A Multi-Dimensional Exploration of Black-Body Radiation: Laws, Applications and Significance

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Abstract. This paper studies the black-body radiation and related laws, providing a background inquiry into black body radiation as one of the important sources of modern physics. The problems to be tackled include derivation of Planck's formula, and derivation of Wien's displacement law and Stefan-Boltzmann law from Planck's formula. This work also mentions applications in cosmology, such as cosmic background radiation. In a methodical way, it introduces the derivation of the black-body formula, provides calculation of average energy and number of vibrations of vibrating particles inside the cavity with a certain frequency, and comes to the black-body radiation formula. Limit method falls into place in order to arrive at Wien formula, and Rayleigh-Jeans formula from Planck's law. The results and conclusions are illustrated on the derivation of Stefan-Boltzmann law and Wien's Law, along with applications in cosmology. This work is significant in that it will help readers get a clear view of the black body radiation formula and its importance. It serves as a guide for the new physicists in their study and seems to be helpful for professionals in other fields.

Keywords: Black-body radiation; Planck's formula; Wien's displacement law; Stefan-Boltzmann law; cosmic background radiation.

1. Introduction

Numerous laws and phenomena are current and persistent challenge to the development and advance of physics in the extensive and large disciplines. Black body radiation takes center stage among them and represents an interesting and broadly relevant study issue that can be in one way or another directly related to a number of other areas.

Situated firmly on the physics board, blackbody radiation is a pivotal topic that attracts a large number of researchers to delve deeply into it. According to the review of relevant literature, blackbody radiation has a wide range of applications and is helpful in explaining the interactions between matter and energy. For instance, Marr J M and Wilkin F P [1] presented a better presentation of Planck's radiation law. In astronomy, blackbody radiation spectrum was derived without quantum assumptions, and made it possible to better comprehend the universe's heat evolution and the luminous process in stars. Blackbody radiation also has a wide range of applications in materials science. Chen and Su [2] explored the Infrared Thermometer and Black-Body Radiation Law. It enables energy conversion and heat management systems, as well as the optimal operation of thermal radiating materials. On the other hand, empirical blackbody radiation is crucial to the advancement of quantum energy quantization principles in quantum mechanics. For example, Agassi J considered the Kirchhoff-Planck Radiation Law and its implications for the understanding of the rise of quantum theory. Li , Zhi , and Chen [3] discussed the Black-Body Radiation and Planck's Quantum Hypothesis.

The current work models black body radiation to understand related concepts and laws. It presents the theoretical model, derives Planck's law, and explains Wien's and Stefan-Boltzmann's laws. Further, it discusses their worth in physics and astrophysics and elaborates cosmic microwave background radiation. This paper aims to enhance the understanding of blackbody radiation to readers.

2. Concept of Blackbody Radiation and Derivation of Planck's Formula

The paper takes black-body radiation as the object of research to explore related concepts and laws. The author, first, introduces the research model and derives Planck's law. Later, the author explains

Wien's displacement law and Stefan-Boltzmann radiation law. Finally, the author discusses in detail the value and significance of these laws for physics, especially their application in astrophysics. The overall contribution of this research is a deeper understanding of the nature and implications of black-body radiation [3].

2.1. Concept of Blackbody Radiation

Planck law's derivation constitutes an unquestionable milestone in the understanding of blackbody radiation. Three basic assumptions on which Planck's law is based are considered with application are introduced. Conversely, a numerical description is provided for the spectral energy distribution of blackbody radiation, while a complete qualitative understanding of electromagnetic radiation is still to be developed [2].

A perfect blackbody - an idealized model like a point mass or a rigid body - is an object that entirely absorbs all incident electromagnetic radiation without any emission. There is no such perfect black body in nature; for instance, for soot, the maximum absorption coefficient is roughly 0.99 [3]. Imagine an object that completely absorbs all frequencies of radiation striking it, without regards to frequency or temperature V,T - every frequency and the absorbed ability being constant and equal to 1 in this case. This is known as an absolute black body, and yet, it remains a concept as no real material exists anywhere as a true absolute black body [4]. In his experimental research, Wien devised a means of creating a perfect black body model using a cavity with a small hole as diagrammatically shown below in Fig. 1.

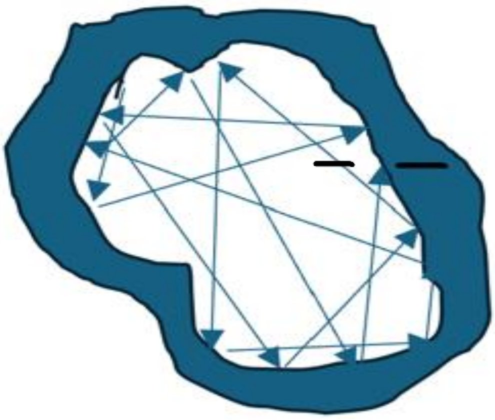

Fig 1. Scheme of Blackbody radiation model with a small hole.

Because radiation from the outside enters through the hole and is subjected to numerous internal reflections, with each reflection causing the cavity wall to absorb part of the energy until, eventually, all incoming radiation is absorbed, the cavity can be thought of as a black body. The ratio of the small hole area to the cavity wall area ensures that virtually no radiation escapes through it. It acts somewhat like an ideal blackbody at low temperatures [1], thereby exhibiting the properties of blackbody radiation.

Planck's hypothesis states that the allowed energies of these resonators are quantized. The exchange of energy between resonators and cavity radiation is also quantized and frequency dependent. At low temperatures, the low-frequency resonators are already in an excited state, emitting long-wave radiation. However, as the temperature rises, higher-frequency resonators are also excited, causing the radiation peak to gradually shift to shorter wavelengths represented in Fig 2 [4, 5]. In 1859, Gustav Kirchhoff proved that black body radiation consists of constants dependent on both the frequency and temperature only, instead of material. This measurement thus relates to radiation energy and frequency at given constant temperature [3]. Laboratory observations showed that the form of the radiation energy density curve behaves as a function of frequency or wavelength, once the thermal radiation of a black body reaches thermal equilibrium, which depends only upon the absolute temperature of the black body and is independent of the shape or composition of the cavity [4] as shown in Fig. 2.

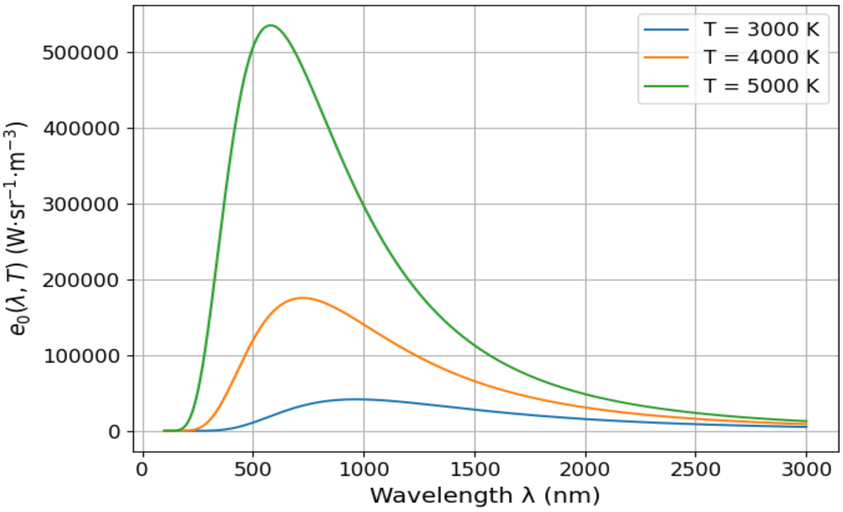

Fig 2. Blackbody radiation experiment results: radiation intensity versus wavelength.

2.2. Derivation of Planck's Formula

At the end of 1900, Planck abandoned the classical physics viewpoint maintained by Wien and Rayleigh, who maintained that "the vibrational energy of charged particles is continuous," and instead proposed that "the energy of vibrating charged particles can only be an integer multiple of a certain minimum energy e." As a result of this, Planck obtained the theoretical formula for intensity distribution of the black body radiation in terms of frequency, which perfectly matched the experimental result [4]. Planck's quantized blackbody radiation law is given in frequency form as:

$$e_0(\nu, T) = g(\nu)\bar{\varepsilon}(\nu, T) = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1} \quad (1)$$

Where $e_0(\nu, T)$ is the radiation power, $\bar{\varepsilon}(\nu, T)$ is the mean energy of resonators, $g(\nu)$ is the number of resonators and h, c are Planck's constant and the velocity of light, k and T are the Boltzmann constant and temperature respectively.

Now, the author moves on to the degenerate form of the Planck formula for blackbody radiation in frequency when one considers the two obvious limits [6, 7]. As before, when frequency ν approaches infinity, $e^{\frac{h\nu}{kT}}$ becomes infinitely large thus the last -1 term will be neglected; thus equation (1) becomes the Wien's law written as:

$$e_0(\nu, T) = g(\nu)\bar{\varepsilon}(\nu, T) = \frac{8\pi h\nu^3}{c^3}e^{-\frac{h\nu}{kT}}. \quad (2)$$

The second condition is that when frequency ν approaches the limit of small values, where $\frac{h\nu}{kT}$ is now small. In that case, expand $e^{\frac{h\nu}{kT}}$ term using Taylor series and will get the expansion as $e^x \approx 1 + x + \frac{x^2}{2} + \ldots$ In terms of expansion, 1 is cancelled with each other in the denominator; thus, the square term of $\frac{h\nu}{kT}$ is negligible and can thus be ignored. Therefore, equation (1) is simplified to:

$$e_0(\nu, T) = g(\nu)\bar{\varepsilon}(\nu, T) = \frac{8\pi h}{c^3}\frac{\nu^3}{\frac{h\nu}{kT}} = \frac{8\pi\nu^2}{c^3}kT \quad (3)$$

This arrived at the Rayleigh-Jeans's equation.

If these three equations above are plotted in the same coordinate for comparison purposes, i.e., if comparing the plots of Planck's Law, Wien's Displacement Law, and Rayleigh-Jeans Formula together, readers can have a more intuitive comparison presented as below [6], see Fig. 3.

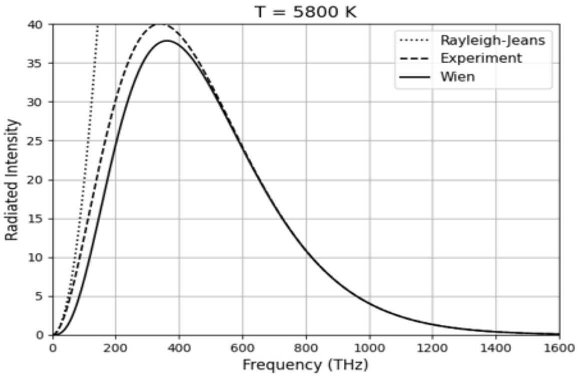

Fig 3. Rayleigh-Jeans distribution, Wien distribution and experimental curve graph comparison.

The graph in Fig. 3 shows that the Wien formula is consistent in the high-frequency area but inconsistent in the low-frequency region when compared to the experimental data. In the low-frequency region, the Rayleigh-Jones formula is found to be more consistent with the experiment; however, in the high-frequency region, there is a significant difference and it is divergent [6]; in the short-wave direction, a disastrous conclusion is drawn which was referred to as the "ultraviolet catastrophe" at the time [3]. The whole field of classical physics is currently experiencing a "disaster" due to the inability of the classical theory to address the blackbody radiation issue. As a result, some started to question classical physics [3]. In order to derive a theoretical formulation that is consistent with the experimental results to solve this problem, Planck derived his equation (1) started from three fundamental assumptions [4]:

Firstly, in the quantum theory of light, energy is discrete, and it is expressed as follows:

$$\varepsilon = nh\nu (n = 1,2,3 \ldots \ldots) \tag{4}$$

Secondly, while distributing harmonic oscillators among their allowed energy states are subject to the usual Boltzmann distribution, i.e., the probability of a harmonic oscillator being in an energy state e is given by $P(\varepsilon) = Ce^{\frac{-\varepsilon}{KT}}$. to ensure normalization, $\sum_0^\infty P(\varepsilon) = 1$, thereby with a coefficient as $C = 1/\sum_0^\infty e^{\frac{-\varepsilon}{KT}}$. Thirdly, Plank accepted the results obtained by Rayleigh where $g(\nu) = \frac{8\pi\nu^2}{c^3}$. Planck's hypothesis on the quantization of energy broke with the traditional view that energy changes continuously and allowed a correct interpretation of phenomena such as photoelectric effects and the Compton Effect. In this regard, it opened an entirely new chapter in the development of physics [4].

3. Deductions of Planck's Formula

3.1. Wien's Displacement Law

The next in line is Wien's Displacement Law; it expresses the relation between the peak wavelength of blackbody radiation and temperature. Two forms of it are put forward, each discussing its implications and utility [5, 8]. The Displacement Law is truly a key in understanding blackbody radiation in respect to other temperatures [6]. This is formulated in 1893 as:

$$\lambda_{max}T = b = 2.898 \times 10^{-3}(m \bullet K) \tag{5}$$

According to Planck's law of blackbody radiation, taking the partial differential and derivative, the specific mathematical derivation process of this formula is as follows. Firstly, according to the expression of the total radiation power $R(T)$ written as [7]:

$$R(T) = \int_0^\infty R(\lambda, T)d\lambda = \int_0^\infty R(\nu, T)d\nu \tag{6}$$

Similarly, there is also the following relation:

$$e_0(\lambda, T)d\lambda = e_0(\nu, T)d\nu \tag{7}$$

Then the distribution formula of the radiation energy density in the range of radiation wavelength from λ too $\lambda + d\lambda$ can be obtained as:

$$e_0(\lambda,T) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{h\nu}{kT\lambda}}-1} \tag{8}$$

Where $\nu = \frac{c}{\lambda}$. According to the relationship between the blackbody radiation power $R(\nu,T)$ and the radiation field energy density $e_0(\nu,T)$ in the thermal equilibrium within the cavity, it is $R(\nu,T) = \frac{c}{4}e_0(\nu,T)$ [7]. By combining the above formulas, one obtains:

$$R(\lambda,T) = \frac{c}{4}e_0(\nu,T) = \frac{c}{4}\frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{h\nu}{kT\lambda}}-1} = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{\frac{h\nu}{kT\lambda}}-1} \tag{9}$$

Alternatively, according to Equation (3), and because $\nu = \frac{c}{\lambda}$, differentiating both sides of the equation gives $|d\nu| = \frac{c}{\lambda^2}d\lambda$. Substituting this equation and Planck's blackbody radiation frequency formula back, one obtains another wavelength form of Planck's blackbody radiation formula as:

$$e_0(\lambda,T)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{\frac{h\nu}{kT\lambda}}-1} \tag{10}$$

Let $x = \frac{h\nu}{kT\lambda}$, one gets $e_x(\lambda,T) = \frac{8\pi k^5T^5}{c^4h^4}\frac{x^5}{e^x-1}$. if one takes the derivative and sets them equal to 0 to obtain the followings:

$$\frac{\partial R(\lambda,T)}{\partial\lambda} = 2\pi hc^2\left[-\frac{5}{\lambda^6}\frac{1}{e^{\frac{h\nu}{kT\lambda}}-1} + \frac{1}{\lambda^5}\left(\frac{-(-\frac{h\nu}{kT\lambda^2}e^{\frac{h\nu}{kT\lambda}})}{(e^{\frac{h\nu}{kT\lambda}}-1)^2}\right)\right] = 0 \tag{11}$$

And

$$5\left(e^{\frac{h\nu}{kT\lambda}}-1\right) = \frac{h\nu}{kT\lambda}e^{\frac{h\nu}{kT\lambda}} \tag{12}$$

Similarly let $x = \frac{h\nu}{kT\lambda}$, then all these equations would become as [7]

$$5(e^x-1) = xe^x \tag{13}$$

This is a transcendental equation, so its fixed solution cannot be obtained by the general solution method. Here, the "bracketing method" is used to estimate its value. First, let $f(x) = 5(e^x-1) = x$. from the equation, it is obvious that when$x = 0$, $f(x) = 0$, and$\lambda \to \infty, R \to 0$. Secondly, the derivative of the equation is found and set equal to 0:

$$\frac{df(x)}{dx} = 5e^x - (e^x + xe^x) = 4e^x - xe^x = 0 \tag{14}$$

Finally, it is concluded that $x = \frac{h\nu}{kT\lambda} = \frac{h\nu}{kT\lambda_m} \approx 4.9651$, approximately 4.97. Computational method has been used here. At this time, if one substitutes the values of these numbers,$x = \frac{hc}{\lambda_m kT} = 4.9651$ one obtains $\lambda_m T = \frac{hc}{4.9651k} = b \approx 0.28984\ cm \cdot K$. this is the Wien displacement formula under normal circumstances [9].

Another form of Wien's displacement law is derived from the Planck's law in terms of circular frequency, thereby obtaining another equivalent form of the Planck's law [10,11]

$$e_0(\omega,T)=\frac{\hbar\omega^3}{\pi^2c^3}\frac{1}{e^{\frac{\hbar\omega}{kT}}-1} \tag{15}$$

If the derivative of the above equation is taken and set equal to 0:

$$\frac{3\hbar\omega^2}{\pi^2c^3}\frac{1}{e^{\frac{\hbar\omega}{kT}}-1}-\frac{\hbar\omega^3}{\pi^2c^3}\frac{\hbar}{kT}e^{\frac{\hbar\omega}{kT}}\frac{1}{\left(e^{\frac{\hbar\omega}{kT}}-1\right)^2}=0 \tag{16}$$

Here, let $x=\frac{\hbar\omega}{kT}$, similarly, the transcendental equation is obtained as $e^x=\frac{1}{1-\frac{x}{3}}$ with solution $x=2.82114$ generated by computer [12]. And because there is a formula connecting the circular frequency and the wavelength as $\omega=2\pi\nu=\frac{2\pi c}{\lambda}$, Therefore, one gets $x=\frac{\hbar\omega}{kT}=2.82114$. And finally:

$$\frac{T}{\omega_{max}}=2.708\times10^{-12}K\cdot Hz^{-1} \tag{17}$$

Now, if the author attempts to divide the equations, it will be strangely found

$$\lambda_{max}\omega_{max}=1.071\times10^9 m\cdot s^{-1}\neq 2\pi c=1.884\times10^9 m\cdot s^{-1} \tag{18}$$

The traditional mathematical formulas are not applicable here as $\lambda_{max}\neq\frac{2\pi c}{\omega_{max}}$.

Wien’s displacement law has been experimentally verified [6] which relates the peak wavelength or angular frequency of blackbody radiation to its temperature [6, 11, and 13]. Two forms can be derived from Planck's law, but surprisingly they cannot be interconverted as explained above. Wien's displacement law can express frequency at which the total energy density of blackbody radiation has its maximum value as a function of temperature [13, 14].

This law has different practical applications together with its theoretical significance. For example, it introduces infrared thermometer brightness temperature measurements based on Wien's displacement law. Also, by complying Wien's and Stefan-Boltzmann's law, it makes a diagram of the quantitative relationship between the absolute temperature (K), radiation power density (W/m^2), and peak wavelength (m), which would be important in practical applications within infrared technology [5]. Wien's displacement law also has many great applications in astronomy, was mainly used to measure the temperature of the cosmic background radiation. In 1964, Penzias and Wilson received a microwave signal noise that is uniformly distributed in space, which is called the cosmic background radiation. This is consistent with the result predicted by the Big Bang theory. The temperature of the cosmic background radiation estimated experimentally is$T = 2.73K$. According to the position of the peak of the spectrum, combined with Wien's displacement law:$\lambda_{max}T\approx 0.28984cm\bullet K$ this temperature is estimated [15, 16] as shown in Fig. 4.

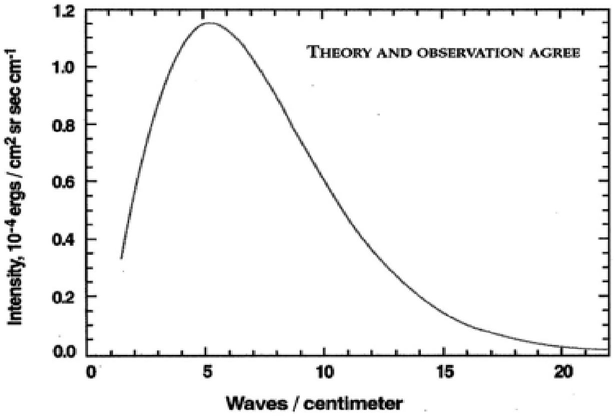

Fig 4. The spectral radiant power density distribution of a blackbody [17].

3.2. Derivation and Application of Stefan–Boltzmann Theory

The Stefan-Boltzmann law being described here explains the quantity emanating from a blackbody that tells of its radiation in area per unit time and measures itself through its temperature. The derivation, verification of this law and its various applications, from astrophysics to the science of materials, are discussed. The aim of these explorations is a synthesis of a comprehensible discussion of all the basic laws revealing their significance to physics [17, 18].

Stefan-Boltzmann law is one of the original fundamental theorems of radiation theory. In 1879, Stefan, during his research on the radiation of substances, experimentally discovered this conclusion: the emission of total radiant energy from a surface was proportional to the fourth power of the absolute temperature of that surface and was independent of wavelength [19,20]. This law is written mathematically as:

$$j^* = \sigma T^4 \tag{19}$$

Where $j^*$the radiant power density of a blackbody is $\sigma = (5.67032 \pm 0.00071) \times 10^{-12} W/(cm^2K^4)$ is the Stefan-Boltzmann constant, T is the absolute temperature, with unit of Kelvin. Boltzmann's contribution was in 1884 when he integrated thermodynamics and Maxwell's electromagnetic theory to theoretically verify the accuracy of Stefan's experimental conclusion, thereby establishing this mathematical expression of the famous Stefan-Boltzmann radiation law i.e. the general equation of total radiant energy, also known as the quartic law.

The Stefan-Boltzmann law can be derived in various ways, using Carnot's theorem or the second law of thermodynamics [19]. From the Carnot theorem it has a relation as:

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P \tag{20}$$

Next, this equation is utilized to prove the Stefan-Boltzmann law: Here, the internal energy of a radiation field can be expressed as the internal energy per unit volume (internal energy density) multiplied by its volume, that is:

$$U(T,V) = u(T)V \tag{21}$$

Now, taking the partial derivative of the internal energy with respect to the volume $\left(\frac{\partial U}{\partial V}\right)_T = u(T)$.

In the derivation process, it is necessary to mention the relation $P = \frac{1}{3}u(T)$, Where P is the radiation field pressure, $u(T)$ is the internal energy density of the radiation field. This relationship can be obtained from classical thermodynamics, statistical mechanics, or electrodynamics. Now, it can be known that:

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial P}{\partial u}\right)_V \left(\frac{du(T)}{dT}\right) = \frac{1}{3}\left(\frac{du(T)}{dT}\right) \tag{22}$$

Then, substituting equations back, one could obtain:

$$u(T) = \frac{T}{3}\left(\frac{du(T)}{dT}\right) - \frac{u(T)}{3} \tag{23}$$

After sorting, one gets $\frac{du(T)}{u(T)} = \frac{4dT}{T}$. And after integrating it one finally arrives at:

$$u(T) = aT^4 \tag{24}$$

This is the Stefan-Boltzmann theorem to be proved.

Next, in order to prove the Stefan-Boltzmann radiation law rigorously, it is common practice to integrate Planck's radiation law over the entire wavelength region [5, 6]. Rewrite Planck’s radiation law i.e. equation (10) as following:

$$j^*{}_\lambda = c_1\lambda^{-5}\left[e^{\frac{c_2}{\lambda T}} - 1\right]^{-1} \tag{25}$$

Let$x = \frac{c_2}{\lambda T}$, so one gets:$dx = -\frac{c_2}{\lambda^2 T}d\lambda$ or $d\lambda = -\frac{\lambda^2 T}{c_2}$therefore the following equations

$$j^* = \int_0^\infty j^*{}_\lambda d\lambda = \int_0^\infty \frac{c_1}{\lambda^5}\frac{d\lambda}{e^{\frac{c_2}{\lambda T}} - 1} = \int_0^\infty \frac{c_1}{\lambda^5}\frac{\lambda^2 T}{c_2}\frac{dx}{e^x - 1} = \frac{c_1}{{c_2}^4}T^4\int_0^\infty \frac{x^3 dx}{e^x - 1} \tag{26}$$

If expanding the denominator in a series one gets $\frac{1}{e^x-1} = \frac{e^{-x}}{1-e^{-x}} = e^{-x}(1 + e^{-x} + e^{-2x} + \cdots) = \sum_{n=1}^{\infty} e^{-nx}$. Then equation (26) can be simplified as $\int_0^\infty \frac{x^3 dx}{e^x-1} = \sum_{n=1}^{\infty}\int_0^\infty x^3\, e^{-nx}dx$. Looking at the integral table, one gets: $\int_0^\infty x^3\, e^{-nx}dx = \frac{6}{\pi^4}$,and because it is proved that $\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{90}$ Is a convergent function, therefore?

$$j^* = \frac{\pi^4}{15}\frac{c_1}{{c_2}^4}T^4 = \frac{2\pi^5 k^4}{15c^2h^3}T^4 = \sigma T^4 \tag{27}$$

The correctness of the Stefan-Boltzmann law is rigorously verified, and the physical meaning of this law is demonstrated [17].

Applications of Stefan-Boltzmann's radiation theory are made use of in photon detection, where both Stefan-Boltzmann's and Planck's radiation theories prove useful in performance study [17]. The photon detector has characteristics such as high sensitivity and fast response time, and it almost fills the entire infrared spectral region. Among all the applications, the total radiation thermometry method uses the total radiation energy of the object's radiation in the full band to determine the temperature of the object. According to the mathematical formula evolved from the Stefan-Boltzmann radiation law [17]:

$$M = \varepsilon\sigma T^4 \tag{28}$$

Here, the emissivity ε of the measured object is known, and when the value of ε is less than 1, the measurement error will be inevitable, and the smaller the value, the greater the error. When using an infrared thermometer to measure the temperature, it is necessary to know the emissivity of the measured object first. It has many advantages, including a very wide temperature measurement range (from tens of degrees below zero to several thousand degrees above zero), a wide application range, etc. For example, it is often used in metallurgy or the machinery manufacturing industry, which can greatly improve product quality and automation efficiency, etc. [17]. Applications of Stefan's-Boltzmann law in Astronomy includes the calculation of total thermal radiation power: The total power of thermal radiation, that is, the energy that a blackbody radiates to the hemispherical space from the unit area per unit time, can be represented by the area enclosed by the radiation energy distribution curve under different temperatures with the coordinate axes shown in Fig. 4, Stefan also used this theory to reach the first reasonable approximation of the temperature of the solar surface as 5700K [19].

4. Conclusion

On the one side, this paper deals with black-body radiation and associated laws, and as such explains the process of deriving the black body radiation, which includes the derivation of Wien's displacement law and Stefan-Boltzmann law which are derived from Planck's formula. It also discusses applications in cosmology like cosmic background radiation. Following a systematic approach, the authors introduce the derivation of black body formula whereby the average energy and the number of concrete particles which vibrate with a particular frequency in a cavity are determined thereby yielding a formula for black-body radiation. On the other hand, it will be very good to mention that this research is very censurable in its sense that it enables readers to be aware of the black body radiation formula and its significance. Thus, it serves as a stepping stone for the teaching of new physicists and also is quite beneficial for experts from other disciplines. In this case, however, the paper is also susceptible to modifications. For example, future studies may address theory improvement on the application of black-body radiation in a wider scope or coverage. In parallel, modification can be continuously sought in the scope of study in order to enhance the quality as well as reliability of the study carried out.

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