Dynamic Simulation and Optimization Analysis of The Bench Dragon Performance

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Abstract. This study delves into the Bench Dragon, a highly representative traditional folk dance in China's Zhejiang and Fujian regions, aiming to fill the research gap in the quantitative analysis of its movements. During the research process, we adopted the polar coordinate system and used the Newton-Raphson method to accurately calculate the position of the dragon head. To ensure the safety of the dance performance, this study established a collision detection model based on rotation matrices. This model can effectively predict and prevent potential collisions during the dance, thereby ensuring the personal safety of the performers and the integrity of the dance props. Through the optimization of the equidistant helix pitch under non-collision constraints, this study obtained a series of key parameters, including the movement state of the Bench Dragon during the coiling process, collision-free time, and minimum pitch. The acquisition of these parameters not only provides a scientific basis for the performance planning of the Bench Dragon but also lays a solid foundation for its inheritance and development. These findings offer practical support for the Bench Dragon's performance planning, enhance its cultural heritage, and provide valuable insights and methodological references for the research of similar traditional art forms.

Keywords: Analysis Bench Dragon, Collision Detection, Movement Optimization, Traditional Folk Dance.

1. Introduction

In recent years, Chinese traditional folk activities have garnered increasing interest in simulation and modeling research. For example, Boon Siew Han et al. developed a physical robotic system that simulates the traditional Lion Dance performance. Another study by Sarah Ho et al. examined how the behaviors of rower’s influence paddling speed in Dragon Boat racing [1]. Similarly, Liu Dan and collaborators digitally recreated the traditional Huagu Lamp Dance [2]. This paper focuses on simulating the spiraling path of the “Bench Dragon,” a folk dance from China’s Zhejiang and Fujian provinces. The Bench Dragon, a unique performance deeply rooted in the folk traditions of the Zhejiang and Fujian regions, is a traditional art form where participants connect rows of wooden benches to form a continuous dragon shape. In this performance, dozens to hundreds of participants wear dragon head masks and hold decorated benches, each representing a segment of the dragon's body. These segments are connected in such a way that the whole group appears as a sinuous, dragon-like formation moving in a spiral pattern. As the "dragon" progresses in a spiral line, dancers synchronize their movements to the beat of drums and fireworks, mimicking the dragon’s undulating, twisting, and ascending motions, creating an electrifying atmosphere.

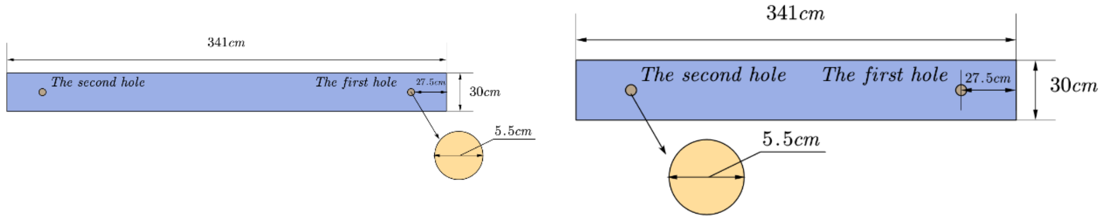
This captivating art form has attracted interest across multiple academic fields, including folklore studies, anthropology, sociology, and aesthetics. Scholars like Liu Wenqi and Mao Ruoyuan have analyzed the cultural role of the Bench Dragon in Anren and Huizhou [3-4], while Min Li has explored the aesthetic value and unique visual appeal of its movements [5]. Yet, quantitative studies focusing on the Bench Dragon’s movement characteristics are scarce. To achieve smooth movement within limited space, collision prevention becomes essential, presenting a dynamic spatial challenge for the performance trajectory. The planning of non-collision paths is critical, particularly when obstacles or spatial restrictions are present. Due to the large scale and complexity of the dance, accurately modeling and simulating the Bench Dragon’s motion and interactions, including movement delays and high-density formations, is challenging. This paper employs mathematical modeling to simulate and optimize the dragon's path, aiming to reduce collision risks and enhance viewing appeal. Other studies have approached similar movement patterns. Nuhu Aliyu Shuaibu et al., for instance, applied a spiral path model to simulate pedestrian flow during the Tawaf ritual in Mecca’s Mataf area, improving crowd control strategies [6]. Another study by Kuanqi Cai et al. used sampling-based path planning in highly congested environments to develop smooth, collision-free pedestrian routes, drawing on social force models and tree search algorithms [7]. Their work provides theoretical support for simulating dense crowd movements, relevant to our study’s Bench Dragon model [8-11].

To accurately capture the Bench Dragon’s dynamic state, this study uses a polar coordinate system and a spiral equation to describe its movement trajectory, calculating the dragon's head position at given time points. Using the equidistant spiral equation, we compute the total arc length through numerical integration, then apply Newton-Raphson to find the head’s polar coordinates. For subsequent points, we approximate distances via bisection. To detect collisions, we translate each bench segment’s vertices into Cartesian coordinates with a rotation matrix and use polygon overlap detection. In optimizing the spiral’s pitch for enhanced visual appeal, we treat pitch as an objective function, applying a bisection method within specified accuracy bounds. This multi-step algorithm supports generating Bench Dragon trajectories, detecting collisions, and optimizing paths based on a spiral pattern. Our approach aids organizers in crafting compelling performance pathways and has further applications in collision detection, congestion analysis, crowd management, and automated navigation, promoting cultural preservation and diversity in traditional performances.

2. Methodology

2.1. Problem Definition

In this paper, the size of the dragon's head, body, and tail needs to be studied in the specified figure 1 below:



(a)Dragon Head (b)Dragon body and dragon tail

Figure 1 Top view of bench dragon parts

The two adjacent benches of the bench dragon are connected by handles, as shown in figure 2 below:

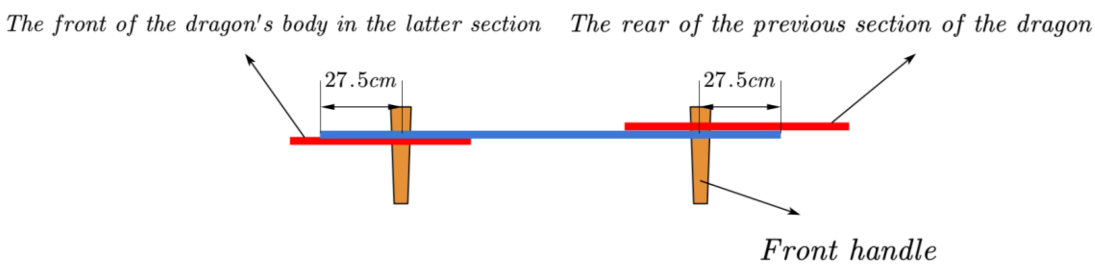


Figure 2 Front view of the bench

2.2. three Research Questions.

This study has three Research Questions:(1) What is the movement state of the coiling process? (2) What is the amount of time it is impossible to continue the disc without a collision? (3) What is the minimum pitch of the spiral that can be turned into a circle with a diameter of nine meters without collision?

Based on the RQs, the main contributions of this paper are:

1.In the polar coordinate system, the Newton Raphson method is used to reverse solve the polar coordinates of the dragon head, combined with the idea of differential approximation, to solve the motion state of the bench dragon

2.A dragon body vertex solving model based on rotation matrix was established, and collision verification was performed using rectangular overlap judgment conditions

3.Using the constraint of no collision and the optimization objective of equidistant spiral, the binary method is used to solve this single objective optimization model and obtain the minimum value of the spiral.

2.3. Calculation and Speed Analysis of Dragon Dance Spiral Movement

This study calculates the mathematical expression of an equidistant spiral under the condition of a pitch of 55cm. To determine the total arc length of the Archimedean spiral, the following formula for curve length calculation is introduced to compute the length L of the Archimedean arc:

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2 d\theta} \tag{1}$$

where α and β are the initial and final polar angles of the curve, respectively.

To calculate the total arc length for the Archimedean spiral $r = a + b\theta$, $d\theta/dr$ need to be found. This is done by substitutingrand $d\theta/dr$ into the curve length formula, resulting in:

$$\begin{cases} r = \dfrac{0.55}{2\pi}\theta \\ L = 442.590256m \end{cases} \tag{2}$$

To maintain a constant forward speed of $1m/s$ for the dragon head's handle, this article proposes two models to calculate the position of the dragon head at different times and to recursively determine the position and speed of the dragon body and tail at 1-second intervals.

2.3.1 Dragon dance head position solving model

Assuming at the kth second, the polar coordinate position of the leading end of the nth section of the dragon's body (with the head being n=0 and the tail being n=223) is denoted as$T_n^k = \left(r_n^k, \theta_n^k\right)$, where$T_0^0 = (r_0^0, \theta_0^0) = (8.8, 32\pi)$.The dragon made of stools moves clockwise towards the center with a constant speed of $v_0^k = 1m/s$ at the leading end of the head (i.e., T_0^k).

The article must first locate the position of the dragon head after it has moved a distance of v_0^k meters along the Archimedean spiral ($r = 0.55\theta/2\pi$) as defined in study. This involves finding a new polar coordinate $T_0^k = \left(r_0^k, \theta_0^k\right)$ such that the curve distance from (r_0^0, θ_0^0) to $\left(r_0^k, \theta_0^k\right)$ is equal to kv_0^kmeters. The formula for the curve distance L is expressed as:

$$L = \int_{\theta_0^k}^{\theta_0^0} \sqrt{b^2 + (b\theta)^2} d\theta = \frac{b}{2}\left[\theta\sqrt{1+\theta^2} + \ln\left(\theta + \sqrt{1+\theta^2}\right)\right] |_{\beta}^{\alpha} \tag{3}$$

To solve for the position of the dragon's head at the kth second, use the curve distance formula. Then, combine this with the straight-line distance formula.

$$d = \sqrt{\left(r_1^k \cos\theta_1^k - r_0^k \cos\theta_0^k\right)^2 + \left(r_1^k \sin\theta_1^k - r_0^k \sin\theta_0^k\right)^2} \tag{4}$$

As shown in figure 3. Iteratively determine the positions of each part of the dragon's body and tail at the kth second based on the head's location.

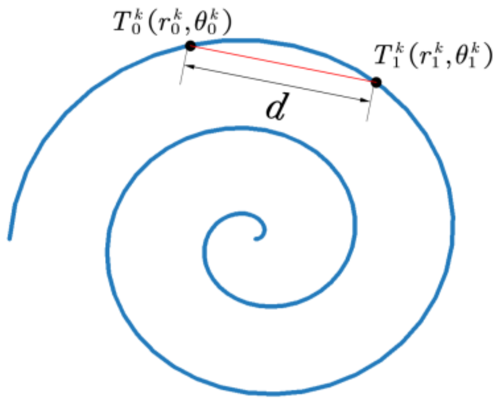


Figure 3 Illustration of the formula for straight-line distance

2.3.2 Dragon dance velocity solving model

Using the binary search method within the time interval $(k-\Delta t, k+\Delta t)]$, calculate the distance traveled by the point, which is expressed as:

$$L = \int_{\theta_n^{k-\Delta t}}^{\theta_n^{k+\Delta t}} \sqrt{b^2 + (b\theta)^2}\, d\theta \tag{5}$$

As the chosen Δt is sufficiently small, the speed v_n^k of the dragon's body can be approximated as the average speed $\overline{v_n^k}$ within the time interval $(k-\Delta t, k+\Delta t)$, with the calculation formula being:

$$\overline{v_n^k} = \frac{v_n^{k-\Delta t} + v_n^{k+\Delta t}}{2\Delta t} \tag{6}$$

Ultimately, the velocity at the head end of the nth section of the dragon's body at the kth second is obtained.

$$v_n^k = \frac{v_n^{k-\Delta t} + v_n^{k+\Delta t}}{2\Delta t} \tag{7}$$

2.4. Collision detection and time analysis of dragon dance hovering motion

To determine the time at which the dragon dance team can no longer continue coiling without the stools colliding (i.e., the termination time of the team's coiling), this article establishes two models: one for solving the vertex of the dragon's body and another for collision testing.

2.4.1 Dragon dance body vertex solution model

Step 1: Determine the center coordinates of the rectangle.

Given the coordinates (x_n^k, y_n^k) of the head end of the nth dragon body at the kth second, and the coordinates (x_{n+1}^k, y_{n+1}^k) of the tail end of the nth dragon body, the midpoint coordinates can be calculated using the midpoint formula as follows:

$$\begin{cases} x_c = \dfrac{x_n^k + x_{n+1}^k}{2} \\ y_c = \dfrac{y_n^k + y_{n+1}^k}{2} \end{cases} \tag{8}$$

Step 2: Use the rotation matrix to calculate the coordinates of the four vertices.

First, connect the head and tail of the dragon body with a directed line segment to obtain a vector:

$$\overrightarrow{T_n^k T_{n+1}^k} = \left(x_{n+1}^k - x_n^k, y_{n+1}^k - y_n^k\right) \quad (9)$$

Then, the angle between the length of the rectangle and the x-axis is denoted as θ:

$$\theta = \arctan \frac{y_{n+1}^k - y_n^k}{x_{n+1}^k - x_n^k} \quad (10)$$

Here, θ is the angle between the rectangle's length and the x-axis, measured counterclockwise from the x-axis to the direction of the rectangle's length. The resulting rotation matrix is as follows:

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad (11)$$

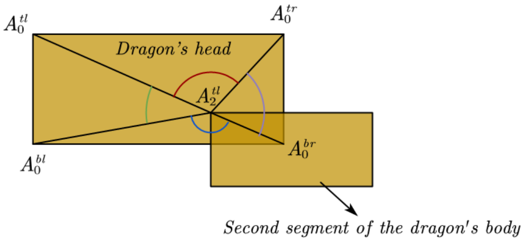
Step 3: Calculate the coordinates of the four vertices.

By multiplying the half-length and half-width of the rectangle by the rotation matrix, you can obtain the offsets of the four vertices relative to the center point. Then, by adding these offsets to the center coordinates of the rectangle, you can find the coordinates of the four vertices: vertex 1 (top right) A_n^{tr}, vertex 2 (bottom right) A_n^{br}, vertex 3 (bottom left) A_n^{bl}, and vertex 4 (top left) A_n^{tl}, with the following expressions:

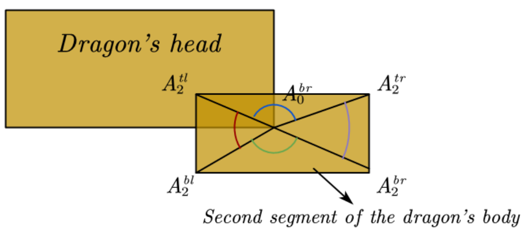
$$\begin{cases} A_n^{tr} = \left(x_n^{tr}, y_n^{tr}\right) = \left(x_c + \frac{l}{2}\cos\theta - \frac{w}{2}\sin\theta, y_c + \frac{l}{2}\sin\theta - \frac{w}{2}\cos\theta\right) \\ A_n^{br} = \left(x_n^{br}, y_n^{br}\right) = \left(x_c + \frac{l}{2}\cos\theta + \frac{w}{2}\sin\theta, y_c + \frac{l}{2}\sin\theta - \frac{w}{2}\cos\theta\right) \\ A_n^{bl} = \left(x_n^{bl}, y_n^{bl}\right) = \left(x_c - \frac{l}{2}\cos\theta + \frac{w}{2}\sin\theta, y_c - \frac{l}{2}\sin\theta - \frac{w}{2}\cos\theta\right) \\ A_n^{tl} = \left(x_n^{tl}, y_n^{tl}\right) = \left(x_c - \frac{l}{2}\cos\theta - \frac{w}{2}\sin\theta, y_c - \frac{l}{2}\sin\theta + \frac{w}{2}\cos\theta\right) \end{cases} \quad (12)$$

2.4.2 Dragon Dance collision detection model

Using the vertex-solving model of the dragon's body, the vertex coordinates of the dragon's head and each body segment can be calculated separately. Taking the following figure as an example, this article will analyze the connection between a specific vertex of the dragon's body (excluding the first body segment) and the four vertices of the dragon's head. Here, we focus on the upper left vertex of the second body segment, as shown in figure 4.



(a)180s



(b)300s

Figure 4 Two types of collisions

If the upper left vertex of the second dragon body does not make contact with the dragon head, meaning point A_2^{tl} is outside the head's rectangle, then the formed angle Arg will be less than $360°$ degrees; conversely, if the upper left vertex of the second dragon body collides with the dragon head, meaning point A_2^{tl} is inside or on the edge of the head's rectangle, then the formed angle Arg will be $360°$ degrees, as follows:

$$\begin{cases} Arg < 360°, A_2^{tl} \text{ does not collide with the dragon head} \\ Arg = 360°, A_2^{tl} \text{ collides with the dragon head} \end{cases} \quad (13)$$

To assess if the second dragon body collides with the dragon head, each vertex of the second body, A_2^{tr},A_2^{br},A_2^{bl},A_2^{tl} must be examined. Perform the previously mentioned connection analysis to calculate the corresponding angle Arg values. If all these angle Ag values are less than $360°$, it indicates no collision between the second body and the head. If any angle Ag reaches $360°$, a collision is present. For the rest of the dragon bodies, excluding the first, repeat this process. If all angle Arg values are less than $360°$, there's no collision at that moment. If any angle Arg reaches $360°$, a collision exists at that time.

$$\begin{cases} \bigcup_{j=2}^{222} \bigcup_{i}^{tr,bt,bl,tl} Arg_j^i = \{Arg|Arg < 360°\}, No\ collision\ occurs\ at\ time\ k \\ \bigcup_{j=2}^{222} \bigcup_{i}^{tr,bt,bl,tl} Arg_j^i = \{Arg|Arg = 360°\}, Collision\ occurs\ at\ time\ k \end{cases} \tag{14}$$

Similarly, repeat the process by switching the dragon head and body. Let Brg represent the sum of the angles from each pair of adjacent vertices on the dragon head to a vertex on the dragon body (as shown in case b in the figure). The formula is expressed as:

$$\begin{cases} \bigcup_{j=2}^{222} \bigcup_{i}^{tr,bt,bl,tl} Brg_j^i = \{Brg|Brg < 360°\}, No\ collision\ occurs\ at\ time\ k \\ \bigcup_{j=2}^{222} \bigcup_{i}^{tr,bt,bl,tl} Brg_j^i = \{Brg|Brg = 360°\}, Collision\ occurs\ at\ time\ k \end{cases} \tag{15}$$

The condition for the dragon's head and body not colliding is as follows:

$$\begin{cases} \bigcup_{j=2}^{222} \bigcup_{i}^{tr,bt,bl,tl} Arg_j^i = \{Arg|Arg < 360°\} \\ \bigcup_{j=2}^{222} \bigcup_{i}^{tr,bt,bl,tl} Brg_j^i = \{Brg|Brg < 360°\} \end{cases} \tag{16}$$

2.5. Minimum pitch optimization of dragon dance circling motion

The diameter of the turning space is set to 9m, so the radius R_d of the turning space in this study is 4.5m, as shown in figure 5.

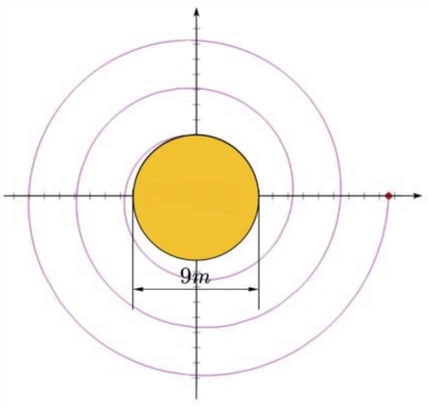


Figure 5 Illustration of the two-dimensional coordinate system for the turning space

To allow the dragon head's handle to coil into the boundary of the turning space along the corresponding spiral, the pitch of the equiangular spiral curve must be minimized. Thus, we define the optimal strategy as the decision plan that results in the smallest possible pitch of the equiangular spiral curve during the dragon dance team's coiling process, and we establish an optimization model for the minimum pitch:

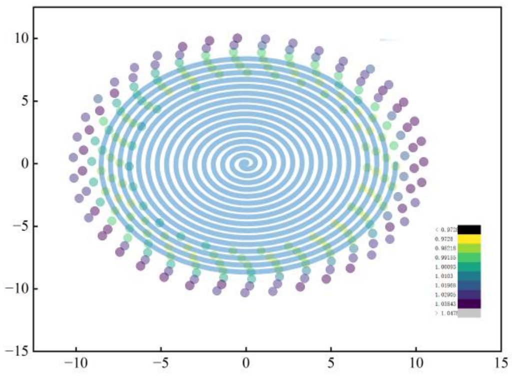
$$\mathbf{min}2\pi b(\mathrm{R_d}, r_n^k, \theta_n^k, Arg_i, Brg_i)$$

$$\begin{cases} r_n^k = b\theta_n^k \\ r_0^k \leq R_d \\ \bigcup_{j=2}^{222} \bigcup_{i}^{tr,bt,bl,tl} Arg_j^i = \{Arg | Arg < 360°\} \\ \bigcup_{j=2}^{222} \bigcup_{i}^{tr,bt,bl,tl} Brg_j^i = \{Brg | Brg < 360°\} \end{cases} \tag{17}$$

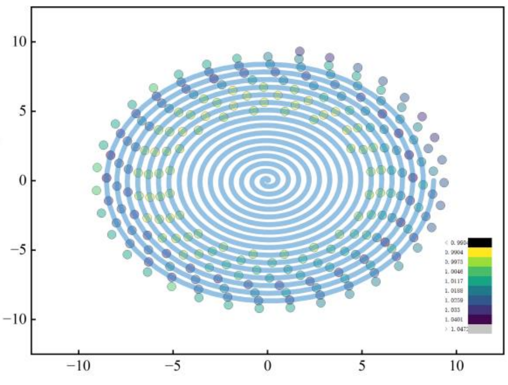
3. Results and discussions

3.1. The results of trajectory calculation and velocity analysis of circling motion of dragon dance

After obtaining the position distribution of the handle on the Archimedean spiral using the bisection method to solve the recursive model of the dragon's body position, the velocity of the leading end of the nth section of the dragon's body at the kth second is derived using the velocity calculation formula. The distribution map of the handle positions combined with velocity is is shown in figure 6.



(a)180s

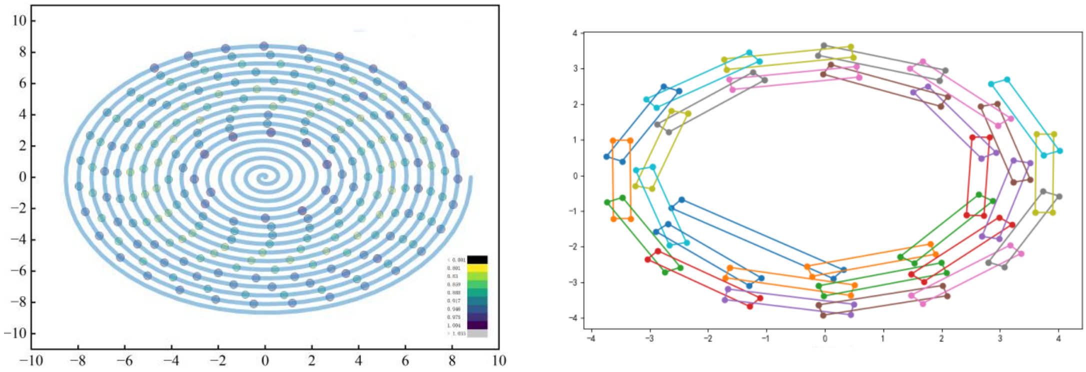


(b)300s

Figure 6 "Bench Dragon" Handle Position and Velocity Distribution

3.2. Calculation result of collision time of dragon dance spiral motion

The result of collision time of dragon dance spiral motion is shown in figure 7.



(a) Position and speed (b) Collision demonstration

Figure 7 The dragon dance team's movement status at 406.43 seconds

3.3. Optimization results of minimum pitch of dragon dance circling motion

Given that both the dragon head and body have a width of 30cm, a pitch less than 0.3m will inevitably lead to collisions between adjacent body sections on the spiral. Based on the calculations from the second question, when the pitch is 0.55m, after the first collision, the dragon head is 2.883m from the center, already within the turning space. This indicates that a pitch of 0.55m is significantly larger than the minimum required to meet the problem's criteria. Therefore, this study sets the search range within $(0.30,0.55)$ to find the minimum pitch, $2\pi b$, that meets the conditions.

This study analyzes different scenarios where the pitch $2\pi b$ falls within the interval $(0.30,0.55)$, based on the model from the second question. It aims to explore the relationship between the distance r_b from the origin to the dragon head at the first collision and the pitch $2\pi b$.

Traverse the solution space. The step size for the pitch $2\pi b$ is specifically controlled as $\Delta(2\pi b) = 0.01$. The pitch values at different step sizes are still solved based on the collision model from the second question. Then, we obtain the figure 8.

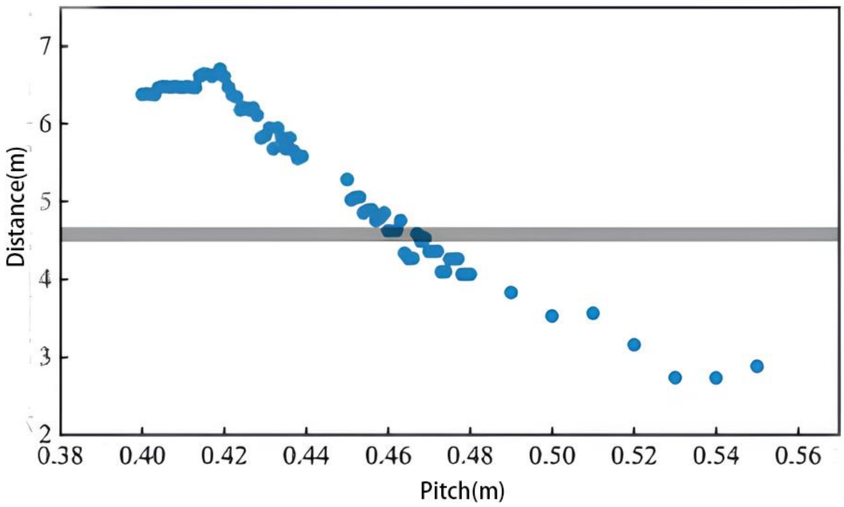


Figure 8 Scatter distribution plot

Observing the scatter distribution in the figure above, it can be seen that as the pitch increases, the distance between the dragon head and the center at the first collision continuously decreases, showing a negative correlation. When the distance between the dragon head and the center at the first collision is 4.5m, the pitch is within the range of $[0.44,0.48]$m.

Given the negative correlation between pitch $2\pi b$ and the initial collision distance r_b of the dragon head revealed earlier, this study employs the binary search method for solution. By iteratively adjusting the pitch, r_b is gradually reduced towards the turning space radius R_d, i.e., $r_b \to R_d^-$, until the error converges to the order of 10^{-10}, continually refining to obtain the minimum pitch as $0.474924\mathrm{m}$.

4. Conclusions

This study focused on the Bench Dragon, a Chinese folk dance. Using math modeling and algorithms, it achieved multiple results. In the polar coordinate system, the Newton - Raphson method with differential approximation determined the motion state precisely. A rotation matrix-based model detected collisions accurately. And an optimization model found the minimum pitch for circling. Overall, it enhanced understanding and offered practical help for the dance's performance and inheritance, while suggesting future exploration in other folk activities.

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