

Research on the Effect of Lamprey Sex Ratio on Ecosystem Diversity Based on Predator-Prey Model

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Abstract. Predator-prey models are valuable in predicting population dynamics and revealing the feedback effects of factors such as sex ratio on ecosystem diversity. Specifically, this study innovatively constructed a small-scale ecosystem differential equation model with a quadratic correction term for the sex ratio. It approximately linearly fitted the functional relationship between sex ratio and food availability. Subsequently, several sets of computer simulations were designed to iteratively model this ecosystem under several externally uncertain factors. The simulation results showed that the biodiversity index H peaked at about 0.74 when the sex ratio reached 0. while the ecosystem possessed the highest species richness. In addition, when the sex ratio varied between 0.56 and 0.78, the number of predators increased by an average of 15% per generation as the proportion of males increased, while the number of prey correspondingly decreased by about 10%. Meanwhile, the Shannon-Wiener index showed that the H -value increased nonlinearly with the increase of sex ratio less than 0.74, while the H -value began to decrease slowly when the sex ratio exceeded 0.74. The results show that the sex ratio has a significant effect on the level of ecosystem diversity.

Keywords: Food supply, Lampreys sex ratio, Ecosystem diversity, Predator-prey model.

1. Introduction

In ecology, the predator-prey system is an important ecological relationship model that is widely used to describe the dynamic relationship between predators and prey. The model not only has an important predictive value for the quantitative changes of biological populations, but also has a profound impact on the understanding of ecosystem stability, species diversity, and energy flow. With the deepening of ecological research, scholars have come to realize that external factors such as population sex ratio and food availability play an extremely crucial role in the predator-prey system.

For this study, the classic Volterra predator-prey model, Logistic population growth model, and Shannon-Wiener index were used to explore the impacts of changes in sex ratios of the lamprey on itself and the larger ecosystem. Among these, the Volterra model was proposed by Italian mathematician Vito Volterra and was originally used to describe fish population dynamics in fisheries. Capturing the dynamic equilibrium between predator and prey through nonlinear differential equations, the model has become an important tool for studying species interactions and is widely used in fields ranging from theoretical ecology to wildlife management. For example, Reynolds et al. used the model to study the interaction between plankton and small crustaceans in marine ecosystems [1]. Meanwhile, to more accurately simulate the growth dynamics of the lamprey population, this paper introduces a logistic model proposed by the Belgian mathematician Pierre François Verhulst. This model takes into account the effects of resource constraints on population growth and is suitable for predicting trends in population size in a finite environment, and is widely used in the fields of biology, environmental science, and socio-economics, and especially proved to be the most effective tool in studying the outbreak cycle of insect populations [2]. In addition, the Shannon-Wiener index was applied in this study to quantify ecosystem diversity, which was used to more accurately assess the ecosystem effects of changes in the sex ratio of lampreys. This physical quantity is an important tool for measuring ecosystem diversity and richness and is widely used in ecological assessment and environmental protection, many studies have used it to monitor the effects of deforestation or climate

change on biodiversity [3]. These models and indicators ensure that the study has a solid theoretical foundation and high application value.

In this paper, the research first extended and constructed an interspecific differential equation model applicable to the ecosystem food chain, and introduced a quadratic correction term to reflect environmental changes in the sex ratio of lampreys. Then it further collected and analyzed data on the food supply and sex ratio of lampreys, and quantified the relationship between the two using Pearson correlation analysis. Based on the results of the correlation analysis, the study defined a sex ratio function to calculate the proportion of males under different food supply conditions. A series of simulation experiments were then designed to solve this non-linear system of ordinary differential equations in combination with the Runge-Kutta method, and a long time series of iterative population simulations were carried out to observe the evolution of the population of each species under different sex ratio conditions over time. Finally, the Shannon-Wiener index was introduced as a measure of ecosystem diversity to monitor the impact of changes in the sex ratio of the lamprey on the species diversity of the whole ecosystem. Throughout the process, the model parameters were continuously adjusted to ensure that the model could truly reflect the ecological processes in reality.

2. Predator Response and Population Regulation Mechanisms in Ecological Models

The data in this paper comes from <https://scholar.google.com> and <http://www.comap.com>.

2.1. The Volterra bait-predator model

The Volterra bait-predator model is common in biology and mathematics, and an increasing number of scholars are focusing on and studying the relationship between bait and predator [4]. The Gaussian-type bait predation system studied in this paper is as follows.

$$x' = xf(x, K) - y\varphi(x) \quad (1)$$

$$y' = y(-d + cq(x)) \quad (2)$$

Where K represents the environmental maximum holding capacity of the lampreys, $x(t)$, $y(t)$ denote the densities of bait and predators, respectively, at the t time [5].

$f(x, K)$ is continuously differentiable (math.), represents the growth rate of the species in the absence of predators, satisfying the following conditions:

$$f(0, K) = r > 0 \quad (3)$$

$$f(K, K) = 0, f_x(K, K) < 0 \quad (4)$$

$$\text{For any } x, f_x(x, K) > 0 \quad (5)$$

The classical logistic type growth function $f(x, K) = r(1 - x/K)$ satisfies all the above conditions and has also been applied by many scholars to various bait predator models. $\varphi(x)$, the functional response function of a predator, describes the amount of bait taken by a single predator per unit time. Moreover, $\varphi(x)$ is also a continuously differentiable function $\varphi(0) = 0$. In general, the functional response function depends on many factors, e.g., the density of the bait, the efficiency of the predator in finding and killing the bait, and the species of the predator and bait.

The predator functional response function $\varphi(x)$ consists of four main types, as shown in Table 1 ($a > 0, m > 0, b > -2\sqrt{a}$).

Table 1. The predator functional response function $\varphi(x)$

Functional Response Function Type	form	broad form
Holling I-type	$\varphi(x) = mx$	/
Holling II-type	$\varphi(x) = \frac{mx}{a+x}$	/
Holling III-type	$\varphi(x) = \frac{mx^2}{a+x^2}$	$\varphi(x) = \frac{mx^2}{ax^2 + bx + 1}$
Holling IV-type	$\varphi(x) = \frac{mx}{a+x^2}$	$\varphi(x) = \frac{mx}{ax^2 + bx + 1}$

In particular, the simpler Holling I-type function from Table 1 was introduced into the lamprey ecosystem model to save time and computational costs.

2.2. Logistic population model

Populations usually grow exponentially with a continuous Malthus population model:

$$\frac{dN_t}{dt} = rN_t \tag{6}$$

Where N_t denotes the population size at one time, and r is the endogenous growth rate of the population. Intraspecific competition is a common phenomenon, e.g. male locusts competing for females and female locusts competing for egg-laying sites, affecting their biological potential and ultimately leading to population declines [6]. Considering density constraints and crowding effects, a logistic population model can be obtained by adding a density constraint factor $(1 - N_t / K)$, to model (1):

$$\frac{dN_t}{dt} = rN_t(1 - \frac{N_t}{K}) \tag{7}$$

Where K is the environmental capacity. When $N_t < K$ the population of species is increasing; When $N_t > K$ the population of species is decreasing.

2.3. Shannon-Wiener Index (ecosystem diversity)

The Shannon-Wiener index is a biodiversity index that measures the diversity and abundance of species in an ecosystem, based on the principle that the higher the species diversity, the higher the entropy and vice versa [7]. The higher the entropy, the higher the diversity of species and the higher the diversity of the ecosystem.

In this paper, the Shannon-Wiener index was extended to the ecosystem in which the lampreys are located and was used to reflect the effect of changes in the sex ratio of the lampreys on the species diversity of the ecosystem.

The Shannon-Wiener index (H) can also be called the ecosystem diversity index and can be expressed by equation (5).

$$H = -\sum P_i \ln(P_i) \tag{8}$$

Where P_i is the relative frequency of species i , which can be calculated from equation (6).

$$P_i = \frac{N_i}{N} \tag{9}$$

Where N_i is the number of individuals of species i and N is the total number of individuals in the ecosystem.

3. Impact of Sex Ratio Variation on the Ecosystem Food Chain

As a part of the ecosystem, the lampreys play an important role in the biome. The sex ratio of lampreys changes in response to changes in the external environment, a phenomenon called adaptive sex ratio variation. Changes in sex ratios will not only affect the reproduction and survival of the lampreys themselves but will also affect the entire food chain and the wider ecosystem.

Analogous to the Gause-type bait-predator system in the Holling Type I function, the study extends the classical Volterra bait-predator model and the logistic model by adding a quadratic correction term to the model and modeling the interspecific relationships in the ecosystem food chain.

Based on the data reviewed, a heat map is shown in Figure 1:

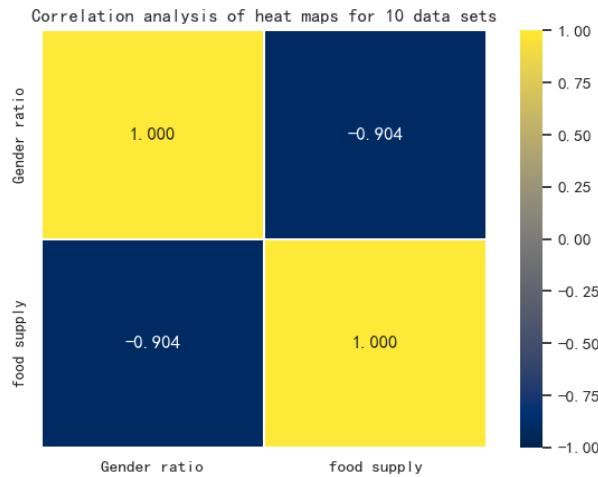


Figure 1. Heat Map of Sex Ratio-Food Supply Correlation Coefficient

Define the Pearson correlation coefficient [8]:

$$r = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{\sum_{i=1}^{10} (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{10} (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^{10} (Y_i - \bar{Y})^2}} \quad (10)$$

According to the correlation analysis in Figure 1, it's discovered that the Pearson coefficient of the two sets of data is -0.904, close to -1. Therefore, it can be concluded that the sex ratio of lampreys is negatively correlated with food supply.

A specific curve graph is also drawn, as shown in Figure 2. It can be seen that the function relationship curve between the sex ratio of lampreys and food supply is basically a straight line with a slope of -4545. As the food supply increases, the proportion of females gradually increases, while the proportion of males decreases accordingly.

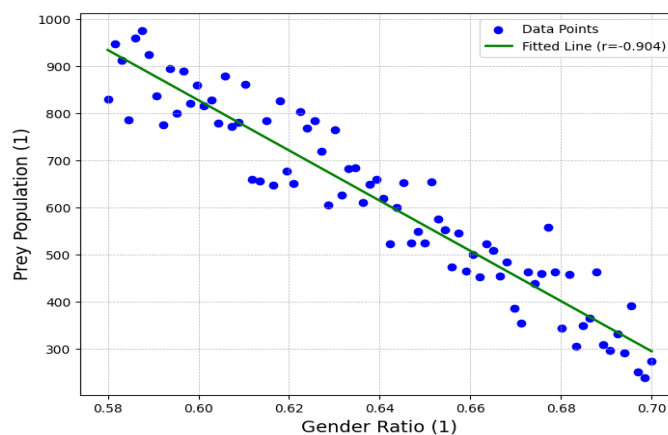


Figure 2. Sex Ratio - Food Supply Curve

Assuming that the biomass of the lampreys is L , the biomass of the prey is F , the biomass of the predators is P , the predation loss rate of all the populations is α (related to the predation success of a single predator, the prey's fitness, etc.), and the energy conversion rate of the populations other than the lampreys is β . Since lampreys are affected by sex ratio, define β_1 to denote the predation rate of male lampreys and β_2 to denote the predation rate of female lampreys. β_1 is greater than β_2 because it can be seen from the data that males have a higher proportion when food supply is low, which means that they can adapt to the harsh environment better than females, in other words, the energy conversion rate of males is higher, so β_1 is greater. Then, define the growth rate of the lampreys population as r_L , their environmental carrying capacity, k_L the population growth rate of the prey as r_F , the environmental carrying capacity k_F , the population growth rate of the predators as r_P , their environmental carrying capacity k_P , and the sex ratio (percentage of males) as G , the percentage of male lampreys at low food availability as G_L , and the percentage of male lampreys at high food supply as G_H . Based on the logistic model, equations (8), (9), (10) are obtained as follows.

$$\frac{dF}{dt} = r_F \times F \times \left(1 - \frac{F}{k_F}\right) - \alpha \times F \times L \quad (11)$$

$$\frac{dL}{dt} = r_L \times L \times \left(1 - \frac{L}{k_L}\right) + \beta_1 \times G \times F \times L + \beta_2 \times (1-G) \times F \times L - \alpha \times L \times P \quad (12)$$

$$\frac{dP}{dt} = r_P \times P \times \left(1 - \frac{P}{k_P}\right) + \beta \times L \times P \quad (13)$$

The gender proportionality function can be obtained:

$$G = G_L - (G_L - G_H) \times S \quad (\text{assuming a linear relationship between } G \text{ and } S) \quad (14)$$

$$S = \frac{F}{k_F} \quad (S \text{ is the food supply}) \quad (15)$$

Food chain: $F \rightarrow L \rightarrow P$.

Meanwhile, the complementary design of the model and the iterative process of solving it are shown below:

Firstly, after giving the initial data, this research then designed a function to calculate the sex ratio, which calculates the current food supply based on the availability of food resources and the current percentage of males based on food availability.

Then, a function was written for the growth of a population of lampreys that used equation (8) above and the time step to calculate the change in the population of lampreys at the next time step, taking into account factors such as food supply and predation rates and energy conversion ratio.

Immediately following this, a function of prey population growth was written to calculate the change in the prey population at the next time step using equation (9) above and the time step, taking into account natural growth and loss to predation by the lampreys.

Subsequently, a function of predator population growth was written to calculate the change in the predator population at the next time step based on equation (10) above and the time step, taking into account food intake and self-growth.

Finally, this research used computers to simulate population iterations.

In solving differential equations, Runge-Kutta Methods were used. It is an important iterative method for solving nonlinear ordinary differential equations [9]. Its high precision is very suitable for solving this model.

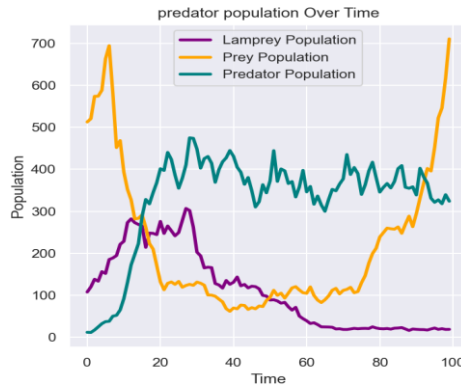


Figure 3. Numbers of different species - t curves

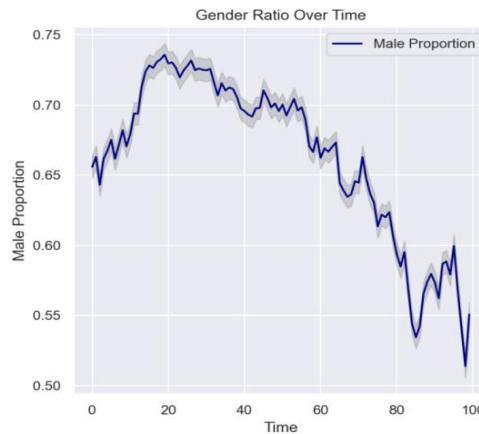


Figure 4. Sex ratio of lampreys - t curve

From Figures 3 and 4, it can be seen that close to 20 generations ago, as the sex ratio increased, the predator population showed an increasing trend and the prey population showed a decreasing trend, and after 20 generations, as the sex ratio decreased, the predator's population shows a decreasing trend and the prey population shows an increasing trend. This suggests that the sex ratio of lampreys affects the ecosystem through the food chain, with an increase in sex ratio leading to an increase in predators and a decrease in prey, and a decrease in sex ratio leading to a decrease in predators and an increase in prey [10].

Figure 5 more accurately and intuitively reflects the relationship between the number of predators, the number of lampreys, and the number of prey under the condition that the independent variable is the sex ratio. When the sex ratio was less than 0.72, the number of lampreys and predators showed a more obvious growth trend with the increase of sex ratio, and the number of prey showed a decreasing trend; After the sex ratio is greater than 0.72, when the sex ratio continues to increase, the number of lampreys and predators remains near a relatively stable population number value, with a certain small fluctuation, while the number of prey continues to decrease.



Figure 5. Curves of the number of different species and the sex ratio of lampreys

In addition to this, the model was used to perform simulation iterations to obtain the following images of changes in the ecosystem diversity index H.

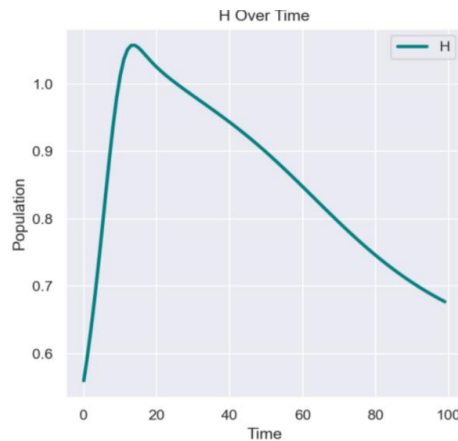


Figure 6. H-Time relationship curve

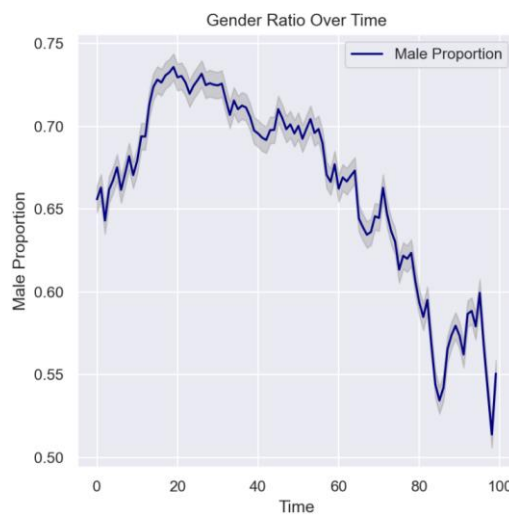


Figure 7. Sex Ratio - Time Curve

From Figures 6 and 7, it can be analyzed that before 20 generations, both the sex ratio (male ratio) and H increase; after the peak at 20 generations, both H and sex ratio decrease:

When the sex ratio changes, the species diversity of the ecosystem will also be affected and change significantly, and the basic trend of change is very similar.

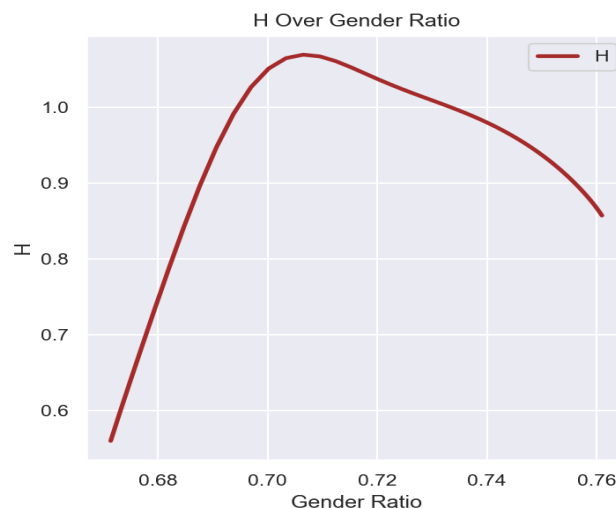


Figure 8. H-Sex Ratio Curve

From the quantitative relationship between sex ratio and H in Figure 8, it can be seen that when the sex ratio is less than 0.74, H increases with the increase of sex ratio (non-linear with the same trend); when the sex ratio (proportion of males) is 0.74, H reaches the peak, and the level of ecosystem species diversity is high; when the sex ratio is greater than 0.74, H begins to slowly decline, and the level of ecosystem species diversity also declines.

4. Conclusion

Overall, to consider the influence of interspecific interactions between male and female lampreys and other creatures in the ecosystem, this study has improved on the Volterra bait-predator model by adjusting the equations between two species to multispecies differential equations based on Logistic equations. Meanwhile, considering that lampreys have a unique sex ratio variable with the environment, the research also set different male and female correction terms in the differential equation to reflect their different environmental fitness. Then, through our Pearson correlation analysis, the male ratio given in the background is used to approximate linearly fit the sex ratio as a function of food supply. After that, the computer is used to design the iteration period and number and simulate the population reproduction iteration process of each ecosystem under the interference of some external uncertain factors. Through the differential equation model, the study obtains the impact of sex ratio change on each species in the food chain. Meanwhile, the study also determines the Shannon-Wiener index to obtain a specific quantitative index of ecosystem diversity and uses it to measure another comprehensive impact of lampreys sex ratio change on the larger ecosystem. However, due to time and data volume constraints, the study overlooks the effects of some external environmental factors on the lamprey ecosystem. In future studies, data collection can be further expanded and experimental design can be improved. At the same time, factors such as disease mortality, habitat changes, and interspecific competition will be taken into account in the model to further improve the understanding and prediction of the lamprey ecosystem and its dynamics. This paper presents a research methodology and framework that can be used in the field of population ecology. By modeling this predator-prey system to depict the relationship between predators, lampreys, and prey, it not only demonstrates its important role in predicting changes in species populations but also emphasizes the importance of external conditions, such as sex ratios and food resources, to ecosystem stability and biodiversity. The validity of this approach and its significant research value were fully demonstrated.

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