# Research On Population Dynamics of Lamprey Based on Differential Equations

Zhiyuan Shi [1,†], Zhongda Xu [2,†], Jialin Zou [3,*,†]

[1] Houston International Institute, Dalian Maritime University, Dalian, China, 116026

[2] School of Science, Dalian Maritime University, Dalian, China, 116026

[3] Marine Engineering College, Dalian Maritime University, Dalian, China, 116026

* Corresponding Author Email: Fengzizou.0127@outlook.com

† These authors contributed equally.

**Abstract.** The sea lamprey is a species with unique ecological characteristics, and its population dynamics are influenced by various factors, including sex ratio strategies, resource availability, and interactions with other species. This study constructs two types of models to investigate the dynamic changes in sea lamprey populations. Firstly, a model for male and female individuals based on Logistic differential equations is developed, considering the impact of sex ratio transition rates on population size. The results indicate that, after accounting for the sex ratio transition rate, the proportion of male sea lampreys increases from 60% to 68.8%. Secondly, a food availability parameter is defined to explore the impact of larval food resources on sex transition rates. A differential equation model considering the food chain between zooplankton and fish is constructed. The results show that when food resources are insufficient, the proportion of male sea lampreys rises significantly (to 80.7%), while the numbers of parasitic fish and competing species increase by 73.0% and 23.2%, respectively. This research provides a scientific basis for a deeper understanding of the population dynamics of sea lampreys and offers valuable insights for their ecological management and conservation.

**Keywords:** Sea Lamprey, Population Dynamics, Differential Equations, Sex Ratio.

## 1. Introduction

The Sea Lamprey is a parasitic species of lamprey, primarily distributed in certain regions of Northeastern North America and Northern Europe, and is classified as an endangered species. However, due to an incidental population invasion via Niagara Falls, the Sea Lamprey has proliferated in the Great Lakes Basin in the United States. From the 1930s to the 1950s, the Sea Lamprey population increased rapidly, posing a serious threat to various large cold-water fish species, especially trout. In 1961, following interventions by local governments, the Sea Lamprey population began to decline. Since the 1970s, the control of Sea Lamprey bioinvasion has remained a key ecological concern in the Great Lakes region. The population dynamics of the Sea Lamprey are influenced by a combination of factors, including sex ratio strategies, resource availability, and interactions with other species. The sex ratio strategy refers to the ability of the Sea Lamprey to adjust the sex ratio of its offspring in response to environmental changes, in order to adapt to fluctuations in resource availability. For instance, in resource-rich environments, the Sea Lamprey tends to produce more female offspring, thereby promoting population expansion; whereas in resource-limited environments, it tends to produce more male offspring to increase reproductive success.

Decision analysis of Integrated Pest Management: a case study on invasive sea lamprey in the Great Lakes Basin develops an adaptive management framework to evaluate lamprey control strategies through a decision analysis approach that optimizes social acceptability and technical feasibility [1]. A Study on Lamprey's Population Based on Sex-Ratio-Related Growth-Balance Model” developed a sex-ratio-related growth-balance model to simulate the effects of sex ratio and food quantity on the growth of lamprey populations, and emphasized the role of sex-adjustment on population stability [2]. Form-assortative mating behaviors of individuals from parasitic and non-parasitic populations of Arctic lamprey explored the pairing behavior of lampreys and found that males tend to select large migratory females, not by sex ratio, but by competition and size constraints [3]. The analysis revealed that there are fewer studies on population dynamics based on sex ratio and resource availability.

The data were obtained from the open source website (https://www.comap.com/contests/mcm-icm) and contain data on ecology, species distribution, environmental impacts, population dynamics, and other relevant data categories. Therefore, this study aims to enhance the understanding of the mechanisms behind sea lamprey population dynamics and provide a scientific basis for their ecological management and conservation. By constructing models, this research seeks to thoroughly analyze the impact of gender ratio strategies on sea lamprey populations and ecosystems, offering scientific support for ecological conservation and resource management.

## 2. The Basic Fundamental of Research

In order to study the effects of sex ratio strategies on lamprey populations in relation to other species and ecosystems under the constraints of the availability of resources such as food, the following is what we did.

In Population Model of Sea Lampreys, the sex ratio transformation parameter is defined as a way to study the variation in the population of lamprey of different sexes. Based on the sex ratio transformation parameter, the differential equations for the number of female lampreys and the number of male lampreys are given separately. Then, the relationship between the population change of lamprey and sex ratio was obtained by solving the set of differential equations.

In Ecosystem Model Based on Resource Availability, competitors and predators of lamprey were introduced. The Volterra predation-competition model was used to represent predation and competition among different species. Meanwhile, the resource availability parameter was defined to represent the effect of food availability on the sex ratio transition rate of lamprey during the larval stage, which in turn affects its population size. This was solved by forming a new system of differential equations through Model I with the newly added differential equations. The effect of the lamprey population on other organisms in the ecosystem is also inferred.

## 3. Results

### 3.1. Population Model of Sea Lampreys

Prior to the development of this model, the text proposed the ideal hypothesis. The ideal hypothesis is defined by us as the sea lamprey population is not influenced by other species and resources in the ecosystem and can be considered as an ideal sea lamprey population under stable feeding conditions. In this model the text will focus on the effect of sex ratio on sea lamprey populations.

First, considering the limited resources and space, the text build logistic model based on differential equations to describe the growth retardation of the population. The differential equation is as follows:

$$\frac{dN_1^{(1)}}{dt} = r_1 N_1^{(1)}\left(1 - \frac{N_1^{(1)}}{K}\right) \tag{1}$$

Where $N_1^{(1)}$ represents Population size of sea lamprey in the hypothesis above.

Considering the particular sex ratio strategy of the sea lamprey and the correlation between sex ratio and population size, the text first quantified the effect of sex ratio on the population size of the sea lamprey and defined the sex ratio and the sex ratio conversion rate as follows:

$$\begin{cases} \gamma^{(1)} = \dfrac{N_m^{(1)}}{N_f^{(1)}} \\ \sigma^{(1)} = \dfrac{\gamma^{(1)} - 1}{\gamma^{(1)} + 1} \end{cases} \tag{2}$$

Where $\gamma^{(1)}$ represents the sex ratio, while $\sigma^{(1)}$ represents sex ratio conversion rate in this hypothesis.

Then, based on the definition of (2), the text transform the differential equation (1) into a system of differential equations with the following improvement based on gender differentiation:

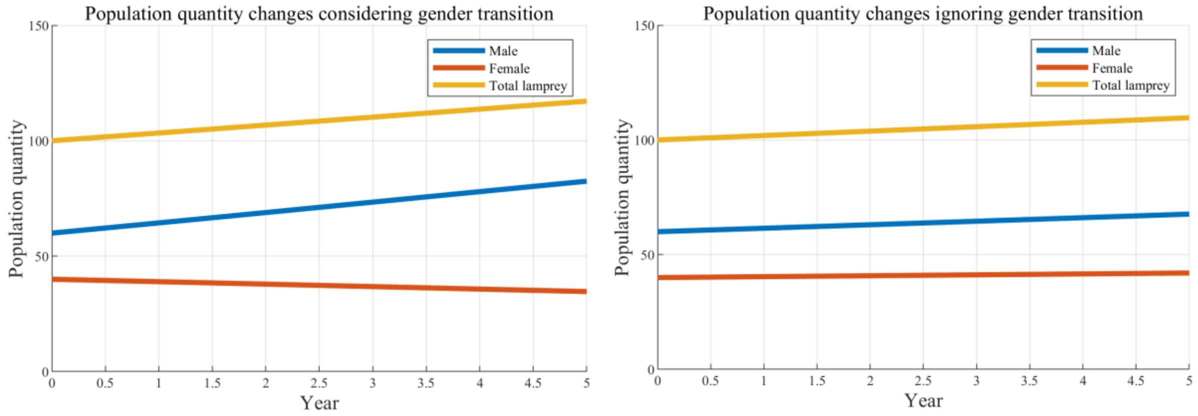
$$\begin{cases} \dfrac{dN_m^{(1)}}{dt} = r_m N_m^{(1)} (1 - \dfrac{N_m^{(1)}}{K}) + r_m \sigma^{(1)} N_f^{(1)} \\ \dfrac{dN_f^{(1)}}{dt} = r_f N_f^{(1)} (1 - \dfrac{N_f^{(1)}}{K}) + r_f \sigma^{(1)} N_f^{(1)} \\ \dfrac{dN_1^{(1)}}{dt} = \dfrac{dN_f^{(1)}}{dt} + \dfrac{dN_m^{(1)}}{dt} \end{cases} \tag{3}$$

Where $N_m^{(1)}$ and $N_f^{(1)}$ represents sea lamprey male and female population in this hypothesis respectively.

Then the text associate (2) (3) to obtain the sea lamprey population model based on a system of differential equations as follows:

$$\begin{cases} \gamma^{(1)} = \dfrac{N_m^{(1)}}{N_f^{(1)}} \\ \sigma^{(1)} = \dfrac{\gamma^{(1)} - 1}{\gamma^{(1)} + 1} \\ \dfrac{dN_m^{(1)}}{dt} = r_m N_m^{(1)} (1 - \dfrac{N_m^{(1)}}{K}) + r_m \sigma^{(1)} N_f^{(1)} \\ \dfrac{dN_f^{(1)}}{dt} = r_f N_f^{(1)} (1 - \dfrac{N_f^{(1)}}{K}) + r_f \sigma^{(1)} N_f^{(1)} \\ \dfrac{dN_1^{(1)}}{dt} = \dfrac{dN_f^{(1)}}{dt} + \dfrac{dN_m^{(1)}}{dt} \end{cases} \tag{4}$$

The text brought the initial value condition $N_m^{(1)} = 60, N_f^{(1)} = 40, N_1^{(1)} = 100$ into (4) and solved it by Matlab for the system of differential equations, and simulated the population size-time relationship line of the sea lamprey as follows (Figure 1):



(a). Population considering gender transition (b). Population ignoring gender transition

**Figure 1.** Sea lamprey Population quantity changes

After establishing and solving the population model of the sea lamprey, the text made a yearly prediction of the population size and sex ratio of the sea lamprey, and obtained the corresponding visualization results as follows (Figure 2).

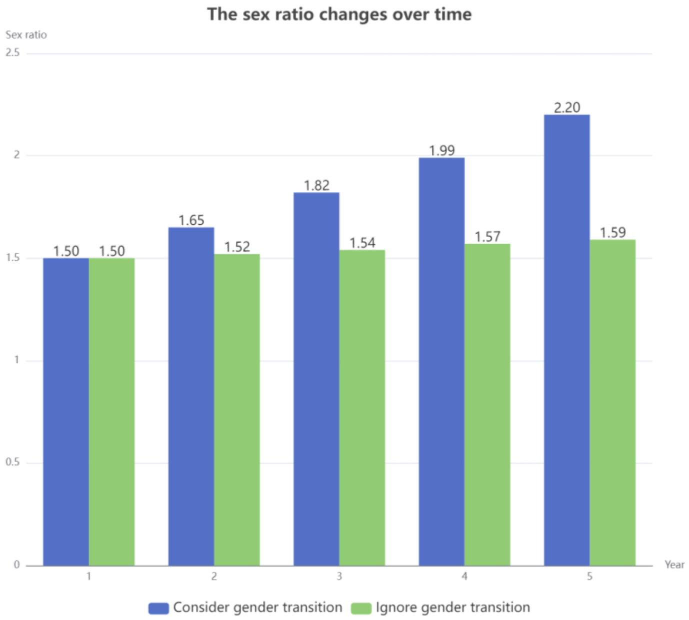


**Figure 2.** Sex ratio changes over time

### 3.2. Analysis of results of Population Model

Based on the above relationship line with the visualization results, it is shown that the following conclusions hold.

The results of the year-by-year prediction of the population size of the sea lamprey according to the presence or absence of the sex ratio strategy show the values of the sea lamprey population size after 5 years were 117 and the number increased by 17% (Figure 1(a)) when the sex ratio strategy was adopted. In comparison, when the sex ratio strategy was ignored, the population of the sea lamprey increased by 10% in 5 years (Figure 1(b)).

Based on the results of sex ratio prediction of the sea lamprey from year to year, it indicates that the ratio of males to females will increase year by year if the sex ratio transformation is considered. After 5 years, the ratio of males to females will reach 2.2, at which time males will account for 68.8% of the total number of sea lampreys (Figure 2).

Qualitatively analyzing the relationship, it is reasonable to assume that the sex ratio strategy affects the population size of the sea lamprey. With limited environmental capacity, the sex ratio strategy favors the population to slow down the stagnation of population growth by elevating the sex ratio.

### 3.3. Ecosystem Model Based on Resource Availability

Based on the discussion above, we generalized the effect of sex ratio on sea lamprey populations under ideal conditions to the effect of sex ratio on the abundance of individual species in the Great Lakes Basin and modeled negative ecosystems.

The sea lamprey has a unique ecological niche in the Great Lakes basin. It is known that its adults form a competitive relationship with parasitic competitors and a parasitic relationship with their hosts, while the larvae are one of the food sources for their hosts [4-6]. To introduce the model, we simplified the role of sea lamprey with the host as weak parasitism.

Meanwhile, based on the description [7], the text combine common sense to suggest that resource availability is positively correlated with the growth rate of larvae. Moreover, the text can reasonably assume that the sex ratio of larvae is related to resource availability [8].

In a word, in order to discuss the advantages and disadvantages of the sex ratio strategy for the survival of sea lamprey populations and the influence to the other species in the food chain, the text introduce the ecosystem model to simulate the changing population of sea lampreys and the other species based on the resource availability and focus on the change of sex ratio and population size at different resource levels on a time axis.

First, the text introduced the negative host hypothesis for which the number of hosts as the lowest level of the food chain is underestimated due to the lack of resource supplementation since the text only discusses the role of sex ratio for the ecosystem and does not take into account the role of resource level (Figure 3). The specific modeling is as follows based on Volterra predation-competition model:

$$\begin{cases} \gamma^{(1)} = \dfrac{N_m^{(1)}}{N_f^{(1)}} \\ \sigma^{(1)} = \dfrac{\gamma^{(1)} - 1}{\gamma^{(1)} + 1} \\ \dfrac{dN_m^{(1)}}{dt} = r_m N_m^{(1)} \left(1 - \dfrac{N_m^{(1)} + \alpha_3 N_3^{(1)}}{K_m}\right) + r_m \sigma^{(1)} N_f^{(1)} + b_{12} N_m^{(1)} N_2^{(1)} \\ \dfrac{dN_f^{(1)}}{dt} = r_f N_f^{(1)} \left(1 - \dfrac{N_f^{(1)} + \alpha_3 N_3^{(1)}}{K_f}\right) + r_f \sigma^{(1)} N_f^{(1)} + b_{12} N_f^{(1)} N_2^{(1)} \\ \dfrac{dN_1^{(1)}}{dt} = \dfrac{dN_f^{(1)}}{dt} + \dfrac{dN_m^{(1)}}{dt} \\ \dfrac{dN_2^{(1)}}{dt} = r_2 N_2^{(1)} \left(1 - \dfrac{N_2^{(1)}}{K_2}\right) - b_{21} N_1^{(1)} N_2^{(1)} - b_{23} N_2^{(1)} N_3^{(1)} \\ \dfrac{dN_3^{(1)}}{dt} = r_3 N_3^{(1)} \left(1 - \dfrac{N_3^{(1)} + \alpha_3^{-1} N_1^{(1)}}{K_3}\right) + b_{32} N_2^{(1)} N_3^{(1)} \end{cases} \tag{5}$$

Where the penultimate differential equation represents the host population, and the last differential equation represents the competitor population.

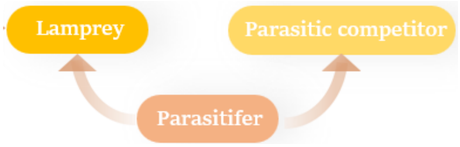


**Figure 3.** Food chain of parasitifer in the negative host hypothesis

Then, the text introduces plankton, including zooplankton and phytoplankton, as a model of resource availability in the Great Lakes Basin and modeled active plankton populations. In this modeling hypothesis, the number of parasitifers, as the highest level of the food chain, is overestimated because the depletion of parasitifer populations by sea lampreys and other camp-parasitic competitors is not considered. The specific modeling is as follows (Figure 4).

**Figure 4.** Food chain of the parasitifer in the positive host hypothesis

According to the study [9-10], the growth of phytoplankton follows the Monod-type nutrient limitation, the predation rate of zooplankton depends on the concentration of phytoplankton with the HollingII-type functional reflection, and the predation of parasitifers on animals follows the HollingIII-type functional reflection. As a result, the text functionally improves the system of logistic Steele's equations for the above three species feeding on each other as follows:

$$\begin{cases} \dfrac{dP^{(2)}}{dt} = \alpha P^{(2)}(1 - \dfrac{P^{(2)}}{K_p}) - \dfrac{\beta\ P^{(2)} Z^{(2)}}{P^{(2)} + C} \\ \dfrac{dZ^{(2)}}{dt} = \dfrac{\lambda P^{(2)} Z^{(2)}}{P^{(2)} + C} - dZ - \dfrac{\theta\ {Z^{(2)}}^2 N_2^{(2)}}{{Z^{(2)}}^2 + e^2} \\ \dfrac{dN_2^{(2)}}{dt} = r_2 N_2 (1 - \dfrac{N_2^{(2)}}{K_2}) + \dfrac{\xi\ {Z^{(2)}}^2 N_2^{(2)}}{{Z^{(2)}}^2 + e^2} \end{cases} \tag{6}$$

Where $P^{(2)}$ represents the number of zooplankton, $Z^{(2)}$ represents the number of phytoplankton, $N_2^{(2)}$ represents the number of hosts in different modeling hypotheses, and $N_3^{(2)}$ represents the number of parasitic competitors in this modeling hypothesis.

After modeling the active zooplankton population as described above, the text discuss the hypotheses corresponding to the model as follows:

The loss term of parasitifers in the positive host hypothesis under the parasitism of camping parasitic competitors with sea lampreys is not considered as $-(b_{21} N_1^{(1)} N_2^{(1)} + b_{23} N_2^{(1)} N_3^{(1)})$.

The complementary terms are also not considered for parasitifers in the negative host hypothesis when feeding on zooplankton $+(\dfrac{\xi {Z^{(2)}}^2 N_2^{(2)}}{{Z^{(2)}}^2 + e^2})$.

At the same time the text makes the following assumptions based on the dependence of the sea lamprey on resource levels:

Sea lamprey larvae prey on zooplankton. Therefore, the model should also introduce the additional term of zooplankton predation by sea lamprey larvae $+(b_{1z} N_m^{(3)} Z^{(3)}) + (b_{1z} N_Z^{(3)} Z^{(3)})$, and the loss of zooplankton to predation by sea lamprey larvae $+(b_{Z1} N_1^{(3)} Z^{(3)})$.

Sea lamprey larvae consume organic detritus. So the text introduced unit values for organic detritus content $f$ ( $f = 1$ is the standard organic debris content unit value). In order to discuss the effect of organic detritus content on the sex conversion rate of sea lampreys, the following relationship was established based on the literature [7-8]:

$$\begin{cases} \gamma^{(3)} = \dfrac{N_m^{(3)}}{N_f^{(3)}} \\ \sigma^{(3)} = \dfrac{1}{f}\dfrac{\gamma^{(3)}-1}{\gamma^{(3)}+1} \end{cases} \tag{7}$$

Based on the above hypotheses and summaries, the text have established the ultimate ecosystem model as follows (Figure 5).

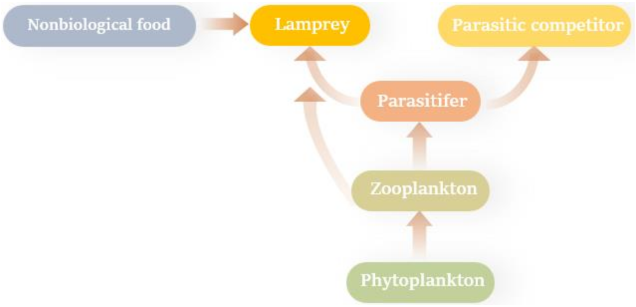


**Figure 5.** Food chain with resource dependence in the ecosystem model

In the modeling process, the text integrated the models and considered the complementary and loss terms in the full hypothesis to obtain the ecosystem model based on resource availability as follows:

$$\begin{cases} \gamma^{(3)} = \dfrac{N_m^{(3)}}{N_f^{(3)}} \\ \sigma^{(3)} = \dfrac{1}{f}\dfrac{\gamma^{(3)}-1}{\gamma^{(3)}+1} \\ \dfrac{dN_m^{(3)}}{dt} = r_m N_m^{(3)}(1-\dfrac{N_m^{(3)}+\alpha_3 N_3^{(3)}}{K_m}) + r_m \sigma^{(3)} N_f^{(3)} + b_{12} N_m^{(3)} N_2^{(3)} + b_{1Z} N_m^{(3)} Z^{(3)} \\ \dfrac{dN_f^{(3)}}{dt} = r_f N_f^{(3)}(1-\dfrac{N_f^{(3)}+\alpha_3 N_3^{(3)}}{K_f}) + r_f \sigma^{(3)} N_f^{(3)} + b_{12} N_f^{(3)} N_2^{(3)} + b_{1Z}^{(3)} N_f^{(3)} Z^{(3)} \\ \dfrac{dN_1^{(3)}}{dt} = \dfrac{dN_f^{(3)}}{dt} + \dfrac{dN_m^{(3)}}{dt} \\ \dfrac{dN_2^{(3)}}{dt} = r_2 N_2^{(3)}(1-\dfrac{N_2^{(3)}}{K_2}) - b_{21} N_1^{(3)} N_2^{(3)} - b_{23} N_2^{(3)} N_3^{(3)} + \dfrac{\xi\, {Z^{(3)}}^2 N_2^{(3)}}{{Z^{(3)}}^2 + e^2} \\ \dfrac{dN_3^{(3)}}{dt} = r_3 N_3^{(3)}(1-\dfrac{N_3^{(3)} + \alpha_3^{-1} N_1^{(3)}}{K_3}) + b_{32} N_2^{(3)} N_3^{(3)} \\ \dfrac{dP^{(3)}}{dt} = aP^{(3)}(1-\dfrac{P^{(3)}}{K_P}) - \dfrac{\beta\, P^{(3)} Z^{(3)}}{P^{(3)}+C} \\ \dfrac{dZ^{(3)}}{dt} = \dfrac{\lambda P^{(3)} Z^{(3)}}{P^{(3)}+C} - dZ^{(3)} - \dfrac{\theta\, {Z^{(3)}}^2 N_2^{(3)}}{{Z^{(3)}}^2+e^2} - b_{Z1} N_1^{(3)} Z^{(3)} \end{cases} \tag{8}$$

Similar to the previous model, the text substituted the initial value conditions into the ecological model based on resource availability and solved the system of differential equations by Matlab, and simulated the following population-temporal relationship of each species in the ecosystem when local resource dependence was taken into account (Figure 6).

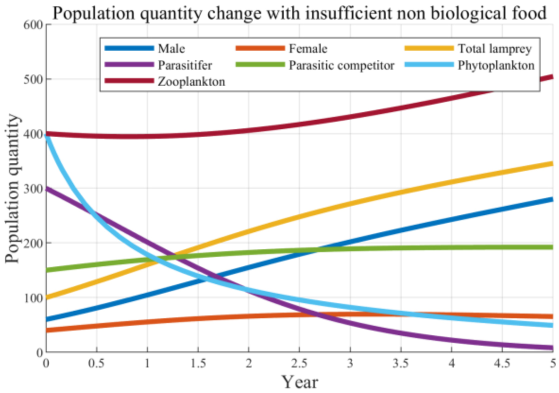
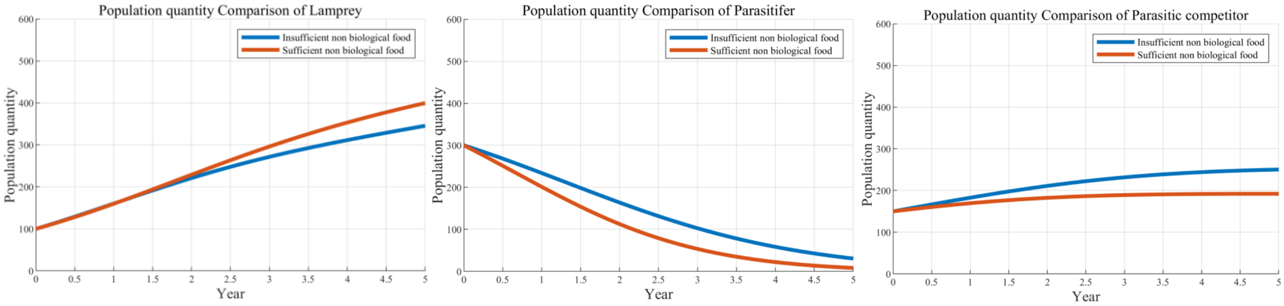


**Figure 6.** Population quantity change with insufficient food

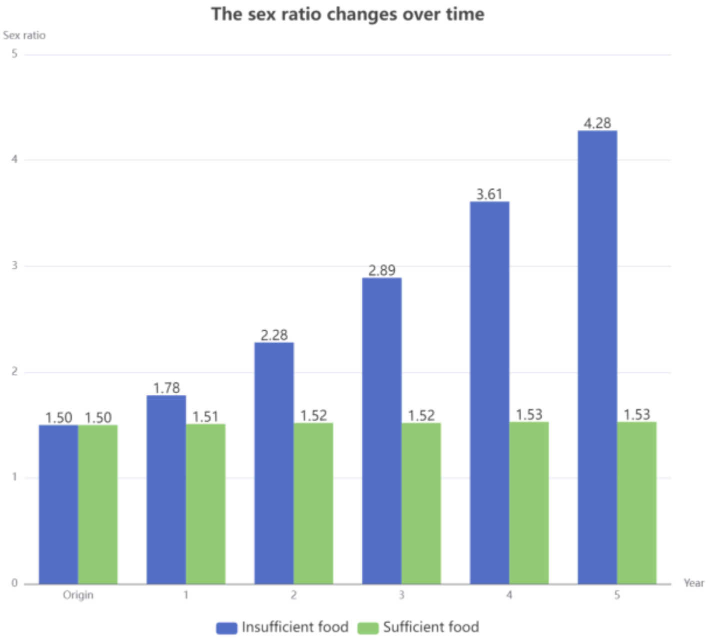
After establishing and solving the new ecosystem model, the text simulated the corresponding sex ratio changes of sea lampreys and the year-by-year prediction of the number of sea lampreys and other species under higher and lower levels of organic debris content by varying the unit value of the organic debris content, and obtained the corresponding visualization results.

### 3.4. Analysis of results of Ecosystem Model



(a). Lamprey population (b). Parasitifer population (c). Parasitic competitor population

**Figures 7.** Population quantity comparison



**Figures 8.** The sex ratio changes over time

Based on the above relationship lines with the visualization results, it is shown that the following conclusions hold.

According to the new ecosystem at different levels of organic detritus resources, the results of the year-by-year prediction of the population of each species shows that the population size of the sea lamprey decrease by 13.6% in response to changes in organic detritus resource levels, whereas the number of hosts and competitors increased by 73.0% and 23.2% respectively (Figure 7).

Year-by-year projections of the corresponding sex ratios of sea lamprey in the new ecosystem under different levels of organic detritus resources shows:

After considering the sex ratio transformation, the ratio of males to females will increase year by year. After 5 years, the ratio of males to females will reach 4.2 (Figure 8), at which time males account for 80.7% of the total number of sea lamprey. This suggests that the role of sex strategy in sea lampreys will be more pronounced when resource levels are taken into account.

Qualitatively analyzing the relationship between the two, it is reasonable to speculate that the sex ratio strategy of the sea lamprey can slow down the reproduction chances of the sea lamprey under resource scarcity by increasing the proportion of males, and reduce the waste of energy caused by extra mortality. However, the disadvantage of this strategy is that it gives a survival advantage to other competitors and hosts, especially when the proportion of hosts increases. At the same time, excessive changes in the sex ratio can be easily exploited by humans, and the release of sterilized males and females is one of the ways in which humans can control sea lamprey populations by the imbalance of their sex ratio.

## 4. Conclusion

The population dynamics of lamprey, a fish with unique ecological characteristics, are influenced by a variety of factors including sex ratio strategies, resource availability, and other species interactions. Firstly, this paper considered the effect of sex ratio conversion rate on population size by constructing a model of male and female individuals based on Logistic differential equation, and the results showed that the proportion of male lamprey would increase from 60% to 68.8% after considering the sex ratio conversion. The population size of lamprey grows faster when sex ratio conversion strategy is considered. Secondly, this paper explored the effect of food resources on the sex-ratio conversion rate of lamprey during the larval period, and further constructed a set of differential equations for the ecological chain by considering a wider range of ecological environments. The results showed that when food resources were insufficient, the proportion of male lampreys would increase significantly (80.7%), while the number of parasitic fish and competing species increased by 73.0% and 23.2%, respectively. The study in this paper provides a scientific basis for an in-depth understanding of the mechanism of lamprey population dynamics as well as its ecological management and conservation.

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