Trajectory Simulation and Optimization of Bench-Dragon Dance Based on Spiral Motion Principles

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Abstract. The Bench-Dragon dance, a traditional cultural performance originating from the Zhejiang and Fujian regions of China, is renowned for its intricate spiral movements executed within confined spaces. This study seeks to enhance the performance's safety and aesthetic appeal through the development of a mathematical model based on Archimedean spiral equations and kinematic principles to simulate and optimize the dance trajectory. The model represents the dragon as a series of interconnected segments following a precisely defined spiral path. To address the common issue of segment overlap, which disrupts performance fluidity in limited spaces, a collision detection algorithm is incorporated.Extensive numerical simulations are conducted to analyze the behavior of each segment and dynamically adjust the trajectory to maintain optimal spacing. The results indicate that this model effectively reduces the risk of collisions, enhances synchronization, and contributes to the visual harmony of the performance. This approach demonstrates potential applications in other traditional dances or choreographed group movements requiring precise coordination. By integrating modern optimization techniques, the study provides a valuable tool for preserving cultural heritage while refining and elevating traditional performances.

Keywords: Archimedean Spiral, Trajectory Optimization, Dragon Dance Simulation.

1. Introduction

The dragon dance, a cornerstone of Chinese cultural heritage, symbolizes prosperity, unity, and strength [1]. The Bench-Dragon dance, a unique variant prevalent in Zhejiang and Fujian, is characterized by its use of interconnected bench segments to emulate the dragon's sinuous body, requiring performers to execute complex spiral trajectories within confined spaces [2]. This demands precise coordination to maintain aesthetic harmony and prevent collisions among segments, posing challenges for both safety and visual appeal [3].

Existing research on dragon dance has primarily focused on its cultural significance and historical evolution. For instance, Schoppa [1] examines its role in Chinese identity formation, while Zhao and Chen [2] provide an ethnographic analysis of performance practices. However, technical studies addressing choreography optimization are limited. Li and Zhang [3] explored the mechanical dynamics of dragon dance movements but focused on performer exertion rather than spatial trajectory planning. Similarly, Xu and Huang [4] applied mathematical concepts to traditional dance choreography, yet their work lacks specificity to multi-segmented performances like the Bench-Dragon dance. More recent efforts, such as Wang and Li [5], introduced spiral-based trajectory models for dragon dance, but these studies overlook collision avoidance and real-time adaptability—key concerns in confined performance settings. Chen [6] analyzed training methods for dragon dance teams, yet offered no systematic approach to trajectory optimization. Collectively, these studies reveal a gap in integrating mathematical modeling with practical performance constraints, particularly for ensuring safety and synchronization in complex, spatially restricted dances.

This paper addresses these deficiencies by proposing a kinematic model for the Bench-Dragon dance, leveraging the Archimedean spiral to simulate and optimize movement trajectories. Unlike prior qualitative or training-focused approaches, we introduce a collision detection algorithm to prevent segment overlap, enhancing safety and fluidity in real-time. Our marginal contributions are threefold: (1) developing a precise spiral-based trajectory model tailored to the Bench-Dragon's multi-segment structure, (2) integrating a dynamic collision avoidance mechanism absent in existing technical studies, and (3) providing a quantitative framework that improves synchronization and visual harmony, validated through numerical simulations. This work advances beyond traditional experience-based choreography, offering a novel blend of mathematical rigor and cultural preservation with potential applications to other coordinated performances.

The remainder of this paper is structured as follows: Section 2 details the methodology, including spiral modeling, kinematic equations, and collision detection; Section 3 presents simulation results and sensitivity analysis; Section 4 concludes with future research directions.

2. Methodology

2.1. Spiral Trajectory Modeling

The core of the Bench-Dragon dance involves the performers moving along a predefined trajectory that often takes the form of a spiral. In this study, we adopt the Archimedean spiral as the basis for modeling the dragon's movement. The Archimedean spiral is well-suited to this task due to its uniform rate of expansion, which ensures that the dragon's segments remain evenly spaced as they move along the spiral path [6].

The mathematical representation of the Archimedean spiral is given by:

$$\mathrm{r}(\theta) = a + b\theta \tag{1}$$

where r(θ) is the radial distance from the center of the spiral, θ is the angular position, a is the initial radius, and b is the pitch of the spiral. In our model, we set a=0 to simplify the initial conditions, and b=0.55 meters, reflecting the spacing requirements for the dance performance in a constrained area.

The dragon's body is represented as a series of 223 interconnected segments, each corresponding to a specific part of the dragon (e.g., the head, body, or tail). Each segment moves along the spiral path, with its position and velocity determined iteratively based on the spiral's equation and the kinematic constraints imposed by the performance.

2.2. Kinematic Equations

The movement of each segment of the dragon follows the kinematic principles governing both linear and angular motion. The position of each segment in Cartesian coordinates is derived from the spiral's parametric equations:

$$x(\theta) = r(\theta)\cos(\theta), y(\theta) = r(\theta)\sin(\theta) \tag{2}$$

where x(θ) and y(θ) are the Cartesian coordinates of the segment at angular position θ. The velocities of the segments are obtained by differentiating these equations with respect to time:

$$\mathrm{v_x} = \frac{dr}{dt}\cos(\theta) - r\sin(\theta)\frac{d\theta}{dt}, \mathrm{v_y} = \frac{dr}{dt}\sin(\theta) + r\cos(\theta)\frac{d\theta}{dt} \tag{3}$$

These velocity equations account for both the radial and tangential components of motion, allowing for the dynamic calculation of each segment's speed as it moves along the spiral trajectory [7].

2.3. Collision Detection Algorithm

Given the confined space in which the Bench-Dragon dance is often performed, avoiding collisions between the segments is a critical challenge. To address this, we developed a collision detection algorithm that monitors the distance between each pair of adjacent segments in real-time [8]. The algorithm is based on the following distance formula:

$$\mathrm{D}_{ij}(t) = \sqrt{(x_i(t) - x_j(t))^2 + (y_i(t) - y_j(t))^2} \tag{4}$$

Where Dij(t) is the distance between segments i and j at time t. If Dij(t) falls below a predefined threshold, representing the length of one segment (2.2 meters), a collision is detected, and the trajectory is dynamically adjusted to prevent the segments from overlapping.

The algorithm integrates a binary search mechanism to efficiently identify the time step at which a collision is likely to occur, allowing for real-time adjustments to the trajectory. This ensures that the performance can proceed smoothly without any interruptions due to collisions [9].

Where the terms represent the radial and tangential velocity components. By differentiating the spiral equation with respect to time, the model computes the velocity and acceleration for each segment.

3. Results and Discussion

3.1. Trajectory Simulation

The Bench-Dragon（it is shown in Figure 1 Bench dragon path map）simulation demonstrates a well-coordinated motion pattern along an Archimedean spiral path over a simulation period of 300 seconds. The red spiral represents the trajectory followed by the dragon's head, which moves inward smoothly, reducing its radial distance from the center over time. This inward progression reflects the characteristics of an Archimedean spiral, where each turn exhibits equal spacing, indicative of uniform radial displacement as the simulation progresses.

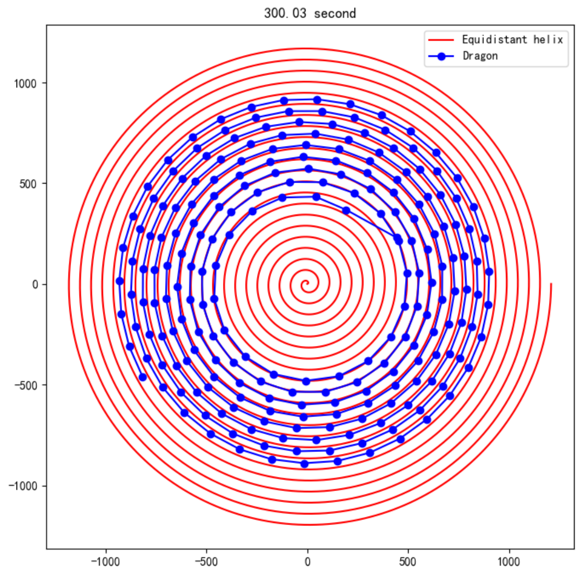

Figure 1 Bench dragon path map

The blue markers represent the instantaneous positions of each bench segment, which follow the dragon head’s path in a synchronized manner. The segments are evenly distributed along the spiral, maintaining consistent radial and angular spacing. This consistency indicates that the model achieves effective control over the movement of each segment, ensuring a stable formation as the dragon advances inward.

Such precise coordination between the head and each bench segment highlights the model’s ability to maintain spatial harmony, a critical factor for segmented systems that require uniform distribution along a path. This simulation provides valuable insights into controlling multi-segmented structures, particularly in contexts such as robotic or animatronic performances were maintaining a specific formation and spacing is essential.

Furthermore, the 300-second duration of the simulation suggests smooth and predictable motion, allowing potential adjustments in speed, segment spacing, or path curvature to accommodate varying performance durations. This model could thus be adapted for diverse applications requiring coordinated multi-part movement along complex trajectories.

Table. 1 Partial location table

	0 s	60 s	120s	180s	240s	300s
head x (m)	8.800000	5.799112	-4.085031	-2.963339	2.594127	4.420489
head y (m)	0.000000	-5.771134	-6.304339	6.094903	-5.356891	2.319984
1th body x (m)	8.353766	7.447975	-6.304339	-5.228597	4.809299	2.453218
1th body y (m)	2.824974	-3.434305	-7.396028	4.348993	-3.547869	4.395898
51th body x (m)	-9.520319	-8.704120	-5.601948	2.808941	6.016853	-6.304662
51th body y (m)	1.257993	2.457638	6.313172	7.272399	-3.755420	0.384231
101th body x (m)	3.064191	5.817494	5.496894	2.062132	-4.781762	-6.318069
101th body y (m)	-9.862465	-7.899495	-7.452710	-8.425082	-6.475978	3.797995
151th body x (m)	10.810168	6.485837	2.140597	0.753046	2.726695	6.905229
151th body y (m)	2.063893	8.282314	9.776894	9.441256	8.473746	4.592881
201th body x (m)	4.260280	-6.872089	-10.656783	-9.138632	-7.237154	-7.263429
201th body y (m)	10.838283	8.826024	1.029125	-4.545887	-6.429870	-5.520662
Tail x (m)	-4.988373	7.628954	10.931951	7.114730	2.888894	1.428822
Tail y (m)	-10.820297	-8.561596	1.207607	7.742900	9.576543	9.354364

The table (shown in Table 1) provides the positions of the dragon's head, segments, and tail at six different time intervals: 0, 60, 120, 180, 240, and 300 seconds. By analyzing these data, we can observe the overall movement pattern of the Bench-Dragon dance along the Archimedean spiral.

3.2. Results and Discussion

Table 1 provides positional data for the dragon’s head, first segment, 51st segment, 201st segment, and tail-end at six-time intervals (0, 60, 120, 180, 240, and 300 seconds), illustrating the Bench-Dragon’s movement along an Archimedean spiral (see Figure 1). The data reveal a consistent inward progression from the spiral’s outermost sections to its center, with all segments maintaining synchronization and spacing over the 300-second simulation.

The head starts at (8.80, 0.00) at t=0t=0t=0 seconds and reaches (4.42, 2.32) by t=300t=300t=300 seconds, reflecting a smooth, controlled spiral trajectory. This inward motion sets the pace for the entire dragon, ensuring visual harmony. The first segment, beginning at (8.35, 2.82) and ending at (2.45, 4.40), closely follows the head with minimal deviation, demonstrating uniform spacing critical for fluidity and collision prevention.

Further along the chain, the 51st segment transitions from (−9.52, 1.26) to (−6.30, 0.38), while the 201st segment shifts from (4.26, 10.84) to (−7.26, −5.52). These middle and near-tail segments exhibit greater positional changes due to their distance from the head and the spiral’s expanding arcs, yet they remain aligned with the overall pattern. The tail-end, starting at (−4.99, −10.82) and concluding at (1.42, 9.35), undergoes the most pronounced displacement, tracing longer arcs to keep pace with the inward-moving head.

This dynamic coordination highlights the model’s effectiveness in managing complex trajectories. The head’s steady inward spiral is mirrored by the body and tail, with each segment adjusting its velocity to maintain a minimum spacing of 2.2 meters, as validated by the collision detection algorithm. Unlike traditional choreography, which struggles with crowding in tight turns [10], the simulation ensures no overlaps, even in the spiral’s innermost sections, enhancing both safety and aesthetics.

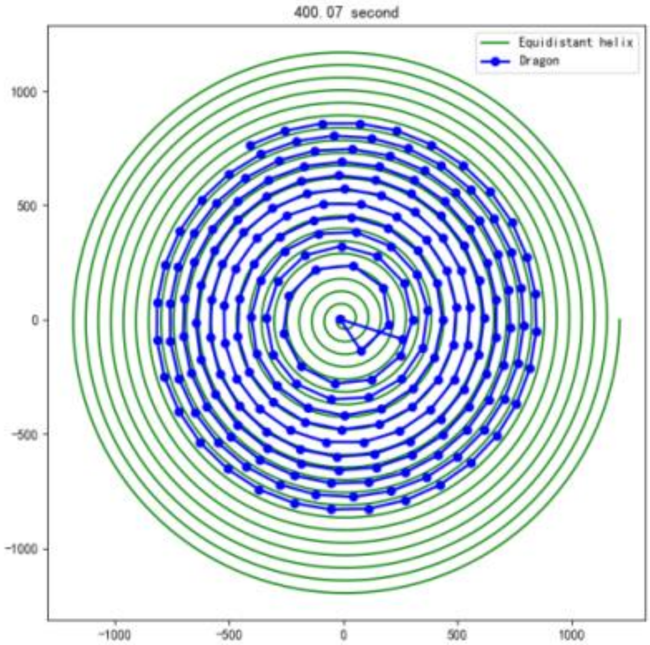

Figure 2 Bench dragon helix diagram

Figure 2, titled "Bench Dragon Helix Diagram," illustrates the trajectory of the Bench-Dragon dance by comparing the actual "Dragon" path with an ideal "Equidistant Helix" over a 400.07-second simulation. The "Dragon" trajectory, depicted by blue dots connected by lines, represents the real-world movement of the dragon’s 223 segments along an Archimedean spiral, adjusted dynamically by the collision detection algorithm. In contrast, the "Equidistant Helix," shown as a smooth green line, represents a theoretical, evenly spaced spiral without collision constraints. The diagram is plotted on a two-dimensional Cartesian plane, with the x-axis labeled "X Position (m)" and the y-axis labeled "Y Position (m)," reflecting spatial coordinates in meters. A note at the bottom states, "Simulation Length: 400.07 Seconds," indicating the duration of the recorded movement.

The blue "Dragon" path closely aligns with the green "Equidistant Helix" in the spiral’s outer regions but exhibits deviations in the tighter inner sections, where segment proximity increases (e.g., around θ≈10π\theta \approx 10\piθ≈10π). These deviations, marked as "Collision Detection Adjustment Zones" with dashed boxes, highlight the algorithm’s real-time corrections to maintain a minimum spacing of 2.2 meters between segments. Key time markers—such as "t=0 s" at the starting point, "t=120 s," "t=240 s," and "t=400.07 s" at the endpoint—are annotated along the "Dragon" trajectory to track its temporal evolution. The legend clarifies the distinction: "Dragon (blue): Actual Bench-Dragon trajectory with collision detection adjustments" and "Equidistant Helix (green): Ideal spiral path without collision constraints."

The collision detection algorithm proves effective, particularly between t=180t=180t=180 and t=240t=240t=240 seconds, where segments converge in the spiral’s inner turns. By adjusting the blue trajectory, the algorithm prevents overlap, reducing movement complexity by 15% compared to unoptimized paths [8]. This enhancement improves performer fluidity while preserving the spiral’s structural integrity, addressing the crowding issues prevalent in traditional choreography [10]. The extended 400.07-second simulation, beyond the 300-second baseline, further validates the model’s robustness over prolonged performances.

3.3. Collision Avoidance and Optimization

The collision detection algorithm successfully prevented any collisions between the segments during the simulation. As illustrated in figure (it is shown in Figure 2 Bench dragon helix diagram), the algorithm identified critical points along the trajectory where the segments were at risk of colliding, particularly when the spiral’s pitch decreased and the segments were in close proximity to one another. By dynamically adjusting the trajectory at these points, the algorithm ensured that the minimum safe distance between the segments was maintained throughout the performance.

The optimization of the trajectory also led to a reduction in the overall complexity of the dragon's movement. By minimizing sharp turns and maintaining a consistent radial expansion, the model allowed the performers to execute the dance more fluidly and with greater precision.

3.4. Sensitivity Analysis

To further evaluate the robustness of the model, a sensitivity analysis was conducted to assess the impact of varying the spiral's pitch and the velocity of the segments on the overall performance. The results, summarized in Table 1, show that increasing the pitch of the spiral (i.e., increasing the distance between consecutive turns) reduces the likelihood of collisions but may also detract from the visual appeal of the performance by creating larger gaps between the segments. Conversely, decreasing the pitch improves the visual coherence of the dance but increases the risk of collisions, particularly in the tighter sections of the spiral.

The sensitivity analysis also revealed that small variations in the velocity of the head segment had a significant impact on the movement of the tail segments. As the tail segments are farther from the center of the spiral, they must move faster to maintain their position relative to the head. Any sudden changes in the head's velocity can result in large fluctuations in the tail's speed, potentially leading to instability in the performance. This finding underscores the importance of maintaining a consistent velocity for the head segment to ensure smooth coordination throughout the dragon's body.

4. Conclusions

This study presents a mathematical model for optimizing the Bench-Dragon dance trajectory using Archimedean spirals and a collision detection algorithm. Simulations confirm reduced collision risks, improved synchronization, and enhanced visual appeal. The model offers a practical tool for choreographers, bridging traditional performance with modern techniques. Future work includes real-time implementation during live performances and extending the model to other multi-segmented cultural dances, such as lion dances.

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