# Research On Spherical Multi-Point Path Optimization Based on Simulated Annealing and Ant Colony Algorithms

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**Abstract.** In practical application scenarios such as logistics distribution and UAV cruise, the spherical multi-point path optimization problem has significant research value and practical significance. This research focuses on solving the spherical multi-point path optimization problem. The study first establishes a spherical distance calculation model, deriving the distance formula between any two points using spherical geometric principles. Taking eight specific points as research objects, the problem is transformed into a graph theory problem, and a single-objective linear programming model is constructed. The study employs Simulated Annealing Algorithm and Ant Colony Algorithm for comparative analysis. Through parameter optimization, the Simulated Annealing Algorithm achieved a shortest path length of 7.0381 kilometers. In comparison, the Ant Colony Algorithm obtained an optimal path (44-61-158-83-147-100-31-115) with a length of 4.0635 kilometers and a total time of 390.2 minutes, showing a 2.2% improvement in efficiency compared to the Simulated Annealing Algorithm. The results demonstrate that the Ant Colony Algorithm exhibits better performance and stability in this spherical multi-point path optimization problem.

**Keywords:** Spherical distance calculation, shortest path planning, Simulated Annealing algorithm, Ant Colony algorithm.

## 1. Introduction

With the acceleration of globalization and the growing demands of logistics transportation, multi-point path optimization in spherical space has become a significant research topic[1, 2]. Traditional planar path planning methods show apparent limitations when applied to actual transportation problems on the Earth's surface, primarily due to distance calculation deviations caused by the Earth's spherical characteristics[3, 4]. Against this background, how to accurately calculate spherical distances and find optimal transportation paths has become a crucial problem that needs to be addressed.

In recent years, with the rapid development of artificial intelligence algorithms, intelligent optimization methods such as Simulated Annealing and Ant Colony algorithms have demonstrated excellent performance in path planning[5-7]. These algorithms possess strong global search capabilities and local optimization abilities, effectively avoiding local optimal solutions[8]. However, how to apply these algorithms to path optimization problems in spherical environments still requires in-depth research and exploration.

This study aims to address the spherical multi-point path optimization problem by establishing a spherical distance calculation model and combining the advantages of Simulated Annealing and Ant Colony algorithms to propose a path optimization method suitable for spherical environments. This has significant theoretical implications and practical value for improving logistics transportation efficiency and reducing transportation costs.

## 2. Spherical Distance Calculation Between Points on Earth's Surface

The data in this paper originates from http://www.mcm.edu.cn. Considering that there are no missing values or anomalies in the longitude and latitude data of the 222 points provided in the dataset, this paper does not perform any data processing.

Given the longitude and latitude of two points on Earth, select two additional points with the same longitude and latitude as these two points respectively. Using the 0-degree meridian as the reference, the following figure is constructed (see Figure 1). The equatorial radius of the Earth is approximately 6378.140 kilometers.

Consider the points $A(\phi_1,\lambda_1)$, $B(\phi_2,\lambda_2)$, $C(\phi_2,\lambda_1)$, $D(\phi_1,\lambda_2)$, $E(0,\lambda_1)$ and $F(0,\lambda_2)$. Firstly, this study notes that:

$$\because \angle AOC = \phi_2 - \phi_1 \ \ and \ \ OA = OC = R \tag{1}$$

Using the chord formula on a sphere, the length AC can be expressed as:

$$AC = 2R\sin(\frac{\angle AOC}{2}) = 2R\sin(\frac{\phi_2 - \phi_1}{2}) \tag{2}$$

By analogy, for points E and F, this paper similarly has:

$$EF = 2R\sin(\frac{\lambda_2 - \lambda_1}{2}) \tag{3}$$

Next, consider the angle at A. This study has:

$$\because \angle AOG = \angle AOG = \phi_1 \tag{4}$$

Since OA = R, it follows that:

$$OG = OA\cdot\cos(\phi_1) = R\cdot\cos(\phi_1) \tag{5}$$

Furthermore, because $\angle GOO' = \angle AGO = 90^\circ$ and OG=O′A, this study obtains:

$$O'A = R\cdot\cos(\phi_1) \tag{6}$$

Herein, because $\angle AO'd = \lambda_2 - \lambda_1$, then this study can get the following equation:

$$AD = 2\cdot O'A\cdot\sin(\frac{\lambda_2 - \lambda_1}{2}) = 2R\cdot\cos(\phi_1)\cdot\sin(\frac{\lambda_2 - \lambda_1}{2}) \tag{7}$$

$$BC = 2\cdot OC\cdot\sin(\frac{\lambda_2 - \lambda_1}{2}) = 2R\cdot\cos(\phi_2)\cdot\sin(\frac{\lambda_2 - \lambda_1}{2}) \tag{8}$$

Then, this paper can convert it into finding the length of a plane trapezoid and use the law of cosines to find the distance between two points on the sphere. See Figure 1.

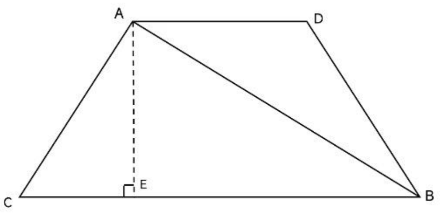


**Figure 1:** ladder diagram

From the geometric properties of a circle, the following relationship holds:

$$\because CE = \frac{CB - AD}{2}, EB = \frac{CB + AD}{2}, AE^2 = AC^2 - CE^2, \tag{9}$$

$$AB^2 = AC^2 - \frac{(CB - AD)^2}{4} + \frac{(CB + AD)^2}{4} = AC^2 + CB\cdot AD \tag{10}$$

$$AB^2 = 4R^2 \cdot \sin^2(\frac{\phi_2 - \phi_1}{2}) + 4R^2 \cdot \cos(\phi_1) \cdot \cos(\phi_2) \cdot \sin^2(\frac{\lambda_2 - \lambda_1}{2}) \tag{11}$$

Finally, find $AB$ in the circle, as shown in figure 2:

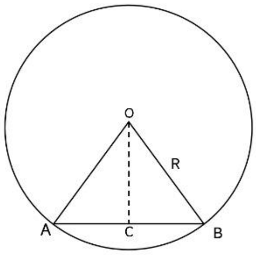


**Figure 2:** Triangle inside a circle

$$\because OA = R, AC = \frac{AB}{2}, \tag{12}$$

$$AC^2 = R^2 \cdot \sin^2(\frac{\phi_2 - \phi_1}{2}) + R^2 \cdot \cos(\phi_1) \cdot \cos(\phi_2) \cdot \sin^2(\frac{\lambda_2 - \lambda_1}{2}) \tag{13}$$

$$\because \sin \angle AOC = \frac{AC}{R} \tag{14}$$

$$\angle AOC = \arcsin(\frac{AC}{R}) \tag{15}$$

$$\because AB = R \cdot \angle AOB = 2R \cdot \angle AOC \tag{16}$$

So, the distance formula between two points on a sphere is:

$$AB = 2R \cdot \arcsin(\sqrt{\sin^2(\frac{\phi_2 - \phi_1}{2}) + \cos(\phi_1) \cdot \cos(\phi_2) \cdot \sin^2(\frac{\lambda_2 - \lambda_1}{2})}) \tag{17}$$

Here, this study aims to solve the distances of eight points coded 31, 44, 61, 83, 100, 115, 147, and 158 as shown in Table 1.

**Table 1:** Spherical distance between any two points of 8 points

| Point | 31 | 44 | 61 | 83 | 100 | 115 | 147 | 158 |
|---|---|---|---|---|---|---|---|---|
| 31 | 0 | 3.4743 | 3.0248 | 1.7152 | 0.3859 | 0.2658 | 0.5376 | 2.5876 |
| 44 | 3.4743 | 0 | 0.4496 | 1.8390 | 3.2136 | 3.7344 | 2.9930 | 1.0459 |
| 61 | 3.0248 | 0.4498 | 0 | 1.3989 | 2.7649 | 3.2852 | 2.5440 | 0.6695 |
| 83 | 1.7152 | 1.8390 | 1.3989 | 0 | 1.4051 | 1.9810 | 1.1940 | 0.8755 |
| 100 | 0.3859 | 3.2136 | 2.7649 | 1.4051 | 0 | 0.6260 | 0.2232 | 2.2801 |
| 115 | 0.2658 | 3.7344 | 3.2852 | 1.9810 | 0.6260 | 0 | 0.7994 | 2.8533 |
| 147 | 0.5376 | 2.9930 | 2.5440 | 1.1940 | 0.2232 | 0.7994 | 0 | 2.0695 |
| 158 | 2.5876 | 1.0459 | 0.6695 | 0.8755 | 2.2801 | 2.8533 | 2.0695 | 0 |

## 3. Path Planning Based on Simulated Annealing and Ant Colony Algorithms

Transform the relationships between each point into a graph theory problem by establishing a directed graph G = (V, A). Here, V = {$V_1$, $V_2$, ..., $V_8$} is the vertex set of graph G, where each element $V_1$, $V_2$, ..., $V_8$ in V is a vertex of the graph. A = {$a_1$, $a_2$, ..., $a_8$} is the arc set of graph G, where each element $a_{ij}$ = ($V_i$, $V_j$) in A represents an arc from vertex $V_i$ to $V_j$.

The purpose of this question is to find the shortest path between points. First, find the sum of the shortest paths of the closed curve. Combined with the straight-line speed of 20 km/h, we get the total shortest time. Finally, add the sum of the working time at each point to get the shortest path on all paths. The following single-objective linear programming model is established:

$$\min z = (3\sum_j \sum_i d_{ij} \cdot x_{ij} + \sum_{i=1}^{8} t_i) \quad i = 1, 2, \cdots, 8 \tag{18}$$

$$s.t. \sum_{\substack{j=1 \\ j \neq i}}^{8} x_{ij} = 1, i = 1, 2, \cdots, 8 \tag{19}$$

$$\therefore \sum_{\substack{i=1 \\ j \neq i}}^{8} x_{ij} = 1, j = 1, 2, \cdots, 8 \tag{20}$$

$$u_i - u_j + n x_{ij} \leq n - 1 \tag{21}$$

$$x_{ij} = \begin{cases} 0 & From\ point\ i\ to\ point\ j \\ 1 & Point\ i\ is\ less\ than\ point\ j, and\ i\ is\ before\ j \end{cases} \tag{22}$$

The Simulated Annealing algorithm for finding the optimal path between 8 points was first proposed in 1953. Its foundation lies in the similarity between general combinatorial optimization problems and the annealing process of solid materials in physics. During the annealing process of solid materials, the substance is first heated to allow its particles to move freely, with the system reaching a thermodynamic equilibrium. As the temperature gradually decreases, the system eventually reaches its lowest thermodynamic equilibrium point.

The Simulated Annealing algorithm requires an initial feasible solution. It reduces temperature according to a certain cooling probability, then constructs neighborhood solutions, compares their total objective function values, and selects the set of feasible solutions closest to the target. After meeting certain iteration criteria, it outputs the results.

The basic operational steps are: First, set the initial temperature and cooling coefficient. Second, randomly generate a neighborhood solution from the current state. Third, calculate the energy difference between the current solution and the neighborhood solution. Fourth, if the energy difference is less than 0, accept the new solution; otherwise, generate an acceptance probability P according to the Metropolis criterion. Fifth, if P is less than 0.5, accept the new solution; otherwise, maintain the current solution. Finally, gradually decrease the temperature and repeat steps 2-5 until the temperature reaches 0 or the optimal solution is achieved[9].

For the convenience of calculation, these 8 points are numbered 1-8 in sequence. See Table 2.

**Table 2:** 8 positioning numbers

| Original Positioning | 31 | 44 | 61 | 83 | 100 | 115 | 147 | 158 |
|---|---|---|---|---|---|---|---|---|
| serial number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

The initial temperature and cooling probability were selected multiple times in the algorithm, and the optimal path time was found by adjusting the parameters. See Table 3.

**Table 3:** Optimal path distance

| Initial temperature | Cooling probability | Shortest path length | Path(from left to right) |
|---|---|---|---|
| 10000 | 0.9999 | 7.0381 | 7 6 1 5 2 3 8 4 |
| 1000 | 0.999 | 8.2936 | 8 2 7 1 6 4 5 3 |

According to 147-115-31-100-44-61-158-83, the shortest path time is minutes. The optimal path of the simulated annealing algorithm is shown in Figure 3.

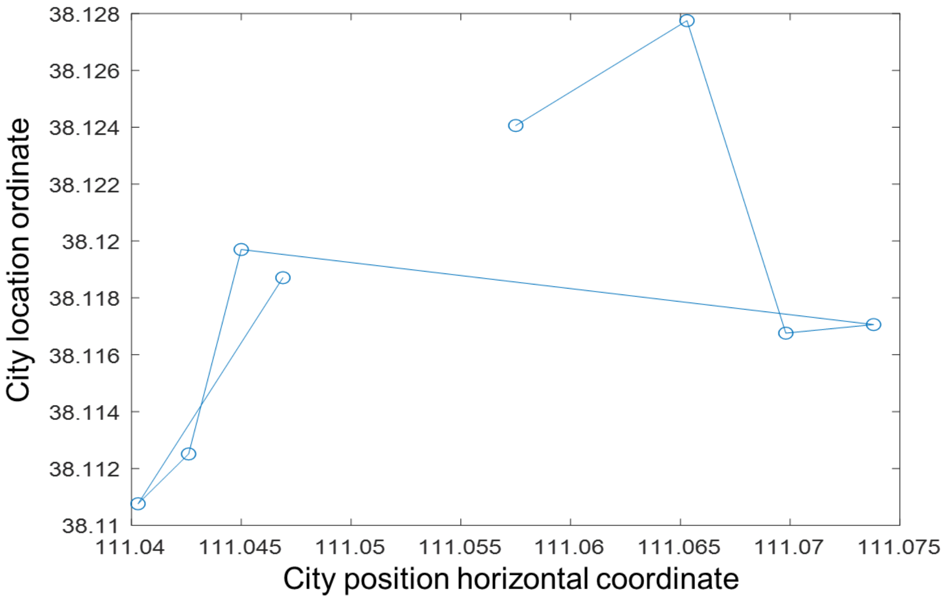


**Figure 3:** Simulated annealing algorithm optimization path

In the figure, two sections of the road are repeated, resulting in a waste of time. The peak time of working at 8 points is 378 minutes, that is, 6 hours and 30 minutes. Therefore, the time on the road should be shortened as much as possible. When the initial temperature is set higher and the cooling probability is greater, the algorithm running time increases and causes repeated routes. This paper uses the ant colony algorithm to solve it.

The Ant Colony Algorithm was first proposed in 1992, inspired by the phenomenon of ants finding the shortest path. When ants search for food, they release pheromones along their paths, and other ants choose their routes based on pheromone concentration. Ants traveling shorter paths release more pheromones, leading to an increasing number of ants choosing these paths. Eventually, through positive feedback, the entire ant colony converges on the optimal path, which corresponds to the optimal solution of the optimization problem.

The specific steps of the Ant Colony Algorithm are as follows[10]: First, initialize parameters including the number of ants, initial temperature, and cooling coefficient. Second, each ant randomly generates a neighborhood state from its current state. Third, calculate the energy values for all ants. Fourth, update the ants' information transmission and pheromone markings based on their energy values. Fifth, each ant selects new neighborhood states based on the updated information transmission and pheromone markings. Finally, gradually decrease the temperature and repeat steps 2-5 until the temperature reaches zero or the optimal solution is achieved. The resulting path diagram obtained through the Ant Colony Algorithm is shown in Figure 4.

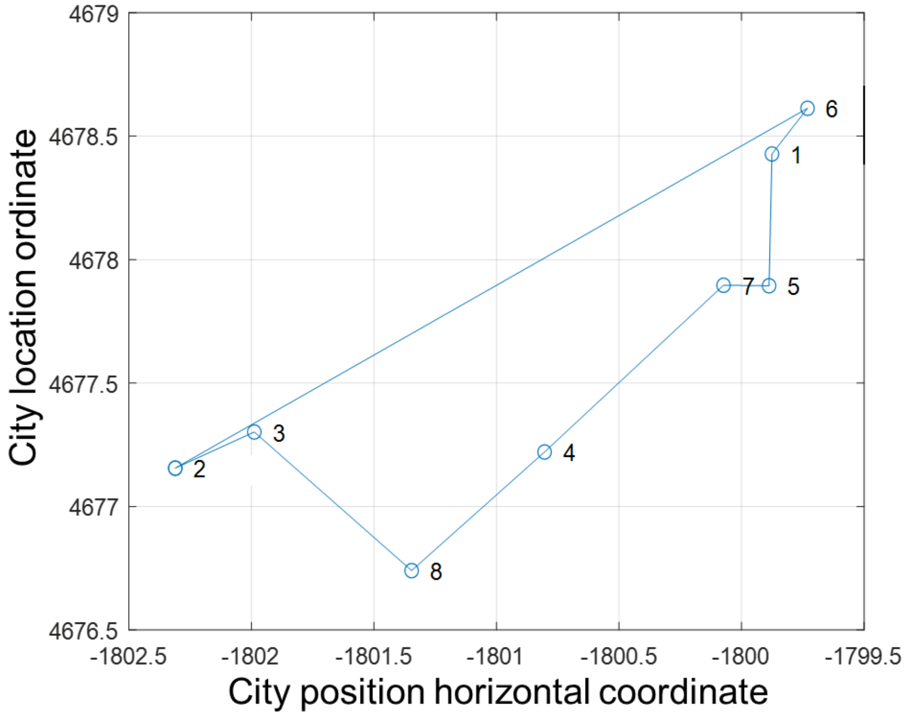


**Figure 4:** Closed Optimization Path

The optimal path selected is: 44-61-158-83-147-100-31-115, the shortest path length is 4.0635 kilometers, and the shortest time is 390.2 minutes, which is 2.2% faster than the simulated annealing algorithm.

## 4. Conclusions

This study has made meaningful progress in spherical path optimization through a comparative analysis of multiple algorithms, successfully developing a comprehensive solution from spherical distance calculation to path optimization. Experimental results demonstrate that the Ant Colony Optimization (ACO) algorithm outperforms the traditional Simulated Annealing (SA) algorithm in this application scenario, not only optimizing the shortest path length to 4.0635 kilometers but also effectively avoiding path repetition issues. However, certain limitations exist in this study. The validation was conducted with only 8 sampling points, representing a relatively limited sample size, and external interference factors such as weather and traffic conditions that might be encountered in practical applications were not fully considered. Additionally, there remains room for improvement in the algorithm's time complexity and computational efficiency.

Looking ahead, the research could be enhanced by expanding the number of sampling points to improve the model's applicability and reliability. The introduction of multi-objective optimization mechanisms would allow for a more comprehensive consideration of multiple dimensions such as time, distance, and cost. Incorporating machine learning methods would help enhance the algorithm's adaptability to complex environments while developing dynamic path planning capabilities that would enable better response to real-time road conditions. These further investigations will contribute to the development of a more sophisticated and practical spherical path optimization system.

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