# Helical Path Simulation and Optimal Turning Design for The Bench Dragon Performance

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**Abstract.** The "Bench Dragon," a traditional folk cultural activity originating from Zhejiang and Fujian, involves participants linking benches to form a dragon shape, symbolizing unity and cooperation. This paper explores the mathematical essence of the Bench Dragon, focusing on its movement trajectory, velocity variations, and collision detection. By establishing a polar coordinate model, the paper simulates the movement of the Bench Dragon along an Archimedean spiral and calculates the positions and velocities of the benches under constant speed conditions. A convex hull algorithm detects collisions, and a variable step-size method precisely identifies the moments of collision. Additionally, the paper analyzes the optimal turning path for the dragon head bench, aiming to minimize the turning curve while maintaining consistent tangency. The results offer new insights into the mathematical modeling of traditional cultural activities and provide a new methodological approach to the preservation and transmission of traditional culture by combining cultural expression with scientific analysis.

**Keywords:** Bench Dragon, Collision Detection, Traditional Culture, Optimization Analysis.

## 1. Introduction

The “Bench Dragon,” also known as “Panlong,” is a traditional folk cultural activity originating from the Zhejiang and Fujian regions of China. In this activity, dozens or even hundreds of benches are joined end-to-end, creating a winding “dragon” shape. This cultural expression holds significant meaning as it symbolizes unity, cooperation, and the traditional values of the community [1-3]. The Bench Dragon is performed in various styles, among which is the Duodeng Dragon, also known in some regions as the Long Bench Dragon, which is the most widespread. The number of participants is not fixed. Each participant holds a bench, with the first person representing the dragon’s head and the last person the tail, while the rest form the body of the dragon. As a traditional custom with a wide audience, previous research has primarily focused on transmitting and preserving the Bench Dragon as an intangible cultural heritage [4-7]. However, there is a noticeable lack of studies exploring its mathematical essence.

Similar to the research in this paper, Nuhu Aliyu Shuaibu et al. studied the crowd distribution of Muslims during the Tawaf ritual at the Kaaba. They discussed the parameterization of the spiral and compared two different proposed models of the Tawaf ritual. However, they represented each person as a point and used density to indicate crowding levels, which does not accurately reflect the collisions of entities with actual volume, making it difficult to determine collision times in specific situations precisely [8]. K Shimura et al. used cellular automata to detect collisions in crowds. However, this discretization approach can also affect the accuracy of collision detection [9]. In this study, the Bench Dragon follows a spiral pattern, moving from the outside inwards, similar to the movement of Muslims during the Tawaf ritual. However, unlike previous studies, this research employs modeling to obtain actual dimensions such as the length and width of the Bench Dragon. Through numerical simulations, the study accurately captures the movement of the Bench Dragon, allowing for precise determination of collision locations and times under different scenarios. By conducting a mathematical analysis of the movements involved in this traditional cultural activity, the study provides performers with easily followable paths and explores the critical importance of different trajectory planning in accident prevention. This paper fills the gap in mathematical research related to Bench Dragon performances and offers a new perspective and methodological foundation for its preservation and transmission as an intangible cultural heritage. It fosters the integration of cultural expression forms with scientific rigor.

In this paper, the main innovations are outlined as follows:

**Innovation 1**: As the Bench Dragon progresses along a helical line 101010, we calculate the relative positions of two adjacent benches on the spiral, as well as the movement paths of various points on the benches. This analysis precisely determines the moment of collision between any two benches.

**Innovation 2**: A circular turning space is provided at the center of the spiral line, where the dragon dance team forms an S-shaped curve by connecting two arcs in the turning path. The goal is to explore whether these arcs can be adjusted to shorten the turning curve while maintaining consistent tangency.

To achieve Innovation 1, a polar coordinate model is established to examine the Bench Dragon's movement along an Archimedean spiral. Iterative calculations are performed to determine the positions and velocities of the handles on the head and body segments under constant speed conditions. A geometric method is used to ascertain the vertex positions of the benches, and a convex hull algorithm is applied to detect potential collisions. A variable step-size method is utilized to accurately pinpoint the moment of collision.

For Innovation 2, this study derives the optimal turning path of the Bench Dragon to identify the shortest possible turning curve. By analyzing the geometric properties of the Archimedean spiral, the study demonstrates that the turning curve is minimized when the tangents of the two arcs are parallel to the spiral. Combining collision detection and the variable step-size method, the radius of the small circle is adjusted to ultimately determine the optimal radius of curvature for the turning arc under collision constraints.

## 2. Methodology

### 2.1. Establishment of the Bench Dragon Polar Coordinate Model

When calculating the motion of the "Bench Dragon," since no data related to thickness is involved, the problem can be simplified to a planar problem. For convenience in calculation and analysis, this paper establishes a Cartesian coordinate system with the center of the spiral as the origin, and takes the line connecting the center of the spiral to the starting point A as the positive direction of the x-axis.

Next, in this Cartesian coordinate system, the equation of the spiral is determined. The spiral is an Archimedean spiral, which is a trajectory generated by a point moving away from a fixed point at a constant speed while rotating around the fixed point with a constant angular velocity. Its polar coordinate equation is expressed as follows:

$$r = a + b\theta \tag{1}$$

In this equation, r is the radius of the spiral (the radial distance in polar coordinates), θ is the polar angle (measured in radians), and a is a constant that determines the initial radius and the spacing of the spiral. In this problem, a=0， $b = \frac{dr}{d\theta} = \frac{0.55}{2\pi}$

Substituting the given data into the equation, the specific equation of the spiral is obtained as:

$$r = \frac{0.55}{2\pi}\theta \tag{2}$$

The exact shape of the spiral in this problem can be shown in Figure 1:

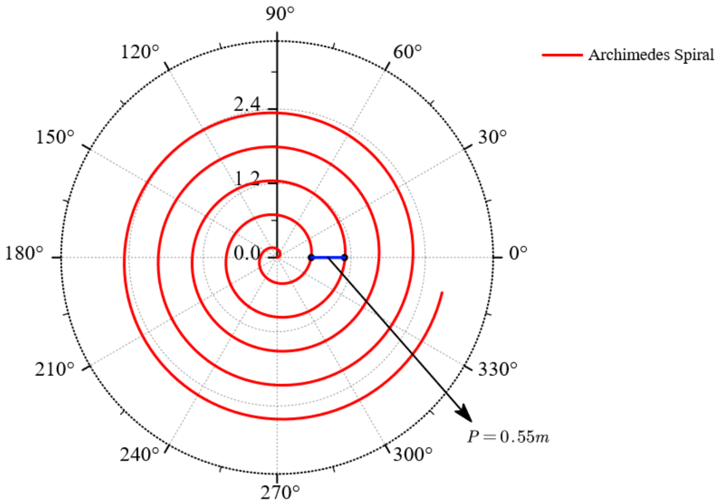


**Figure 1:** Schematic of the Spiral Curve

Next, perform a coordinate transformation for the points on the spiral. According to the polar-to-Cartesian coordinate conversion formulas:

$$\begin{cases} x_i(t)=r_i(t)\cos(\theta_i(t)) \\ y_i(t)=r_i(t)\sin(\theta_i(t)) \end{cases} \tag{3}$$

The polar coordinates of the handle's location $(r_i(t),\theta_i(t))$ will be converted to the corresponding position $(x_i(t),y_i(t))$ in the Cartesian coordinate system.

Next, establish the corresponding recursive model. Based on the geometric relationship between the front and rear handles, the corresponding recursive formula can be derived. The geometric relationship between the front and rear handles is shown in Figure 2:

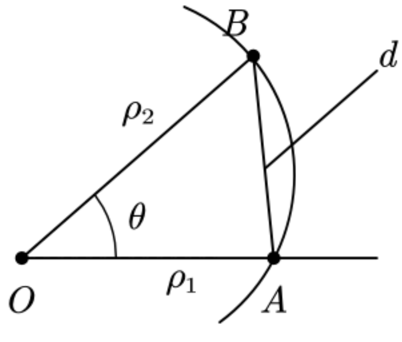


**Figure 2:** Schematic of the Geometric Relationship Between the Front and Rear Handles

By combining the law of cosines with the radius relationship, the equation is as follows:

$$\begin{cases} r_i^2+{r_{i+1}}^2-d^2=2r_ir_{i+1}\cdot\cos\Delta\theta \\ r_{i+1}=r_i+\Delta\theta\cdot\dfrac{0.55}{2\pi} \end{cases} \tag{4}$$

Here, d represents the constant distance between the two handles on the bench. When the bench is at the dragon's head, d=2.86 m, and when the bench is at the dragon's body and tail, d=1.65 m. By using iterative calculations, the coordinates of all the handles on the spiral curve can be determined.

Finally, the motion of a point on the spiral is calculated. The velocity direction can be decomposed into two components: radial velocity and tangential velocity. Based on the geometric properties of the spiral, the magnitude and direction of the velocity are obtained using the time-step integration method.

## 2.2. Collision Detection in the Motion of the Bench Dragon

In this section, an analysis is conducted on the potential collisions between benches during the movement of the Bench Dragon. Since the distance between the benches decreases as they move deeper into the spiral, collisions become more likely as the dragon approaches the inner circles. Simulations have led to the conclusion that the innermost two circles of benches collide first. Next, geometric methods are used to determine the positional conditions of the four corner points of the benches, and the convex hull algorithm is employed to assess whether a collision occurs between the benches.

First, since the benches are rectangular, the position information of the four vertices of the rectangle can be determined based on two adjacent points on the spiral. Then, the convex hull algorithm is applied to address the collision detection problem. The convex hull algorithm is widely used in computational geometry and can effectively determine whether polygons of any shape intersect. For two freely rotating rectangles, the convex hull algorithm can be used to check whether they collide. A convex hull refers to the smallest convex polygon that encloses a given set of points, where any line segment between two points in the set lies entirely within the polygon. For a rectangle, its convex hull is simply the quadrilateral formed by its four vertices.

When solving the problem of intersecting freely rotating rectangles, the first step is to obtain the coordinates of the vertices of each rectangle. Let the four vertices of rectangle 1 be P1, P2, P3, P4, and the four vertices of rectangle 2 be Q1, Q2, Q3, Q4. These vertices are treated as a set of points, and the convex hull algorithm is applied to solve the problem. Common convex hull algorithms include Graham's Scan and the Jarvis March. Graham's Scan has a time complexity of $O(N \cdot logN)$, making it suitable for cases with a large number of points, while Jarvis March has a time complexity of $O(N \cdot H)$, where h is the number of vertices of the convex hull, and is more suitable for scenarios with fewer vertices.

Using these algorithms, the convex hull formed by the vertices of the rectangles can be obtained, which provides a basis for subsequent intersection detection.

After obtaining the convex hull of the rectangles, intersection detection can proceed. First, the edges of the two convex hulls are checked to see if any of them intersect. If any two edges intersect, the two rectangles are determined to be intersecting. If the edges do not intersect, the next step is to check whether a vertex of one convex hull lies inside the other convex hull. This is known as vertex containment detection. If any vertex lies inside the other convex hull, the two rectangles are considered to intersect.

The method used in this study is Graham's Scan. The specific detection process is as follows:

**Step 1**: Select the point with the smallest y-coordinate from the set of points as the starting point. If there are multiple points with the same y-coordinate, select the point with the smallest x-coordinate as the starting point.

**Step 2**: Using the starting point as the reference, calculate the polar angle of each other point relative to the starting point. Then, sort these points in ascending order of their polar angles.

**Step 3**: Using a stack structure, traverse the sorted points in order. According to the counterclockwise rule, gradually add the points to the stack. If a point does not form a convex hull with the top point of the stack, the top point of the stack is popped.

**Step 4**: Repeat the process of adding and removing points until all points are traversed. At this point, the points in the stack represent the vertices of the convex hull.

## 2.3. Establishment of the Turning Model

We assume the existence of an outward-spiraling helix, which is centrally symmetric to the inward-spiraling helix described in Section 2.1. To enable the directional change of the Bench Dragon, a turnaround region is established in the central area of the helix. Through this turnaround region, the

Bench Dragon exits the inward-spiraling helix and simultaneously enters the outward-spiraling helix, thereby achieving a reversal of direction.

For ease of intuitive understanding while aligning with practical scenarios, we assume that the turnaround curve within the turnaround region consists of two tangent circular arcs. The radius of the larger circle is twice that of the smaller circle. Furthermore, to ensure a smooth transition during the turnaround process, the larger circle is tangent to the inward-spiraling curve, while the smaller circle is tangent to the outward-spiraling curve.

It can be proven that when the angular difference between two tangent points on two spirals is 180 degrees, the tangents to the spirals at these points are parallel to each other.

Based on geometric relationships, when the positions of two points of tangency on the spiral are determined, the positions and radii of the two circular arcs can be uniquely defined. The process of determining the U-turn curve is shown in Figure 3:

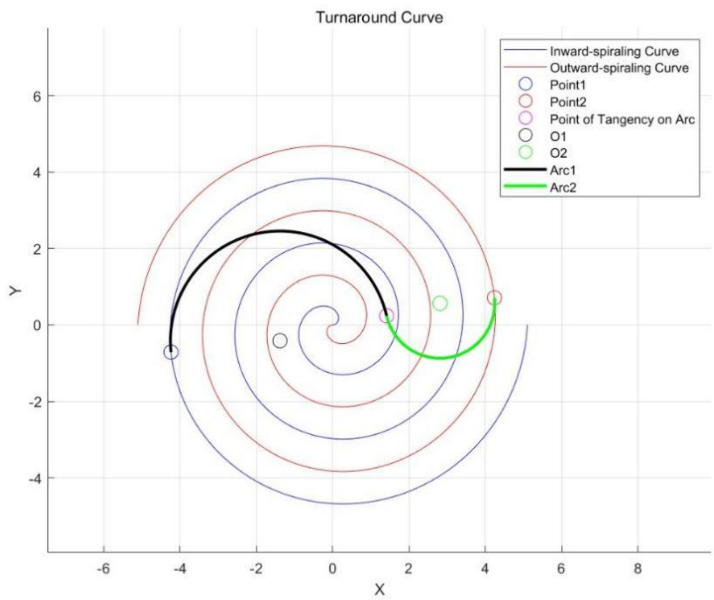


**Figure 3:** Schematic Diagram of the U-Turn Curve

First, two symmetric points located on the center-symmetric spiral are selected as the entry and exit points of the U-turn space. A line segment connecting these two points is drawn, as indicated by the dashed line in the figure. A trisection point closer to the center of the smaller circle is determined along this line segment. Next, tangents are drawn at the two symmetric points, followed by perpendiculars to these tangents passing through the symmetric points. The perpendicular lines are not explicitly marked in the diagram. Subsequently, perpendicular bisectors of the two-line segments, divided by the trisection point, are constructed. According to the properties of perpendicular bisectors, the intersections of the perpendicular bisectors with the perpendiculars to the tangents determine the centers of the two circles. Finally, circular arcs are drawn based on these centers and connected to form the resultant U-turn curve, which is represented by the green and black lines in the figure.

Based on the aforementioned calculations, for any given radius r $r$ of the U-turn curve, the positions of all handles at any given moment can be determined. For visualization, the collection of turnaround curves is shown in Figure 4:

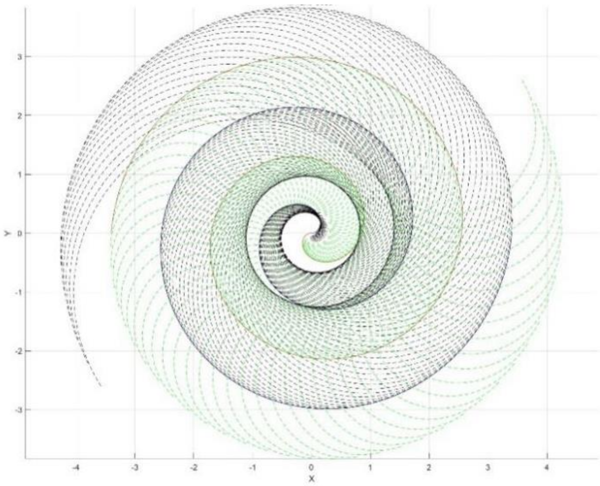


**Figure 4:** Set Diagram of U-Turn Curves

## 3. Results and Discussion

### 3.1. Results of the Polar Coordinate Model

When the dragon dance team coils inward in a clockwise direction along an equidistant spiral with a pitch of 55 cm, the motion states of the bench dragon at 0s, 60s, 120s, 180s, 240s, and 300s are shown in Figure 5-11.

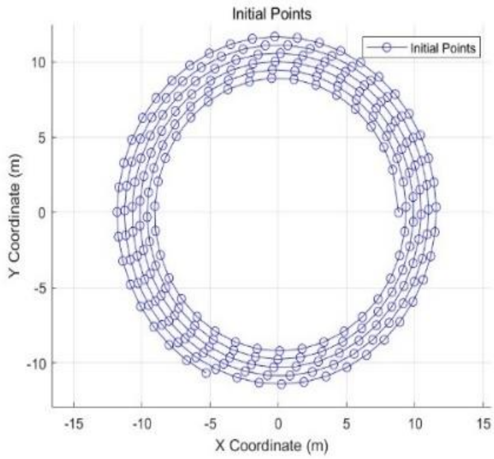


**Figure 5:** Bench position at the start

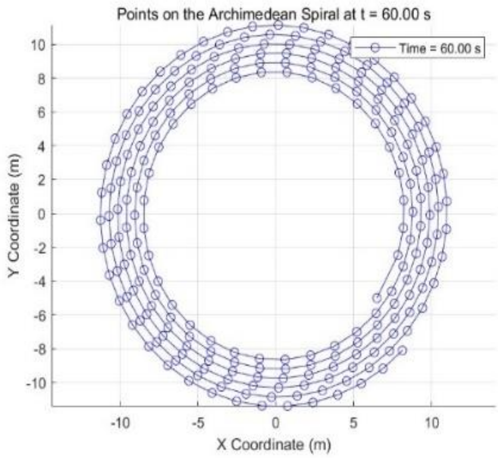


**Figure 6:** Bench position at 60s

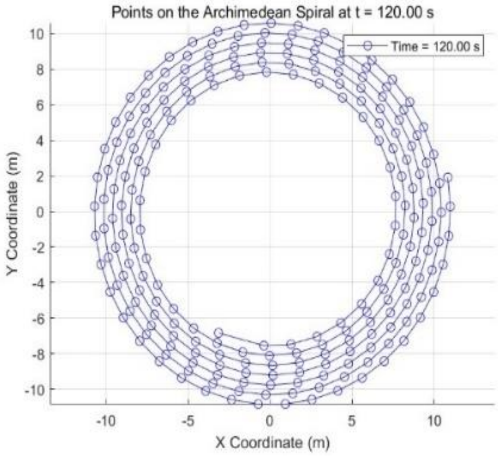


**Figure 7:** Bench position at 120s

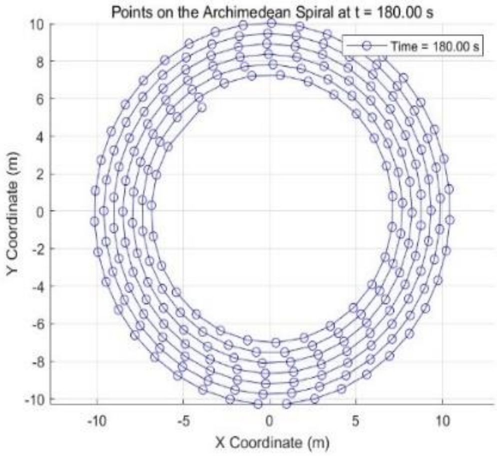


**Figure 8:** Bench position at 180s

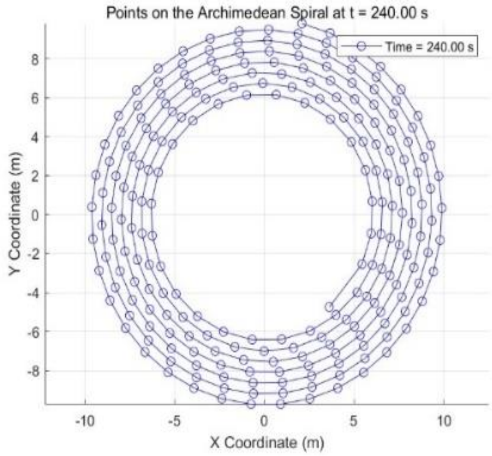


**Figure 9:** Bench position at 240s

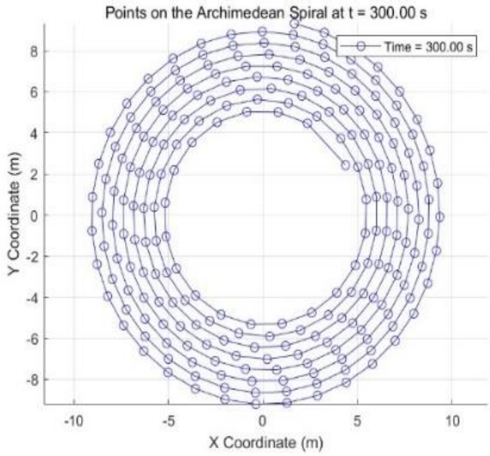


**Figure 10:** Bench position at 300s

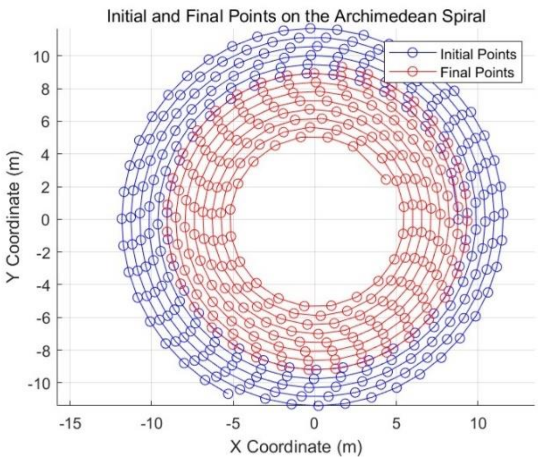


**Figure 11:** Comparison of bench positions before and after

### 3.2. Analysis of the Collision Model Results

In the collision detection process, due to the large computational load, the step size adaptation method was adopted to reduce the time complexity and simultaneously improve the calculation accuracy. By employing the step size adaptation method for the precise calculation of collisions between two freely rotating rectangles, this study successfully determined the specific time of collision. The initial computation utilized a step size of 1 second to traverse the entire time range. Upon detecting a collision, the step size was gradually reduced to 0.1 seconds, ultimately improving the calculation accuracy to meet the required precision. Based on this approach, the collision time obtained in this study was **412.480s**. Figures 12 and 13 show the case when t is 412.5s and 412.48s, respectively:

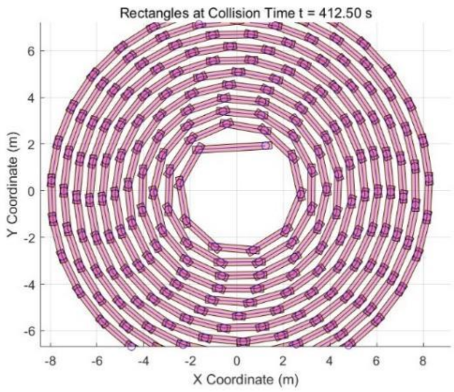


**Figure 12:** Results at 412.5s

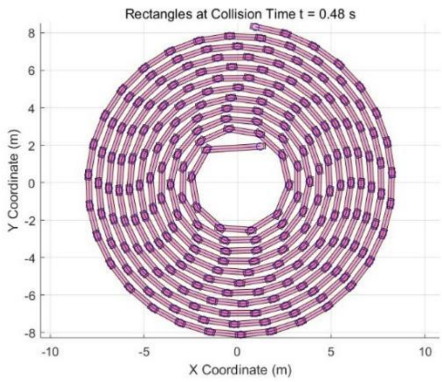


**Figure 13:** Results at 412.48s

### 3.3. Results of the Turning Model

Finally, after simulation analysis, the optimal U-turn curve was obtained. At the same time, the position of the Bench Dragon when no collision occurs is shown in Figure 14.

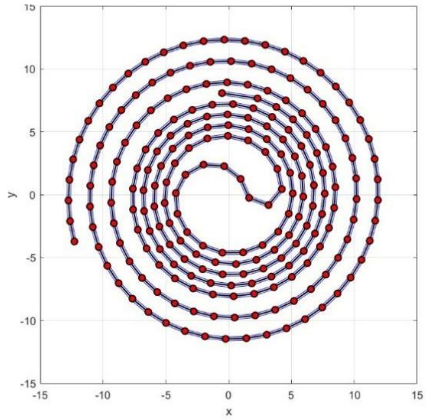


**Figure 14:** Situation when the Bench Dragon is just about to avoid collision

At this point, the relevant data for the two arcs can be obtained as shown in Table 1:

**Table 1:** Relevant Data for the Optimal U-turn Curve Circles

| | Parameters of the Large Circle | Parameters of the Small Circle |
|---|---|---|
| X-coordinate(m) | **-1.382049** | **2.808890** |
| Y-coordinate (m) | **-0.414903** | **0.562976** |
| Radius (m) | **2.869008** | **1.434504** |
| Central Angle (rad) | **3.015768** | **3.015768** |
| Arc Length (m) | **8.65233** | **4.32617** |
| Total Length (m) | **12.9785** | |

## 4. Conclusions

This paper presents a comprehensive mathematical model and computational simulation to analyze the motion trajectory, velocity changes, collision detection, and turning process of the "Benchen Dragon" in traditional folk culture activities. By integrating the principles of the Archimedean spiral and geometry, this study provides a scientific description of the movement of the "Benchen Dragon," while enhancing the precision of the simulation through the convex hull algorithm and variable step-size method. The results show that the positions and velocities of the handles at each segment of the dragon's benches follow specific patterns, and the exact moments of collision can be pinpointed using precise algorithms. Additionally, the optimal turning path was successfully determined, offering valuable insights for the design of more efficient movements in this cultural activity. In conclusion, this research highlights the potential of combining cultural phenomena with scientific analysis and opens new avenues for further studies in cultural research.

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