Agro-ECosystem Studies Based on Lotka-Volterra Modelling and Fourth Order Longe-Kuta Approach

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Abstract. In this paper, an agroecosystem food web model based on the Lotka-Volterra equation was constructed to simulate the ecosystem transition from forest to farmland in the state of Mato Grosso, Brazil, covering the producers, consumers at all levels, and soil organic matter dynamics equations. The model was solved using the fourth-order Ronger-Kuta method (RK4), optimised and evaluated by defining an objective function containing economic benefits and ecological sustainability and related assessment indicators, and determining the weights of the indicators using AHP and EWM analysis. On this basis, two new species, leaf-cutting ants and red-eyed wasps, were introduced to adjust the dynamic equations of the model and solved again. The results showed that the introduction of the new species enriched the ecosystem structure, improved the soybean yield and enhanced the ecosystem stability, and the objective function values of the new model were better than the original model, which provided valuable references for ecosystem management and sustainability research.

Keywords: Lotka-Volterra equation; fourth-order Longe-Kuta method; objective function; species introduction.

1. Introduction

The sustainable development of agro-ecosystems has become a key issue as the demand for agricultural products is surging due to global population growth. The transition from forest to farmland affects the structure and function of ecosystems, and in-depth studies are important for the development of scientific agricultural management strategies. Mathematical modelling provides a powerful tool for ecosystem studies, and the Lotka-Volterra equation effectively describes species interactions. In ecosystem evolution, the introduction of new species can change ecological relationships and affect system stability and economic value.

In this paper, we constructed an agroecosystem model based on the Lotka-Volterra equation, solved it by the fourth-order Longe-Kuta method, and established an objective function evaluation system to optimise the management strategy and explore the impact of new species introduction, so as to provide support for agroecosystem management and sustainable development.

2. Model the current ecosystem

2.1. Development of an Agricultural Ecosystem Model

To simulate the ecosystem transition from forest to farmland, we constructed a food web model based on the Lotka-Volterra equation by substituting the rainforest-farmland transformation in the state of Mado Grosso, Brazil. The model contains producers (soybeans and weeds), primary consumers (aphids), secondary consumers (spiders) and tertiary consumers (birds). The model also considered the effects of agricultural cycles, seasonal variations, soil organic matter, and herbicides and pesticides on the ecosystem [1, 2].

(1) Soybean Growth Equation

Soybean population size $N_{AT}(t)$ was affected by its own growth rate, aphid feeding, competition with weeds, soil organic matter content, and herbicide suppression:

$$\frac{dN_{AT}}{dt} = r_{AT}(t) \cdot N_{AT} \cdot \left(1 - \frac{N_{AT} + N_C}{K_{AT}}\right) - \beta_A \cdot N_A \cdot N_{AT} \\ +0.0001 \cdot SOM \cdot N_{AT} - h(t) \cdot N_{AT} \tag{1}$$

Where, $r_{AT}(t)$ is the growth rate of soybeans over the seasons and $h(t)$ is the rate of plant inhibition by herbicides that changed dosage over the seasons.

$$r_{AT}(t) = 0.1 \cdot \left(0.5 + 0.5\sin\left(\frac{2\pi t}{12}\right)\right) \tag{2}$$

$$h(t) = 0.003 \cdot \left(1 + 0.1\sin\left(\frac{2\pi t}{12}\right)\right) \tag{3}$$

(2) Weed Growth Equation

Weed population size $N_C(t)$ was affected by its own growth rate, aphid feeding, competition with soybeans, soil organic matter content, and herbicide suppression:

$$\frac{dN_C}{dt} = r_C \cdot N_C \cdot \left(1 - \frac{N_C + N_{AT}}{K_C}\right) - h(t) \cdot N_C + 0.0001 \cdot SOM \cdot N_C \tag{4}$$

(3) Aphid Population Equation

Aphid population size $N_A(t)$ was affected by its own growth rate, predation by spiders, and insecticide suppression:

$$\frac{dN_A}{dt} = r_A \cdot N_A \cdot \left(1 - \frac{N_A}{K_A}\right) - \alpha_S \cdot N_S \cdot N_A - u(t) \cdot N_A \tag{5}$$

(4) Spider Population Equation

Spider population size $N_S(t)$ was affected by its own growth rate, aphid feeding rate, natural mortality and insecticide suppression.

$$\frac{dN_S}{dt} = \gamma_S \cdot \left(\frac{\alpha_S \cdot N_A \cdot N_S}{1 + \alpha_S \cdot h \cdot N_A}\right) - d_S \cdot N_S - \alpha_B \cdot N_B \cdot N_S - v(t) \cdot N_S \tag{6}$$

Due to the large aphid populations, we introduce here the Holling II type of functional response in order to make the simulations ecologically consistent with the predation rate equation:

$$\text{predation} = \frac{\alpha_S \cdot N_A \cdot N_S}{1 + \alpha_S \cdot h \cdot N_A} \tag{7}$$

Where *h* is a treatment time factor limiting spider predation rates at high aphid densities.

(5) Bird Population Equation

Bird population size $N_B(t)$ was determined by natural mortality, energy conversion after predation on spiders.

$$\frac{dN_B}{dt} = \epsilon_B \cdot \alpha_B \cdot N_S \cdot N_B - d_B \cdot N_B \tag{8}$$

(6) Soil Organic Matter Dynamic Equation

To further model the complexity of the ecosystem, we introduced a soil organic matter content $SOM(t)$ whose value is determined by direct and indirect inputs of energy from soybeans and weed residues, and organic matter decomposition loss.

$$\frac{dSOM}{dt} = k_1 \cdot N_{AT} - k_2 \cdot SOM + k_3 \cdot N_C \quad (9)$$

2.2. Four-Step Runge-Kutta Method (RK4)

In this section, we will describe the Four-Step Runge-Kutta method (RK4) applied to model the dynamics of an agricultural ecosystem over time. The RK4 method updates the system variables by calculating four intermediate slopes (k1, K2, k3, k4), which are then used to derive the new state of the system [3]. The specific steps for this method are outlined below:

(1) Calculate the first slope k_1

The first slope k_1 is computed using the current time t_n and the current state X_n of the system, which includes key variables such as [N_{AT} , N_B , N_A N_S N_B SOM]. The slope is given by:

$$k_1 = h \cdot F(t_n, X_n) \quad (10)$$

Where F is the system's differential function.

(2) Calculate the second slope k_2

The second slope k_2 is computed using the midpoint of the interval $t_n + \frac{h}{2}$ and the updated state $X_n + \frac{k_1}{2}$:

$$k_2 = h \cdot F\left(t_n + \frac{h}{2}, X_n + \frac{k_1}{2}\right) \quad (11)$$

(3) Calculate the third slope k_3

Similarly, the third slope k_3 is calculated using the updated state $X_n + \frac{k_2}{2}$ and the midpoint time $t_n + \frac{h}{2}$:

$$k_3 = h \cdot F\left(t_n + \frac{h}{2}, X_n + \frac{k_2}{2}\right) \quad (12)$$

(4) Calculate the fourth slope k_4

The fourth slope k_4 is computed using the final updated state $X_n + k_3$ and the time $t_n + h$:

$$k_4 = h \cdot F(t_n + h, X_n + k_3) \quad (13)$$

(5) Update the system variables

The final update to the system's state is computed as the weighted average of the four slopes:

$$X_{n+1} = X_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (14)$$

This weighted average method ensures a balance between accuracy and stability of the numerical solution [4], and the results are shown in Fig. 1.

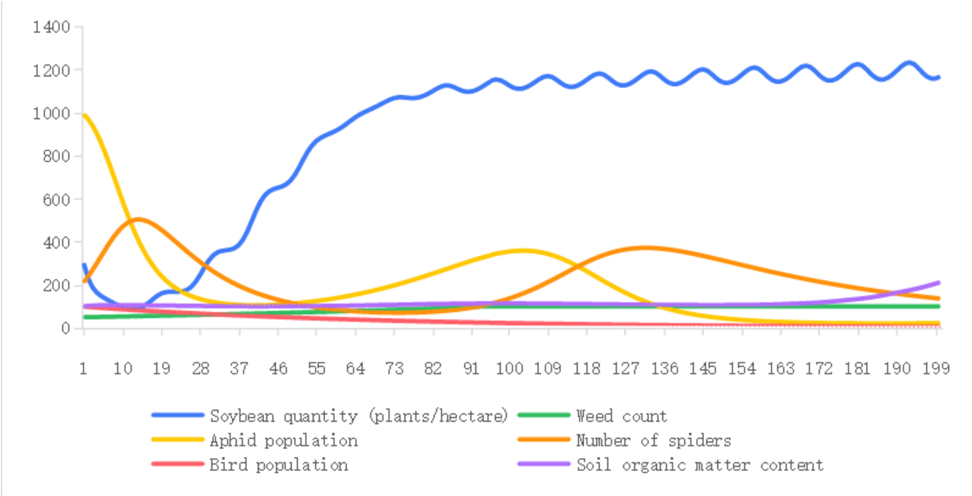

Fig 1. Numerical results for consumers at all levels.

The Four-Step Runge-Kutta (RK4) method allows for the approximation of solutions to ordinary differential equations by evaluating the system's behavior at intermediate points and averaging the results. This method ensures precise and stable computations and is particularly suitable for modeling the dynamic changes in ecosystems affected by agricultural practices, such as the conversion of forested land into agricultural fields. By incorporating the RK4 method, we can track the gradual evolution of species and agricultural impacts over time, providing valuable insights into ecosystem management and sustainability.

2.3. Evaluation system based on the objective function

The objective function is a mathematical expression to be maximized or minimized in an optimization problem and is used to measure the quality or performance of the solution. In order to optimize the agricultural management strategy, the objective function is defined along with three secondary indicators to balance the economic returns and ecological sustainability:

$$\text{Maximize} Z = \int_0^T \left[\lambda_1 \cdot AT(t) - \lambda_2 \cdot (u(t) + v(t)) - \lambda_3 \cdot (1 - \frac{SOM(t)}{SOM_{\max}}) \right] dt \tag{15}$$

In order to simplify the model design, $AT(t)$ is the biomass (yield) of soybean, which represents the economic effect per unit area of farmland; $u(t)$ and $v(t)$ together represent the negative impacts of chemicals; in order to correctly incorporate the optimization objective of soil organic matter content(SOM) into the maximization problem, $SOM(t)$ is replaced by the "insufficiency" of soil organic matter content, which represents ecosystem stability; $\lambda_1 \backslash \lambda_2 \backslash \lambda_3$ the weights of each indicator are derived from the AHP and EWM analyses.

2.3.1. AHP and EWM analysis

Taking into account the subjective experience and objective data, this paper adopts the combination of AHP and EWM to determine the weights [5]. The specific steps are as follows:

Conduct dimensionless processing of data

Calculate the weights of each index by AHP and EWM respectively

Weighted average of the weights of AHP and EWM to get the comprehensive weights:

$$\lambda_j^* = \alpha \lambda_j^{\text{AHP}} + (1 - \alpha) \lambda_j^{\text{EWM}} \tag{16}$$

Where, α is the scaling factor of AHP weights and is taken as 0.5.

(1) Determination of EWM SI weights

Entropy Weight Method (EWM) is an objective assignment method based on information entropy. Its core idea is to calculate the information entropy of each indicator to reflect the degree of dispersion of the indicator, so as to determine the weight of each indicator. The smaller the information entropy, the greater the dispersion of the indicator and the higher the weight. Based on the data of these three indicators for the period 2011-2020, we determined the weights, as shown in Table 1:

Table 1. Indicator weights.

Targets	Economic benefits	Impact of chemicals	Ecosystem stability
Weight (%)	33.843	33.078	33.078

(2) Calculation of AHP PI Weights

In order to take into account the practical significance and relative importance of the indicators, we introduced the AHP to adjust the weights. We considered the economic returns to agricultural production to be the most important, ecosystem stability to be the second most important, and the impacts of chemicals to be at the bottom of the list for weighting, as shown in Table 2:

Table 2. Adjusted weights.

Targets	Economic benefits	Impact of chemicals	Ecosystem stability
Weight (%)	53.529	20.604	25.867

The final weights are obtained from the weighted average, as shown in Fig. 2:

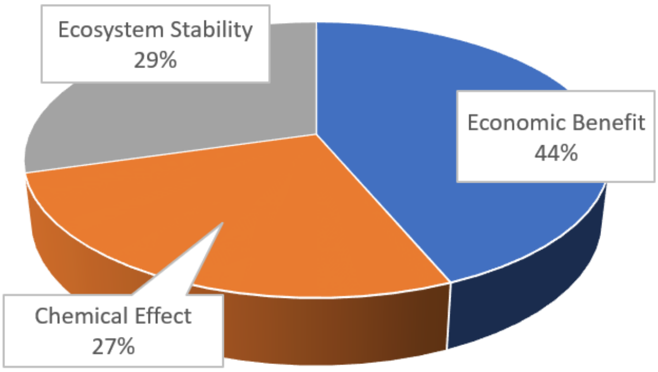

Fig 2. Weighting Schematic.

2.3.2. Calculation of Evaluation Indicators

By solving the Lotka-Volterra equation, we get the changes in the simulated variables. We need to substitute these values into the objective function expression and use numerical integration methods to calculate the value of the objective function.

2.3.3. Constraints are established

1. Population size non-negativity:

$$N_{AT}(t) \geq 0, N_C(t) \geq 0, N_A(t) \geq 0, N_S(t) \geq 0, N_B(t) \geq 0 \quad (17)$$

2. Environmental carrying capacity:

$$N_{AT}(t) \leq K_{AT}, N_C(t) \leq K_C, N_A(t) \leq K_A, N_S(t) \leq K_S, N_B(t) \leq K_B \quad (18)$$

3. Chemical use ceilings:

$$0 \leq u(t) \leq u_{\max}, \quad 0 \leq v(t) \leq v_{\max}, \quad 0 \leq h(t) \leq h_{\max} \quad (19)$$

2.3.4. Data Processing

Due to the large differences in the different secondary indicators, the data were processed for missing values, outliers and positive and negative indicators in order to minimize the impact of the units of the data itself.

$$x' = \frac{x - min(x)}{max(x) - min(x)} \tag{20}$$

$$x' = \frac{max(x) - x}{max(x) - min(x)} \tag{21}$$

2.3.5. Objective Function Solving

Since $N_{AT}(t)$, (h(t)+$\nu(t)$) and $SOM(t)$ are discrete values computed from the dynamic equations, it is not possible to solve the integrals directly analytically, so numerical integration is required, and here the trapezoidal method is used to solve them:

1. Discretized Time Intervals

Divide the time interval [0, T] into n subintervals, each of width $\Delta t = \frac{T}{n}$. Setting $t_i = i\Delta t (i = 0,1,2,\dots,n)$, the value of the product function at t_i is $f(t_i)$.

2. Calculate the Area of a Trapezoid

For each interval $[t_{i-1}, t_i]$ calculate the area of the trapezoid:

$$z_i = \frac{f(t_{i-1}) + f(t_i)}{2} \Delta t \tag{22}$$

Sum the areas of all the small trapezoids to get an approximation of the integral:

$$Z \approx \sum_{i=1}^{n} \frac{f(t_{i-1}) + f(t_i)}{2} \Delta t \tag{23}$$

The value of this objective function is: Z_0 =687.04.

This value will be used as a reference value to evaluate the model optimization results in comparison with the subsequent models.

3. Incorporate the reemergence of species

Over time, edge habitats begin to mature, bringing back native species to the area, and ecosystem structure changes as a result. In this section we introduce two organisms in different ecological niches and explore their impact on the economic value and stability of the original ecosystem.

3.1. Modeling

We introduced two new species to the original food web model: leaf-cutting ants N_L and red-eyed wasps N_{Ww}. Leaf-cutting ants indirectly affect crop growth by promoting soil organic matter accumulation, while red-eyed wasps control pest populations by feeding on aphids.

3.1.1. New species

1. Leaf cutter ants N_L: Leaf cutter ants contribute to the accumulation of organic matter in the soil by decomposing plant residues, which in turn affects crop growth. Leaf cutter ant populations are influenced by weed and soybean populations.

2. Red-eye wasp N_{Ww}: The red-eye wasp is a natural enemy of aphids and controls pest populations by feeding on aphids. Red-eyed wasp populations are closely related to aphid populations.

3.1.2. Dynamic Equation Adjustment

(1) Soybean Population Dynamics Equation:

Soybean feeding inhibition by leaf-cutting ants.

$$\frac{dN_{AT}}{dt} = r_{AT}(t) \cdot N_{AT}\left(1 - \frac{N_{AT} + N_C}{K_{AT}}\right) - \beta_A \cdot N_A \cdot N_{AT} - \delta \cdot N_L \cdot N_{AT} + \eta \cdot \mathrm{SOM} \cdot N_{AT} - h_{\max} \cdot N_C \tag{24}$$

(2) Weed Population Dynamics Equation:

Weeds are inhibited by feeding by leaf-cutting ants.

$$\frac{dN_C}{dt} = r_C \cdot N_C\left(1 - \frac{N_C + N_{AT}}{K_C}\right) - h_{\max} \cdot N_C + 0.0001 \cdot \mathrm{SOM} \cdot N_C \tag{25}$$

(3) Aphid Population Dynamics Equation:

Aphids are inhibited by feeding on red-eye wasps.

$$\frac{dN_A}{dt} = r_A \cdot N_A\left(1 - \frac{N_A}{K_A}\right) - \alpha_S \cdot N_S \cdot N_A - \alpha_W \cdot N_{Ww} \cdot N_A - u_{\max} \cdot N_A \tag{26}$$

(4) Leaf-Cutting Ant Population Dynamic Equation:

Determined by its own growth rate, environmental capacity limitations, weed gain, and energy transformation from feeding on soybeans.

$$\frac{dN_L}{dt} = r_L \cdot N_L\left(1 - \frac{N_L}{K_L}\right) + \kappa \cdot N_C - \delta \cdot N_L \cdot N_{AT} \tag{27}$$

(5) Red-Eye Wasp Population Dynamic Equation:

Determined by energy transformation of predatory aphids, natural mortality, and insecticide action.

$$\frac{dN_{Ww}}{dt} = \gamma_W \cdot \alpha_W \cdot N_A \cdot N_{Ww} - d_W \cdot N_{Ww} - u_{\max} \cdot N_{Ww} \tag{28}$$

(6) Soil Organic Matter Dynamic Equation:

Additional leaf-cutting ant contribution.

$$\frac{d\mathrm{SOM}}{dt} = \min\left(k_1 \cdot N_{AT} + k_3 \cdot N_C + \mu_L \cdot N_L - k_2 \cdot \mathrm{SOM}, \mathrm{SOM}_{\max} - \mathrm{SOM}\right) \tag{29}$$

3.2. Model solution

We used the fourth-order Runge-Kutta method to numerically solve the adjusted set of differential equations. This method calculates the change in population size at each time by iterating step by step to ensure the stability and accuracy of the model, as shown in Fig. 3.

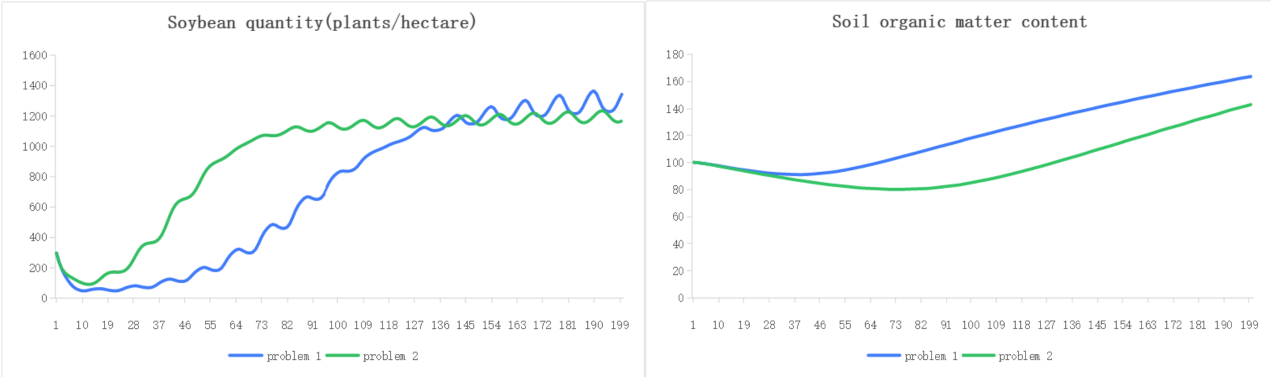

Fig 3. Changes in population size at each time.

Through numerical simulation, we obtained the curves of soybean and soil organic matter over time and compared them with the model in Problem 1, which showed that the addition of the two species enriched the structure of the ecosystem, positively benefited the soybean yield, and made the ecosystem more stable.

3.3. Evaluation system based on the objective function

Based on the evaluation system determined above, we bring in the results of solving the adjusted dynamic equations to obtain the value of the objective function of the model as:

$$Z_0 = 695.56 \tag{30}$$

The model synthesized better compared to the reference value Z_0 =687.04, which shows the positive effect of introduced species in marginal habitats on agro-ecosystems.

Through calculations, we obtained the objective function value Z and analyzed the effects of different parameters on ecosystem stability.

4. Summary

In this study, we successfully constructed an agroecosystem model based on the Lotka-Volterra equation, simulated the ecological transition from forest to farmland, and comprehensively applied the fourth-order Longe-Kuta method, AHP and EWM analyses, and the objective function assessment system to explore the agroecosystem in depth. It was found that the introduction of new species of leaf-cutting ants and red-eyed wasps significantly optimised the ecosystem structure, which not only enhanced the soybean yield, but also strengthened the stability of the ecosystem, and made the model better balanced in terms of economic returns and ecological sustainability.

By comparing the objective function values of different models, the positive effects of new species introduction on the agroecosystem were visually demonstrated, providing a quantitative basis for the optimal management of the agroecosystem. Meanwhile, the models and analysis methods developed in this study are universal and can be expanded and applied to other similar ecosystems in research and management practices.

References

[1] YI Xianyan, Lv Tanghong. Hopf branching analysis of the Lotka-Volterra predator-feeder system with harvest term and double time lag and the influence of Allee effect [J/OL]. Journal of Jili n University (Science Edition),1-10[2025-02-14].https://doi.org/10.13413/j.cnki.jdxblxb.2024201.

[2] Wu D. Research on functional differences and stability of aquatic ecosystems based on food web model [D]. Yangzhou University,2024.DOI:10.27441/d.cnki.gyzdu.2024.002385.

[3] J.F. Li,H. Bi. Stability analysis of implicit-explicit Runge-Kutta ultra-weak locally interrupted Galerkin method for linear fourth order equations [J]. Journal of Natural Science of Heilongjiang University, 2024, 41(06):700-710.DOI:10.13482/j.issn1001-7011.2024.12.246.

[4] Xuezhen Zhao, Yuewen Chen. Atmospheric weighted mean temperature model based on ASO-MLP neural network [J]. Mapping and Spatial Geographic Information, 2025, 48(01):82-85.

[5] JIU Yunyang, LIU Jinzhao, MENG Jinghui, et al. Research on dynamic service quality evaluation method of GSM-R cellular network based on AHP-EWM [J/OL]. Railway Standard Design,1-10[2025-02-14].https://doi.org/10.13238/j.issn.1004-2954.202409060001.