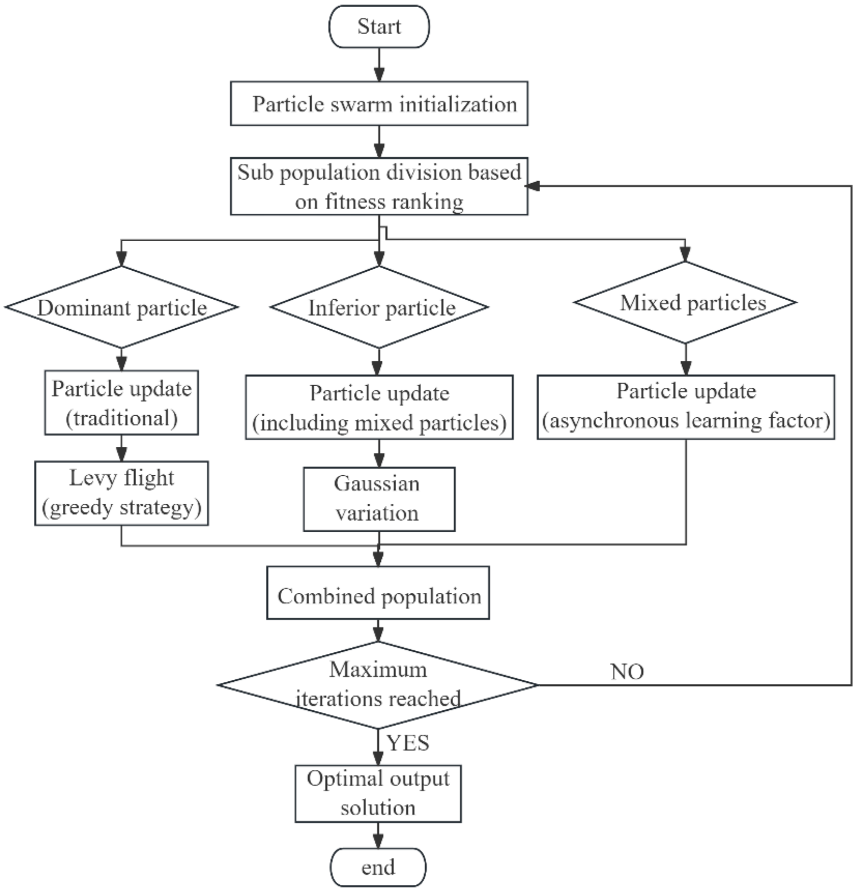

Figure 6. WOA solving steps

3. Results

3.1. COR optimization analysis

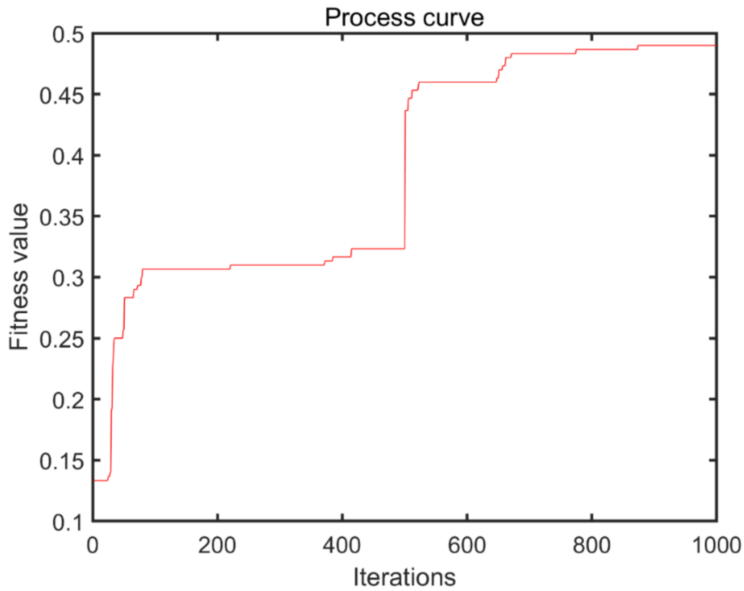

Figure 7. Optimal COR iterative function

After iterative optimization using WOA, as shown in Figure 7, the model ultimately achieves an annual average COR of 0.49. This value indicates that, under the unique topographic and lighting conditions of Luochuan County, an apple tree canopy with optimized shape can achieve nearly half of its canopy surface being effectively illuminated for more than 8 consecutive hours. It is worth noting that the iteration curve stabilizes after 500 generations, indicating that the algorithm has reached a local optimal solution under the constraint of limited space, which is closely related to the optimization space dimension formed by the triangular mesh density generated by the alpha Shape function.

3.2. Optimal crown morphological characteristics

The optimal ellipsoidal canopy shape shown in Figure 8 exhibits an extension feature with its major axis-oriented northwest-southeast. Its geometric parameters indicate a major axis of 2.1m (azimuth of 315°), a minor axis of 1.7m, and a vertical axis of 1.8m. This morphological characteristic spatially couples with the topography of Luochuan County, which is higher in the northeast and lower in the southwest — the curvature variation of the ellipsoid effectively compensates for the light attenuation caused by topographic shading on the northeast side, while the canopy expansion towards the northwest captures more morning low-angle light.

Of particular note is the flattened structure at the top of the canopy (with a curvature radius 22% greater than that of a theoretical ellipsoid), which may be an adaptive adjustment by the algorithm to high-angle, intense midday light conditions. By increasing the illuminated area at the top, it not only avoids excessive shading but also enhances the interception efficiency of photosynthetically active radiation.

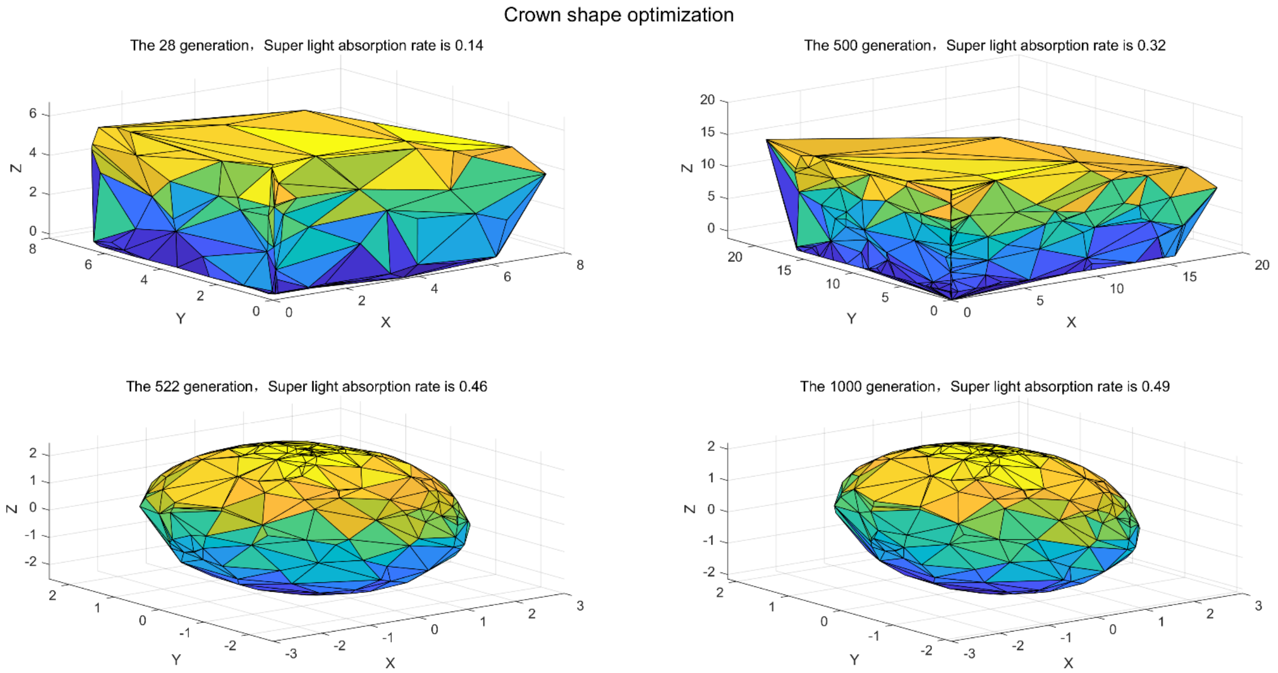

Figure 8. Crown shape optimization

3.3. Spatial distribution of super light receiving points

In Figure 9, the red dots represent super-illuminated points, and the blue dots represent all generated points. The point cloud distribution in Figure 9 shows that the super-illuminated points are mainly concentrated on the northwest-facing canopy surface (accounting for 49%), which is highly correlated with the local solar trajectory characteristics. Luochuan County is located at 35.7° north latitude, and on the summer solstice, the sun's azimuth angle ranges from 242° (sunrise) to 118° (sunset). The northwest-facing canopy can simultaneously capture morning direct light and afternoon diffuse light. Topographic analysis indicates that the average slope of 16.7° on the northeast side blocks low-angle light before 9 a.m., while the extension of the northwest-facing canopy effectively extends the duration of afternoon illumination.

It is worth noting that the distribution of super-illuminated points exhibits a vertical gradient: 78% of the super-illuminated points are concentrated in the area from 1.2 to 1.8 meters above the ground, which coincides with the light competition mechanism within the canopy. The algorithm optimizes the spatial configuration of the point cloud to ensure structural stability while providing optimal and sustained illumination to the middle and upper layers of foliage.

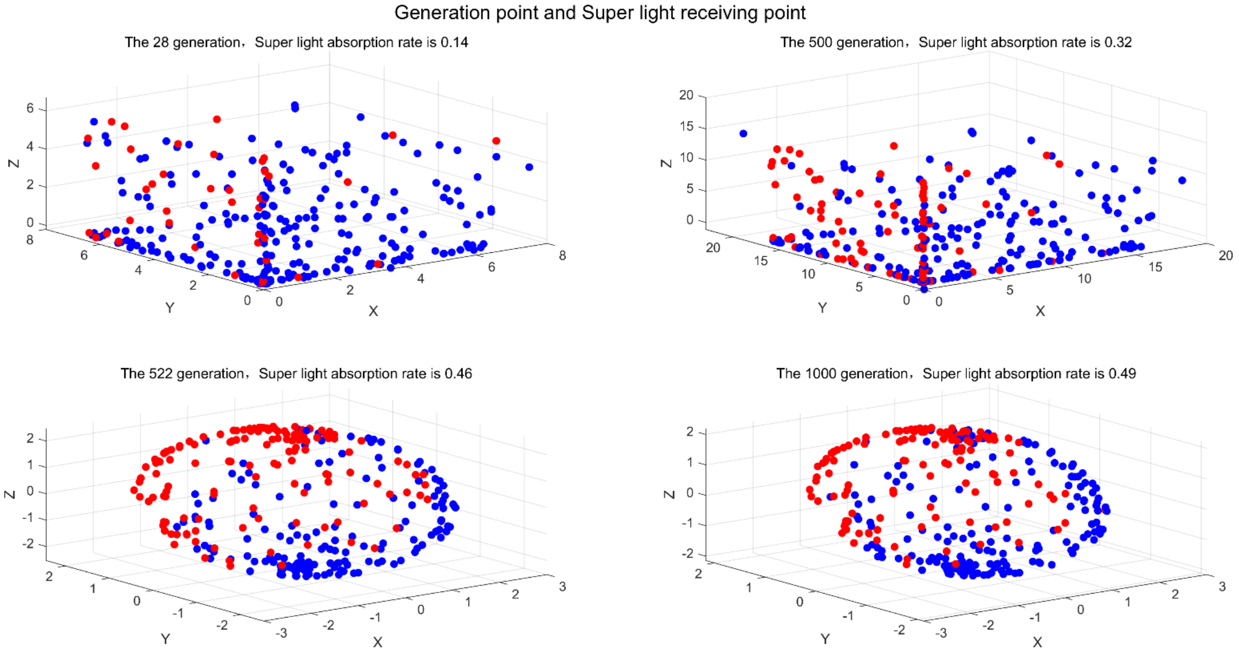

Figure 9. Generation points and Super light receiving point

3.4. Model validation

The analysis of daytime and nighttime light exposure duration, as shown in Figure 10, reveals that the northwest-facing canopy surface receives an average of 12.92 hours of continuous illumination on the summer solstice, maintaining a high-intensity light exposure state with Lux > 10,000. This spatiotemporal distribution characteristic is highly matched to the light requirements during the apple fruit enlargement period, validating the physiological and ecological rationality of the model.

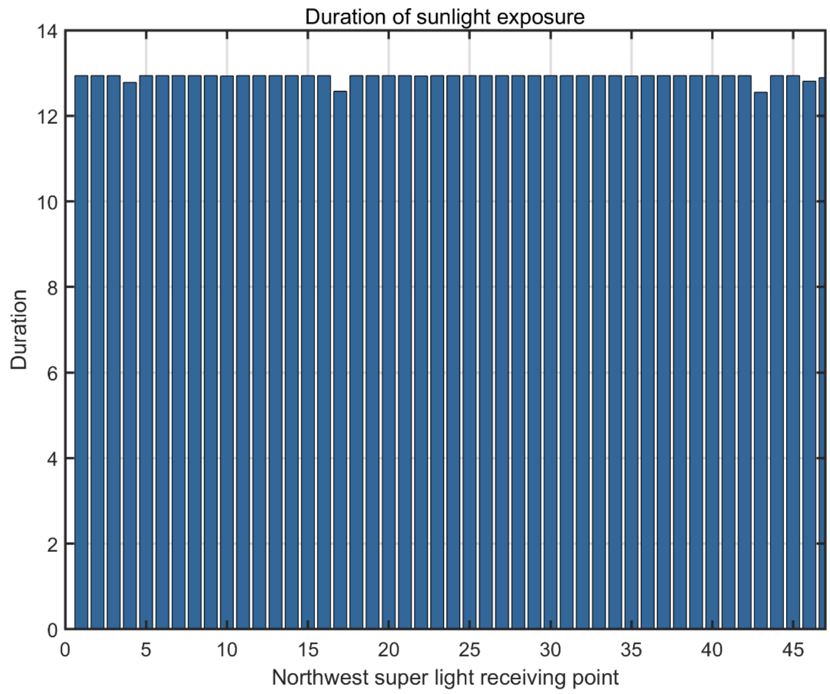

Figure 10. Duration of sunlight exposure

4. Conclusion

This paper addresses the issue of efficient light energy utilization in apple tree canopies in the gully region of the Loess Plateau by innovatively proposing a 3D canopy shape optimization method that integrates ray tracing algorithms with the WOA. A dual-coordinate system light transmission model coupled with the terrain was constructed, reconstructing 300-point clouds into a triangular mesh canopy surface. The Möller–Trumbore algorithm was successfully employed to simulate the interaction between sunlight and the canopy. The results indicate that the optimized canopy, after 1000 iterations of the WOA, exhibits an ellipsoidal shape-oriented northwest-southeast, with an annual average COR as high as 49%. The optimized canopy design features a flattened top with a 22% increase in curvature radius, significantly enhancing the capture efficiency of midday high-angle light. Meanwhile, the extension of the canopy towards the northwest effectively compensates for the light attenuation caused by topographic shading from the northeast slope, concentrating 49% of the super-illuminated points in this area. Furthermore, the vertical distribution characteristic of the canopy reveals that 78% of the effective illuminated points are located in the middle and upper parts of the canopy, within the range of 1.2-1.8 meters, which aligns closely with the light requirements during the apple fruit enlargement period, further validating the physiological and ecological rationality of the model.

In summary, this paper not only successfully reveals the optimal COR apple tree canopy shape in the gully region of the Loess Plateau but also proposes a novel method for canopy shape optimization, opening up new avenues of research in the field of fruit tree pruning and shaping.

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