Research on UAV positioning model based on pure azimuth passive positioning

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Abstract. This paper investigates a pure azimuth passive positioning model for unmanned aerial vehicle (UAV) formations, with a focus on circular formations as an example. The model utilizes the polar coordinate system and geometric principles to derive formulas for calculating the azimuth angle, distance angle, and ideal position coordinates of UAVs. It also accounts for receiving errors and proposes a positioning adjustment method to address deviations from ideal positions. Mathematical induction is employed to unify the relationships between variables under different conditions. The developed model enables the extraction of azimuth angles from received signals and calculates UAV positions within the polar coordinate system. Results demonstrate the model's effectiveness in determining the precise location of UAVs in formations, even under non-ideal conditions. This work fills a gap in the field of UAV formation positioning, providing a theoretical foundation for accurate and reliable positioning. Furthermore, the proposed approach paves the way for future improvements in real-time compensation mechanisms for dynamic errors, with potential applications in diverse UAV formation configurations.

Keywords: Pure Azimuth Passive Positioning, Polar Coordinate System, Plane Geometry Method, Positioning Adjustment.

1. Introduction

In the field of passive positioning, existing research has mainly focused on spaceborne direction finding systems, shortwave direction finding positioning, angle of arrival positioning, lightning positioning, and ground moving target positioning. Huang Kai conducted in-depth research on the direction finding error and positioning error calculation method of spaceborne direction finding systems, but their research mainly focused on satellite platforms and did not involve the complex geometric layout and dynamic adjustment mechanism of unmanned aerial vehicle formations [1]. Zhou Xia analyzed the causes and mitigation methods of shortwave direction finding and positioning errors, but did not consider the multi-source signal interference and dynamic adjustment requirements within the drone formation. Lu Weidong discussed the analysis of dual station arrival angle positioning error, Cai Xinjing studied the positioning accuracy of lightning locator using short baseline time difference method, and Cao Haiyuan analyzed the positioning and accuracy of ground moving targets [3-5]. However, these studies mostly focus on static targets or single platforms, lacking systematic research on dynamic adjustment of drone formations.

The design of pure azimuth passive positioning and adjustment for drone formations is a research direction with important application value. With the rapid development of drone technology, drone formations have been widely used in military reconnaissance, environmental monitoring, logistics transportation, and other fields. However, drone formations require precise positioning and dynamic adjustment during flight to ensure the stability of the formation and efficient completion of tasks. Although the existing passive positioning technology has made significant progress in theory and application, it still faces many challenges in the complex scenarios of unmanned aerial vehicle formations, such as multi-source signal interference, dynamic adjustment requirements, and high-precision positioning requirements.

This article focuses on the design of pure azimuth passive positioning adjustment for unmanned aerial vehicle formation, establishes a positioning model based on polar coordinate system, derives the calculation formulas for ideal position and deviation position, and considers the receiving error angle to complete the establishment of the positioning model. This study not only fills the gap in the design of pure azimuth passive positioning adjustment for drone formations, but also provides new theoretical support for the precise positioning of drone formations, which has important innovation and application value. Through geometric knowledge and mathematical induction, this paper proposes a passive positioning adjustment method suitable for drone formations, which can effectively handle signal reception deviations in drone formations, extract azimuth angles from received signals, and calculate the position of drones in polar coordinates. This method provides a new solution for the dynamic adjustment and precise positioning of drone formations, and is expected to play an important role in the practical application of drone formations.

2. Establishment of Polar Coordinates for Circular UAV Formation

The formation considered in this article is a circular formation consisting of 10 aircraft flying at the same altitude, as shown in Figure 1. A pure azimuth passive positioning and adjustment model is established for the formation of UAVs numbered FY01 to FY09, which are evenly distributed on the circumference, with UAV number FY00 located at the center of the circle. A two-dimensional polar coordinate system is established, and the azimuth angle received by the UAV receiving the signal is defined. Based on the principles of circular geometry and the perpendicularity theorem, a model is developed to derive the calculation formula for the ideal position without deviation. Subsequently, the sine theorem is applied to obtain the calculation formula for the coordinate deviation of the receiving UAV with reception error in the formation with deviation, and further to derive the position coordinate formula of the receiver [6].

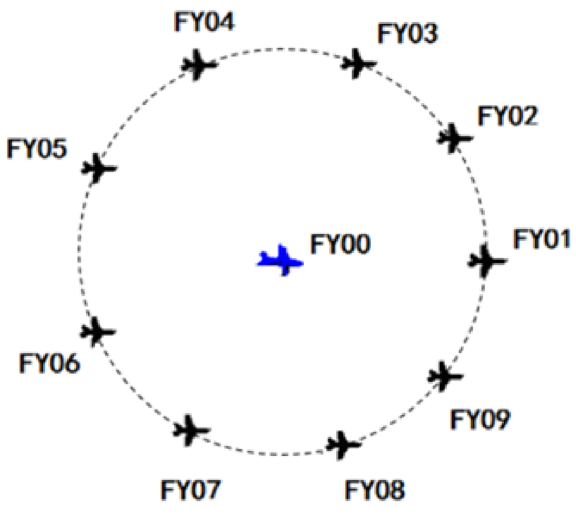

Figure 1. Schematic diagram of circular drone formation

Using drone FY00 as the origin and FY01 as the polar axis, a two-dimensional spatial polar coordinate system is established as shown in Figure 2. To simplify the model, FY01 is fixed as the transmitter, with drone FY01 as point F_1, point F_i as other transmitters on a circular orbit, f_i as their corresponding drone number, point T as the drone receiving the signal, and t as the number of other drones. Since drones cannot simultaneously transmit and receive signals, $f_i \neq t$.

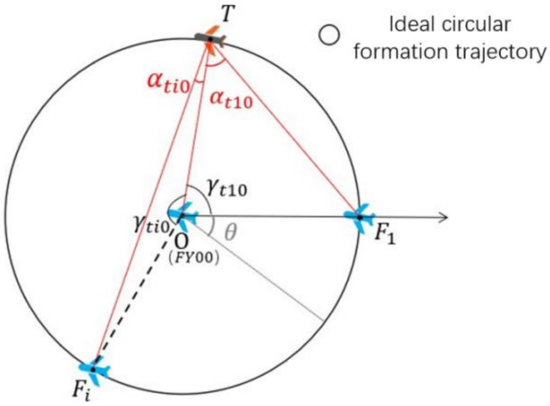

Figure 2. UAV Ideal Circular Formation Flight Coordinate System Setup

3. Calculation of Ideal Position Coordinates

In an ideal formation without deviation, as depicted in Figure 3, the distance angle γ_{ti0} is first determined. Following this, the azimuth angle α_{ti0} is calculated in accordance with the vertical diameter theorem. Subsequently, the position coordinate formula is derived using the sine theorem.

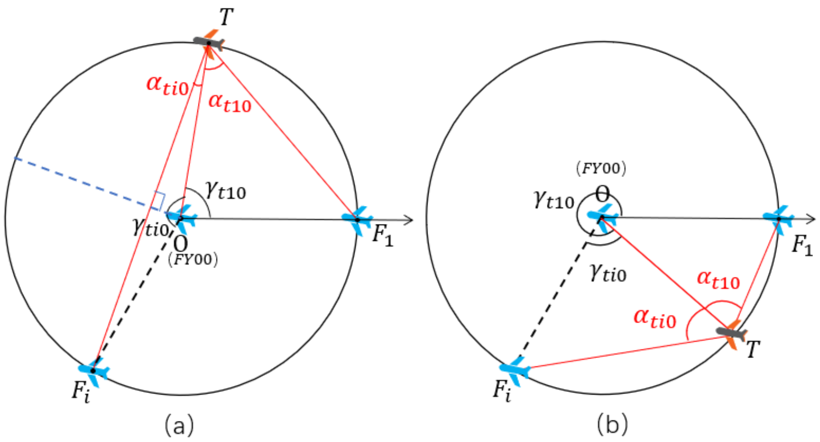

Figure 3. Schematic diagram of ideal formation coordinate system

3.1. Find the ideal distance angle γ_{ti0}

The distance angle $\angle F_iOT$ obtained from the counterclockwise direction of the polar axis with the edge OF_i and the edge OT as the sandwiching edge and O as the vertex, denoted as γ_{ti0}, is shown in Figure 3. For Figure (a):

$$\begin{cases}\gamma_{t10} = (t - f_1)\theta \\ \gamma_{ti0} = (f_i - t)\theta\end{cases} \tag{1}$$

For Figure (b):

$$\begin{cases}\gamma_{t10} = (t - f_1)\theta \\ \gamma_{ti0} = (t - f_i)\theta\end{cases} \tag{2}$$

From equations (1) and (2), it can be seen that γ_{ti0} has different expressions in different situations. To unify the expression of γ'_{ti} in different situations, a variable can be introduced.

$$P_{\gamma i} = \begin{cases}1, f_i > t \\ -1, f_i < t\end{cases} \tag{3}$$

The formula for the distance angle γ_{ti0} is:

$$\gamma_{ti0} = P_{\gamma i}(f_i - t)\theta, \ \ i = 1,2, \dots, k \tag{4}$$

Let f_i be the transmitting drone number, and t be the receiving drone number.

3.2. Find the ideal azimuth angle α_{ti0}

The azimuth angle α_{ti0} is defined as the angle $\angle F_iTO$ with the edge OF_i and the edge OT, and T as the vertex, as shown in Figure 3. For Figure (a), according to the perpendicular radius theorem:

$$\begin{cases} \alpha_{t10} = 90° - \frac{\gamma_{t1o}}{2} \\ \alpha_{tio} = 90° - \frac{\gamma_{tio}}{2} \end{cases} \tag{5}$$

For Figure (b):

$$\begin{cases} \alpha_{t10} = 90° - \frac{360° - \gamma_{t1o}}{2} = \frac{\gamma_{t1o}}{2} - 90° \\ \alpha_{tio} = 90° - \frac{\gamma_{tio}}{2} \end{cases} \tag{6}$$

From equations (5) and (6), it can be seen that α_{tio} has different expressions in different situations. To unify the expression of α_{tio} in different situations, variables can be introduced.

$$P_{\alpha i} = \begin{cases} 1, \gamma_{tio} < 180° \\ -1, \gamma_{tio} > 180° \end{cases} \tag{7}$$

The formula for the distance angle α_{tio} is:

$$\alpha_{tio} = P_{\alpha_i} \cdot \left(90° - \frac{\gamma_{tio}}{2}\right), i = 1,2,\ldots,k \tag{8}$$

3.3. Find the ideal position coordinates (ρ_i, δ_i)

Based on the knowledge of circular geometry, if the radius of the ideal circular formation shape is ρ, the ideal position coordinates of the drone can be obtained as follows:

$$(\rho_i, \delta_i) = (\rho, (i-1)\theta), i = 1,2,\ldots,9, \quad \text{Where } \theta = 40° \tag{9}$$

Obtain the coordinates of each ideal position as shown in Table 1.

Table 1. Ideal Position Coordinate Table for Drones

Drone Number	Drone location（m, °）
FY00	(0, 0)
FY01	(100, 0)
FY02	(100, 40)
FY03	(100, 80)
FY04	(100, 120)
FY05	(100, 160)
FY06	(100, 200)
FY07	(100, 240)
FY08	(100, 280)

4. Calculation of Position Coordinates with Deviation

On the basis of the ideal formation coordinate system, considering the slight deviation of the receiving unmanned aerial vehicle, it is assumed that the deviation of the passive receiving signal unmanned aerial vehicle will not exceed the distance between the two unmanned aerial vehicles. The unmanned aerial vehicles with deviation can be divided into four cases as shown in Figure 4. For the convenience of research, the deviation effect is amplified here. At this time, the unmanned aerial vehicle emitting the signal still has a deviation on the ideal circular formation orbit, and the position coordinates of the unmanned aerial vehicle with deviation receiving the signal are $T'(\rho_t{}', \delta_t{}')$[7]. Deviation coordinates:

$$(\Delta\rho_t, \Delta\delta_t) = (\rho_t' - \rho_{t0}, \delta_t{}' + \delta_{t0}) \tag{10}$$

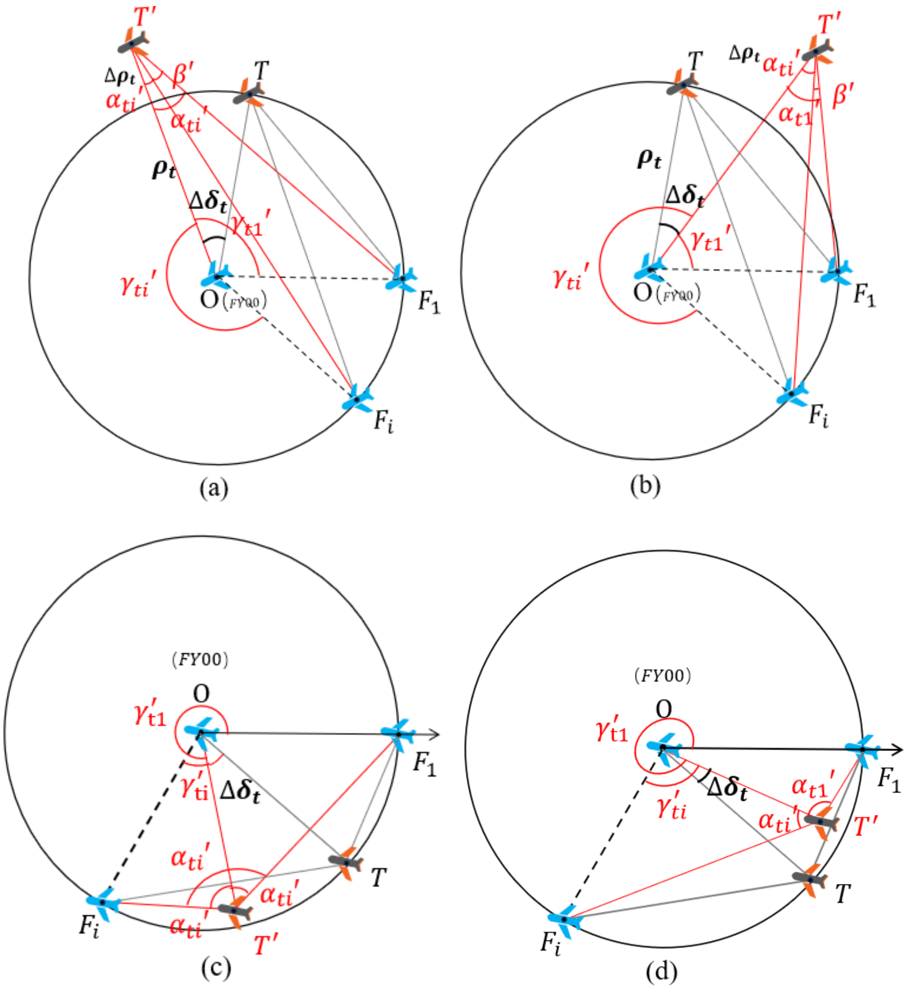

Figure 4. Four Cases of Deviation from Ideal Position

4.1. Calculate the distance angle γ_{ti}' with deviation

The distance angle between the unmanned aerial vehicle with deviation and the transmitter corresponding to the ideal situation is γ_{ti}'. When the unmanned aerial vehicle with deviation deviates clockwise relative to the ideal position, for Figure 4 (a):

$$\Delta\delta_t = \delta_t' - \delta_t > 0 \tag{11}$$

$$\gamma_{t1}' = \gamma_{t10} + \Delta\delta_t \tag{12}$$

$$\gamma_{ti}' = \gamma_{ti0} - \Delta\delta_t \tag{13}$$

When the deviation drone deviates counterclockwise relative to the ideal position, as shown in Figure 4 (b):

$$\Delta\delta_t = \delta_t' - \delta_t < 0 \tag{14}$$

$$\gamma_{t1}' = \gamma_{t10} + \Delta\delta_t \tag{15}$$

$$\gamma_{ti}' = \gamma_{ti0} + \Delta\delta_t \tag{16}$$

According to equations (11) - (16), it can be seen that γ'_{ti} has different expressions in different situations. To unify the expression of γ'_{ti} in different situations, the variable $P_{\gamma i}$ can be referenced to obtain the calculation formula.

$$\gamma'_{ti} = \gamma_{ti0} - P_{\gamma i} \cdot \Delta\delta_t, i = 1,2,\ldots,k, \text{ Where } P_{\gamma i} = \begin{cases} 1, f_i > t \\ -1, f_i < t \end{cases} \tag{17}$$

4.2. Calculate the angle deviation with deviation $\Delta\delta_t$

Take β as the counterclockwise distance angle from the unmanned aerial vehicle F_k to F_1 when transmitting the signal.

$$\beta = (10 - f_k) \cdot \theta \tag{18}$$

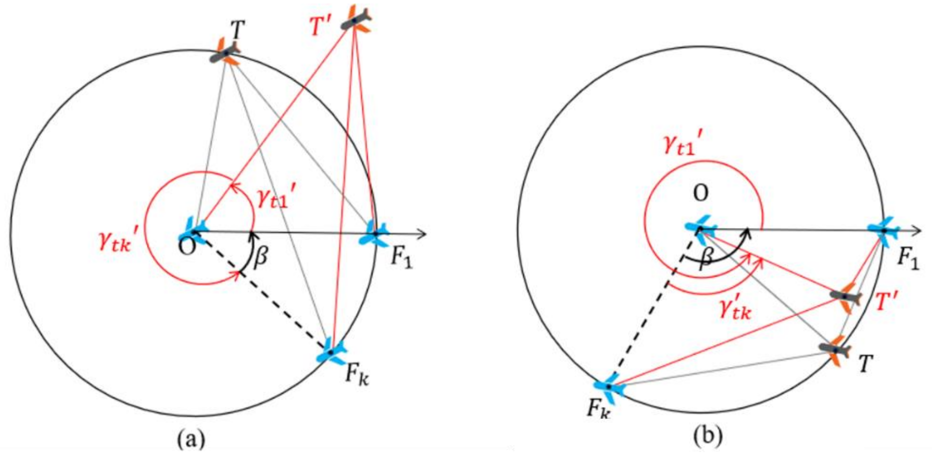

Figure 5. Relationship between β and γ'_{ti}

As shown in Figure 5 (a), when $f_k > t$:

$$\gamma'_{t1} + \beta + \gamma'_{tf_k} = 360° \tag{19}$$

As shown in Figure 5 (b), when $f_k < t$:

$$\gamma'_{t1} + \beta - \gamma'_{tf_k} = 360° \tag{20}$$

To unify the relationship between β and γ'_{ti}, the variable $P_{\gamma k}$ is referenced.

$$\gamma'_{t1} + \beta + P_{\gamma f_k} \cdot \gamma'_{tf_k} = 360°, \ P_{\gamma f_k} = \begin{cases} 1, f_k > t \\ -1, f_k < t \end{cases} \tag{21}$$

To calculate the angular deviation $\Delta\delta_t$ of the unmanned aerial vehicle T'_t receiving information with deviation, solve the following equations simultaneously.

$$\begin{cases} \gamma'_{tf_1} + \beta + P_{\gamma f_k} \cdot \gamma'_{tf_k} = 360° \\ \gamma'_{tf_1} = \gamma_{tf_1 0} - P_{\gamma f_1} \cdot \Delta\delta_t \\ \gamma'_{tf_k} = \gamma_{tf_k 0} - P_{\gamma f_k} \cdot \Delta\delta_t \end{cases} \tag{22}$$

Through variable analysis, three equations can solve for three variables γ'_{t1}、γ'_{tf_k}、$\Delta\delta_t$.

Solution:

$$\begin{cases} \Delta\delta_t = \dfrac{\theta\left[(10 - f_k) + P_{r1}(f_1 - t) + {P_{rf_k}}^2(f_k - t)\right] - 360°}{P_{r1} + {P_{rf_k}}^2} \\ {\gamma_{tf_1}}' = P_{rf_1}\theta \cdot \dfrac{{P_{rf_k}}^2(f_1 - f_k) + (f_k - 10) + 360°}{P_{rf_1} + {P_{rf_k}}^2} \\ {\gamma_{tf_k}}' = P_{rf_k}\theta \cdot \dfrac{P_{r1}(f_k - f_1) + (f_k - 10) + 360°}{P_{rf_1} + {P_{rf_k}}^2} \end{cases} \tag{23}$$

4.3. Calculate $(\rho_t{}', \delta_t{}')$ with deviation from α'_{ti}

It is now required to obtain the position coordinate offset $(\Delta\rho_t, \Delta\delta_t)$ when there is a deviation, and then further calculate the coordinates $(\rho_t{}', \delta_t{}')$ when there is a deviation. Two unknown variables are established and two equations are solved simultaneously[8]. For a certain unmanned aerial vehicle receiving signals, taking $\Delta F_i OT'$ and $\Delta F_j OT'$, there are:

$$\begin{cases} \dfrac{\rho}{\sin\alpha_{ti}{}'} = \dfrac{\Delta\rho + \rho}{\sin[180° - \alpha_{ti}{}' - (\gamma_{ti}{}' - P_{ri}\Delta\delta)]} \\ \dfrac{\rho}{\sin\alpha_{tj}{}'} = \dfrac{\Delta\rho + \rho}{\sin\left[180° - \alpha_{tj}{}' - \left(\gamma_{tj}{}' - P_{rj}\Delta\delta\right)\right]} \end{cases} \tag{24}$$

Calculate the position coordinate offset with deviation $(\Delta\rho_t, \Delta\delta_t)$:

$$\begin{cases} \Delta\delta_t = \arctan G \\ \Delta\rho_t = \rho\left(\dfrac{\sin(M_{ti} + P_{ri}\Delta\delta_t)}{\sin\alpha_{ti}{}'} - 1\right) \end{cases} \tag{25}$$

Where $G = \frac{\sin\alpha_{ti}{}' \sin M_{tj} - \sin\alpha_{tj}{}' \sin M_{ti}}{P_{ri}(\sin\alpha_{tj}{}' \cdot \cos M_{ti} - \sin\alpha_{ti}{}' \cdot \cos M_{tj})}$, the coordinates with deviation can be calculated based on the ideal coordinates (ρ_i, δ_i).

$$(\rho_t{}', \delta_t{}') = (\rho_t + \Delta\rho, \delta_t + \Delta\delta_t) \tag{26}$$

5. Calculation of position coordinates with deviation and reception error

Considering that there is an error in the received signal of the drone, the azimuth angle α'_{ti} extracted by the drone receiving the signal contains the reception error angle α_ε. If the error free azimuth angle is α_{ti}, then adjust α'_{ti} to

$$\alpha'_{ti} = \alpha_{ti} + \alpha_\varepsilon \tag{27}$$

If the error term α_ε is taken as a constant value, the larger the value of α'_{ti}, the smaller the disturbance of the error term α_ε on the error free azimuth angle α_{ti}. Therefore, when selecting $\Delta F_i OT'$ for sine theorem analysis and solution, the corresponding maximum and second largest values of α_{ti0} or α'_{ti} are selected to obtain more accurate positioning information[9-10]. For drone *t*, the azimuth angle $\alpha_{ti}{}'$ is extracted from the received signal, and then the maximum azimuth angle $\alpha_{ti}{}'$ and the second largest azimuth angle $\alpha_{tj}{}'$ are extracted and substituted into the following model to locate the drone *t* receiving the signal. Based on the above analysis, the positioning model for passive signal receiving unmanned aerial vehicles can be obtained as follows:

$$\begin{cases} \rho'_t = \rho_t + \Delta\rho_t \\ \delta'_t = \delta_t + \Delta\delta_t \end{cases} \tag{28}$$

$$\begin{cases} \Delta\rho_t = \rho\left(\dfrac{\sin(M_{ti} + P_{ri}\Delta\delta)}{\sin\alpha_{ti}{}'} - 1\right) \\ \Delta\delta_t = \arctan G \\ G = \dfrac{\sin\alpha_{ti}{}' \sin M_{tj} - \sin\alpha_{tj}{}' \sin M_{ti}}{P_{ri}\left(\sin\alpha_{tj}{}' \cdot \cos M_{ti} - \sin\alpha_{ti}{}' \cdot \cos M_{tj}\right)} \\ M_{ti} = 180° - \alpha_{ti}{}' - \gamma_{ti}{}' \\ M_{tj} = 180° - \alpha_{tj}{}' - \gamma'_{tj} \\ \gamma'_{ti} = \gamma_{ti0} - P_{\gamma i} \cdot \Delta\delta_t \\ P_{\gamma i} = \begin{cases} 1, f_i > t \\ -1, f_i < t \end{cases} \\ \Delta\delta_t = \dfrac{\theta\left[(10 - f_k) + P_{r1}(f_1 - t) + P_{rk}{}^2(f_k - t)\right] - 360°}{P_{r1} + P_{rk}{}^2} \end{cases} \tag{29}$$

This model extracts k azimuth angles $\alpha_{ti}', i = 1,2, \ldots, k$ from unmanned aerial vehicles that passively receive signals, and calculates their positions in the polar coordinate system to complete coordinate positioning.

6. Conclusions

This article successfully solves the accurate positioning problem of unmanned aerial vehicle formations under ideal and deviated conditions by establishing a pure azimuth passive positioning model based on polar coordinate system. The research results indicate that the calculation formulas for ideal and deviated positions derived from geometric knowledge and sine theorem can effectively extract azimuth angles and calculate the position of unmanned aerial vehicles in polar coordinates. In addition, this article also considers the influence of reception error angle and further improves the positioning model to adapt to complex situations in practical applications. This achievement provides new theoretical support for the precise positioning of drone formations and fills the research gap in this field. Future research directions will focus on real-time compensation mechanisms for dynamic error angles to improve positioning accuracy and adaptability to complex environments. At the same time, explore more positioning adjustment methods for non-circular formations to meet diverse application scenarios.

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