Model Of Bench Dragon Based on Equidistant Spiral Trajectories

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Abstract. By studying the movement patterns and constraints of the bench dragon, this folk activity can be made more scientifically standardized. Firstly, a polar coordinate system is established for the movement trajectory, and the integral formula for the arc length of the equidistant spiral and the formula for the node position of the bench dragon on the spiral trajectory are derived. Then, through geometric analysis, it is proven that during the dragon dance team's spiral movement, the outer vertex of the front end of the dragon head bench will first collide with the inner edge of the outer bench. Based on this, the problem is simplified and the objective function for collision judgment is derived. Constraints are listed according to the problem, and a single-objective optimization model is established. The particle swarm optimization algorithm combined with dichotomy is used to solve the termination condition of the bench dragon's spiral movement, and it is concluded that a collision occurs at 412.70s.

Keywords: Isometric spirals, particle swarm optimisation algorithms, dichotomous methods.

1. Introduction

As a deeply - rooted and symbolically significant intangible cultural heritage that has long thrived in the provinces of Zhejiang and Fujian, the Bench Dragon is far more than a simple folk art form. It is a living repository of centuries - old historical chronicles, carrying within its every plank and every movement the rich tapestry of local cultural connotations. It serves as a powerful emblem of the ethnic sentiments that bind the local communities together, representing their shared values, traditions, and a sense of collective identity passed down through generations.As an important intangible cultural heritage in Zhejiang and Fujian provinces, the Bench Dragon carries profound historical and cultural connotations as well as ethnic sentiments. However, with the acceleration of modernization, the protection and inheritance of traditional folk culture are facing unprecedented challenges.

This study conducts an in-depth investigation into the kinematic characteristics of the Bench Dragon using mathematical methods, which not only helps deepen the understanding of this traditional cultural activity but also provides a scientific basis for its inheritance and innovation in modern society. Furthermore, the research results hold far-reaching significance for promoting the protection and utilization of intangible cultural heritage, as well as fostering cultural diversity and confidence.

Given the current scarcity of research on the kinematic characteristics of multi-segment motion models moving along specific geometric paths (such as equidistant spirals), especially the lack of studies on the regularity of collision conditions between individual bench segments, this paper aims to fill this gap [1]. By constructing an accurate mathematical model, the entire process of the Bench Dragon winding along a spiral path during actual performances is simulated, thereby exploring and determining the extreme (termination) conditions for avoiding collisions between benches. This objective is not only related to the scientific analysis of traditional cultural activities but also aims to provide new perspectives and tools for the inheritance and development of traditional folk culture through modern technological means.This study primarily utilizes geometric analysis to simplify the complex multi-segment motion model and establishes a new single-objective optimization model under specific and necessary geometric parameters.

Additionally, this research innovatively combines the Particle Swarm Optimization algorithm with the Bisection Method to study traditional folk sports and solve the research problem, ultimately determining the extreme (termination) conditions for the Bench Dragon's winding motion [2-4]. Furthermore, this paper offers new insights into the kinematic characteristics of the Bench Dragon, providing a theoretical foundation and technical support for future research in related fields (such as the study of kinematic characteristics of multi-segment motion models during curved motion) and potential practical applications.

2. Relevant theories

For convenience of description, the center of the front and rear handles of the bench dragon head is called the front and rear nodes, and each node has two inner and outer edge vertices, which are the inner and outer edge vertices of the front node and the inner and outer edge vertices of the rear node, respectively.The dragon dance team was coiled clockwise along an equally spaced spiral with a pitch of 55 cm, with the center of each handle located on the spiral. The traveling speed of the front handle of the dragon was always 1 m/s. Initially, the dragon was located at point A of the 16th circle of the thread (see Fig. 1).

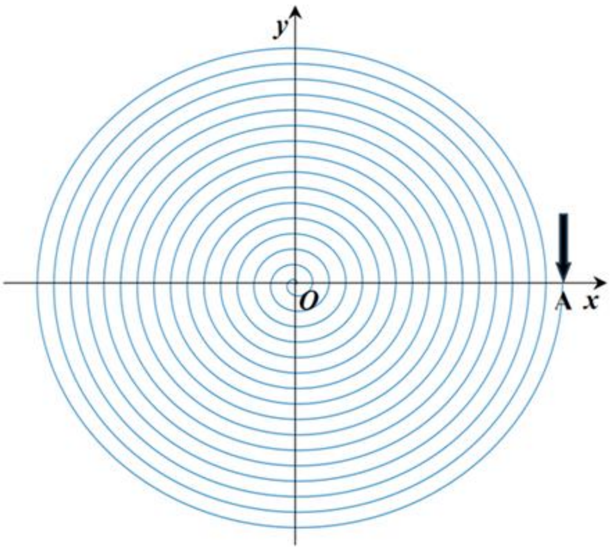

Figure 1. Schematic diagram of a coiled screw thread

2.1. Spiral arc length function

In order to better describe the movement of the bench dragon on the helix, a polar coordinate system is first established, and the equation of the isometric helix in polar coordinates is: $r = a + b\theta$. According to the schematic diagram of Fig. 1 disk into the helix, it can be seen that the helix extends from the origin in the direction of the x-axis i.e., $a = 0$, and the pitch of the helix is $0.55m$, i.e., $b = \frac{0.55}{2\pi}$, that is:

$$r(\theta) = \frac{0.55\theta}{2\pi} \tag{1}$$

For very small solenoidal segment microelements, the length can be approximated:

$$ds = \sqrt{(rd\theta)^2 + (dr)^2} = \sqrt{r^2 + (\frac{dr}{d\theta})^2}\, d\theta \tag{2}$$

For a solenoidal segment starting at θ_1 and ending at θ_2, its length can be expressed as:

$$L = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta \tag{3}$$

For isometric solenoids, perfecting the above equation yields, due to $\frac{dr}{d\theta} = \frac{0.55}{2\pi}$:

$$L = \int_{\theta_1}^{\theta_2} \sqrt{(\frac{0.55\theta}{2\pi})^2 + (\frac{0.55}{2\pi})^2} \cdot d\theta = \frac{0.55}{2\pi} \int_{\theta_1}^{\theta_2} \sqrt{\theta^2 + 1} d\theta \tag{4}$$

First permute θ so that $\theta = \tan t$, then $d\theta = \sec^2(t)dt$, which is obtained by substituting it into the integral formula:

$$L = \frac{0.55}{2\pi} \int_{\theta_1}^{\theta_2} \sqrt{\theta^2 + 1} d\theta = \frac{0.55}{2\pi} \int_{\theta_1}^{\theta_2} \sqrt{\tan^2(t) + 1} \sec^2(t) dt \tag{5}$$

A further treatment of the above equation is given by $\tan^2 t + 1 = \sec^2 t$ and the trigonometric integral formula:

$$L = \frac{0.55}{2\pi} \int_{\theta_1}^{\theta_2} \sec^3(t) dt = \frac{0.55}{2\pi} (\frac{1}{2} \sec(t) \tan(t) + \frac{1}{2} \ln \left| \sec(t) + \tan(t) \right|) + C \Big|_{\theta_1}^{\theta_2} \tag{6}$$

where C is a constant, which can be omitted when seeking a specific solution, substituting $t = arc\tan(\theta)$ back into the above equation gives:

$$L = \frac{0.55}{2\pi} (\frac{1}{2} \theta \sqrt{\theta^2 + 1} + \frac{1}{2} \ln \left| \sqrt{\theta^2 + 1} + \theta \right|) \Bigg|_{\theta_1}^{\theta_2} \tag{7}$$

2.2. Location of nodes of the dragon's head, body and tail

Bench dragon dragon head before the node along that isometric spiral line clockwise disc into the body of the dragon between each section of the bench connected through the node, every two nodes center spacing is fixed, if the location of the previous node is known, the location of the latter node can be determined by the previous node as the center of the circle for the circle and intersect with the spiral line [5].

Assuming that a certain two neighboring nodes of the dragon's body are P_i and P_{i+1}, and their polar coordinates are (r_i, θ_i) and (r_{i+1}, θ_{i+1}) .The positional coordinates of the latter node can be accurately calculated using the fixed distance between the two. In order to better express the distance between those two points, convert these two polar coordinates to right-angle coordinates and find the distance d between them as:

$$\begin{aligned} & P_i : (r_i \cos\theta_i, r_i \sin\theta_i) \\ & P_{i+1} : (r_{i+1} \cos\theta_{i+1}, r_{i+1} \sin\theta_{i+1}) \\ & d = \sqrt{(r_i \cos\theta_i - r_{i+1} \cos\theta_{i+1})^2 + (r_i \sin\theta_i - r_{i+1} \sin\theta_{i+1})^2} \\ & (i = 1, 2, 3 ... 223) \end{aligned} \tag{8}$$

Simplifying the above equation yields:

$$d = \sqrt{r_i^2 + r_{i+1}{}^2 - 2r_i r_{i+1} \cos(\theta_i - \theta_{i+1})} (i = 1, 2, 3 ... 223) \tag{9}$$

where i.e. d is the radius of the circle. Since the length of the dragon's head and body plates are different, their corresponding radii of the circle are different, $2.86cm$ and $1.65cm$ respectively.

3. Experments

3.1. Simplified judgement collision model

From the geometric relationship, the dragon dance team disk into the movement process, the front node of the dragon head bench corresponding to the outer edge of the vertex will be at a certain moment and the dragon body of a bench inside edge will be the first collision, the moment of contact for the bench dragon disk into the movement of the termination of the moment [6-8].

Let the two nodes of the dragon's head be A and B, their polar coordinates are (r_0, θ_0) and $(r_0^{'}, \theta_0^{'})$, respectively, corresponding to the two nodes of a certain bench of the dragon's body are C and D, their polar coordinates are (r_1, θ_1) and (r_2, θ_2), respectively, and the collision point is $E:(\rho_E, \theta_E)$. They are related as follows:

$$\theta_1 < \theta_0 + 2\pi \le \theta_2 \tag{10}$$

The above geometric relationship is shown in Figure 2:

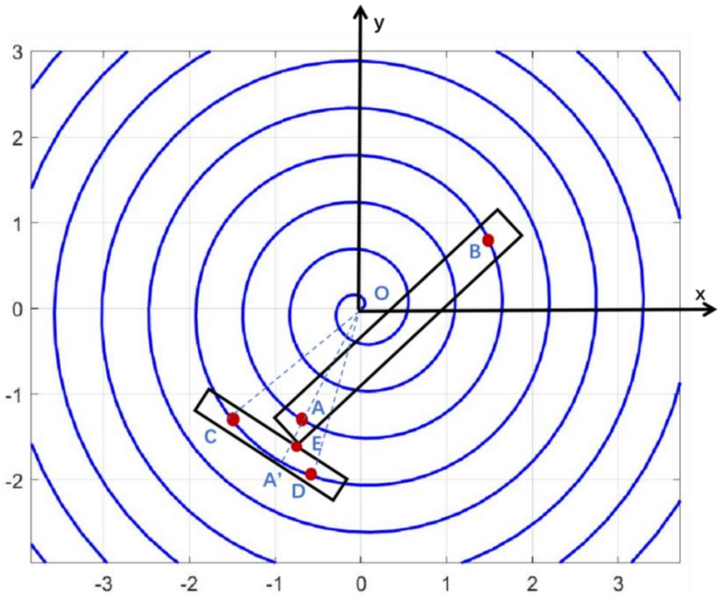

Figure 2. Collision diagram

where A' is the intersection of the center point of the helix and the line where A is located with the helix, and its angular value is A greater than 2π.

3.2. Establishment of the objective function

The equation of a point-slope line where A and B lie through the midpoint $(\frac{r_0 \cos\theta_0 + r_0^{'} \cos\theta_0^{'}}{2}, \frac{r_0 \sin\theta_0 + r_0^{'} \sin\theta_0^{'}}{2})$ of A and B is:

$$y - \frac{r_0 \sin\theta_0 + r_0^{'} \sin\theta_0^{'}}{2} = \frac{r_0^{'} \sin\theta_0^{'} - r_0 \sin\theta_0}{r_0^{'} \cos\theta_0^{'} - r_0 \cos\theta_0} (x - \frac{r_0 \cos\theta_0 + r_0^{'} \cos\theta_0^{'}}{2}) \tag{11}$$

Let the tangent corresponding to the slope of the line be:

$$\tan\alpha = \frac{r_0^{'} \sin\theta_0^{'} - r_0 \sin\theta_0}{r_0^{'} \cos\theta_0^{'} - r_0 \cos\theta_0} \tag{12}$$

From the geometric relationship, the tangent of the angle between the line where A and B are located and the line intersecting the line where the upper edge of the faucet bench is located is:

$$\tan\beta = \frac{30}{341} \tag{13}$$

The equation of the line passing through the lower vertex of the leading bench and the midpoints of A and B is:

$$y - \frac{r_0 \sin\theta_0 + r_0^{'} \sin\theta_0^{'}}{2} = \tan(\alpha+\beta)\left(x - \frac{r_0\cos\theta_0 + r_0^{'}\cos\theta_0^{'}}{2}\right) \tag{14}$$

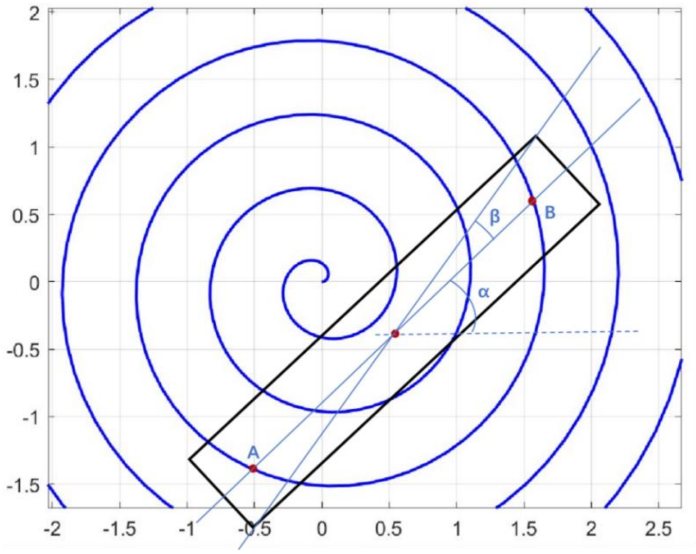

Figure 3. Schematic diagram of the angle for solving for the slope

The schematic is shown in Figure 3 above and converted to a linear equation in the polar coordinate system:

$$\rho\sin\theta - \frac{r_0 \sin\theta_0 + r_0^{'} \sin\theta_0^{'}}{2} = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}\left(\rho\cos\theta - \frac{r_0\cos\theta_0 + r_0^{'}\cos\theta_0^{'}}{2}\right) \tag{15}$$

where $\tan\alpha\tan\beta \neq 1$.

The equation of the line where C and D lie is:

$$y - r_1\sin\theta_1 = \frac{r_1\sin\theta_1 - r_2\sin\theta_2}{r_1\cos\theta_1 - r_2\cos\theta_2}(x - r_1\cos\theta_1) \tag{16}$$

Transform the above equation into the form $Ax + By + C = 0$, i.e..

$$\frac{r_1\sin\theta_1 - r_2\sin\theta_2}{r_1\cos\theta_1 - r_2\cos\theta_2}x - y + r_1\sin\theta_1 - \frac{r_1\sin\theta_1 - r_2\sin\theta_2}{r_1\cos\theta_1 - r_2\cos\theta_2}r_1\cos\theta_1 = 0 \tag{17}$$

where $A = \frac{r_1\sin\theta_1 - r_2\sin\theta_2}{r_1\cos\theta_1 - r_2\cos\theta_2}, B = -1, C = r_1\sin\theta_1 - \frac{r_1\sin\theta_1 - r_2\sin\theta_2}{r_1\cos\theta_1 - r_2\cos\theta_2}r_1\cos\theta_1.$

The polarized linear equation is:

$$\rho = \frac{-C}{A\cos\theta + B\sin\theta} = \frac{-C}{\sqrt{A^2+B^2}\sin(\theta+\phi)} \tag{18}$$

where $\sin\phi = \frac{A}{\sqrt{A^2+B^2}}, \cos\phi = \frac{B}{\sqrt{A^2+B^2}}.$

Since a straight line on the upper side of one of the benches of the dragon's body is parallel to the line where C and D are located and the distance from the center of the circle of the coordinate system is relatively small by $0.15m$, the equation of the line is:

$$\rho = \frac{-C}{\sqrt{A^2+B^2}\sin(\theta+\phi)} - \frac{0.15}{\sin(\theta+\phi)} \tag{19}$$

Joining Eq. 15 and Eq. 19 yields:

$$\begin{cases} \rho\sin\theta - \frac{r_0\sin\theta_0 + r_0^{'}\sin\theta_0^{'}}{2} = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}(\rho\cos\theta - \frac{r_0\cos\theta_0 + r_0^{'}\cos\theta_0^{'}}{2}) \\ \rho = \rho = \frac{-C}{\sqrt{A^2+B^2}\sin(\theta+\phi)} - \frac{0.15}{\sin(\theta+\phi)} \end{cases} \tag{20}$$

Let the polar coordinates of the intersection of the two lines solved for be (ρ_F, θ_F), the right angle coordinates be $(\rho_F\cos\theta_F, \rho_F\sin\theta_F)$, and the distance d_i between the midpoints of A and B and that intersection be:

$$d_i = \sqrt{(\frac{r_0\cos\theta_0 + r_0^{'}\cos\theta_0^{'}}{2} - \rho_F\cos\theta_F)^2 + (\frac{r_0\sin\theta_0 + r_0^{'}\sin\theta_0^{'}}{2} - \rho_F\sin\theta_F)^2} \tag{21}$$

where d_i decreases with disk-in motion.

Bench dragon exactly collision that disk into the termination moment of the movement, that is, when d_i exactly equal to A, B midpoint to the dragon head bench under the vertex of the distance d_0, and $d_0 = \sqrt{0.15^2 + 1.705^2} = 1.711585522257m$, so the optimization objective for d_i and d_0 difference is the smallest:

$$\min\left|d_i - d_0\right| \tag{22}$$

3.3. Establishment of an objective optimisation model

$$\min\left|d_i - d_0\right|$$

$$s.t.\begin{cases} \rho\sin\theta - \frac{r_0\sin\theta_0 + r_0^{'}\sin\theta_0^{'}}{2} = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}(\rho\cos\theta - \frac{r_0\cos\theta_0 + r_0^{'}\cos\theta_0^{'}}{2}) \\ \theta_1 < \theta_0 + 2\pi \le \theta_2 \\ A = \frac{r_1\sin\theta_1 - r_2\sin\theta_2}{r_1\cos\theta_1 - r_2\cos\theta_2} \\ B = -1 \\ C = r_1\sin\theta - \frac{r_1\sin\theta_1 - r_2\sin\theta_2}{r_1\cos\theta_1 - r_2\cos\theta_2} r_1\cos\theta_1 \\ \rho = \frac{-C}{\sqrt{A^2+B^2}\sin(\theta+\phi)} - \frac{0.15}{\sin(\theta+\phi)} \\ d_0 = \sqrt{0.15^2 + 1.705^2} \\ d_i = \sqrt{(\frac{r_0\cos\theta_0 + r_0^{'}\cos\theta_0^{'}}{2} - \rho_F\cos\theta_F)^2 + (\frac{r_0\sin\theta_0 + r_0^{'}\sin\theta_0^{'}}{2} - \rho_F\sin\theta_F)^2} \end{cases} \tag{23}$$

where the decision variable for the above optimization model is d_i.

3.4. Design of MATLAB program flow

Through the simulation of the bench dragon along the isometric spiral coil into the process, to determine the bench dragon parts (dragon head, dragon body, dragon tail) of the position and speed change, the first 300s when the dragon head of the angle of the node as the initial value, along the spiral according to a certain step length traversal to solve for the collision of the approximate angle of the range of the range of the exact collision in this range using particle swarm optimization algorithm combined with the dichotomy to search for the exact collision that is the termination of the moment of the exact angle [9-10], the specific The specific process is shown in Figure 4:

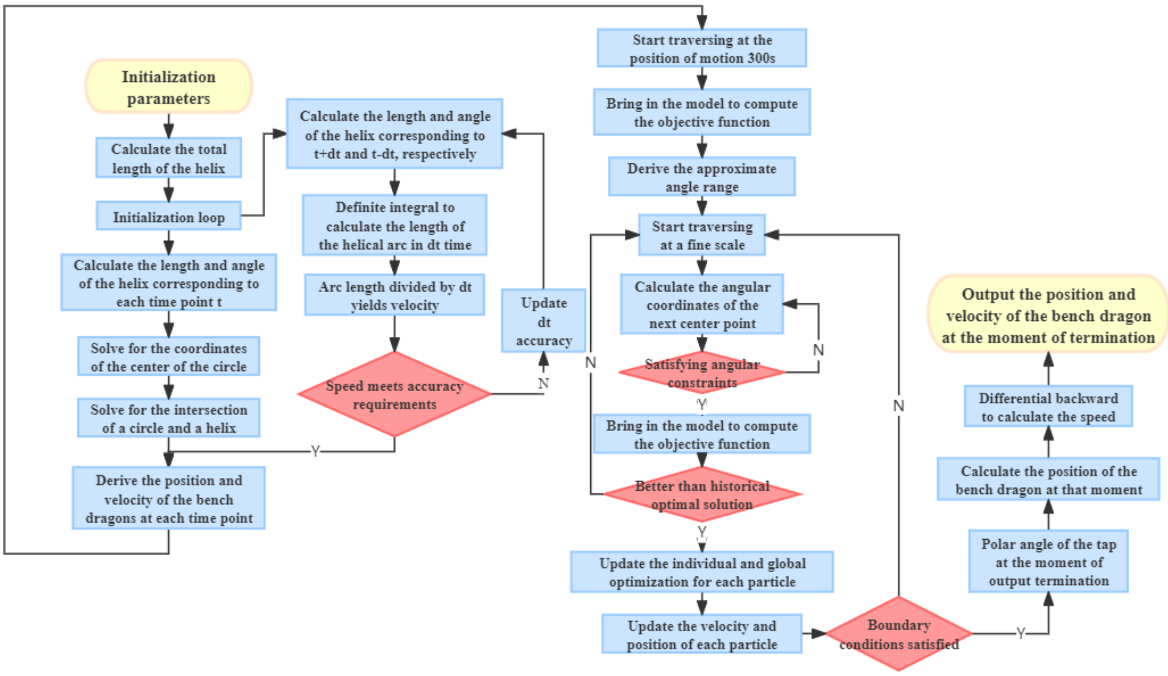

Figure 4. Program flow chart

As a result of the above experiments and research analyses, the maximum safe coiling range and the termination moment for the bench dragon to coil along the equidistant solenoid without collision were derived, which provides an important scientific basis for the understanding and protection of this intangible cultural heritage.

4. Results

When a dragon dance team moves to the moment of collision, the distribution of bench dragon positions is shown in the following figure, and the positions and velocities of the nodes in front of the dragon head, the nodes in front of the dragon body in the 1st, 51st, 101st, 151st, and 201st sections behind the dragon head, and the nodes in front of the dragon body and the nodes in the back of the dragon tail are shown in Table 1:

Table.1. Results of position and speed

	Abscissa x (m)	Ordinate y (m)	Velocity (m/s)
Dragon Head	1.348211	1.841375	1.000592
1st Section of Dragon Body	-1.511725	1.860482	0.992022
51st Section of Dragon Body	1.439907	4.272960	0.977221
101st Section of Dragon Body	-0.702176	-5.860143	0.974908
151st Section of Dragon Body	0.802955	-6.976475	0.973964
201st Section of Dragon Body	-7.915337	-1.065361	0.973451
Dragon Tail (Rear)	1.121559	8.300312	0.973293

The bench dragon happens to collide, that is, the termination moment of the rotary motion is: 412.70s, at this time the extreme diameter of the node in front of the dragon's head is 2.282178m, and the corresponding distribution of the node position is shown in Figure 5:

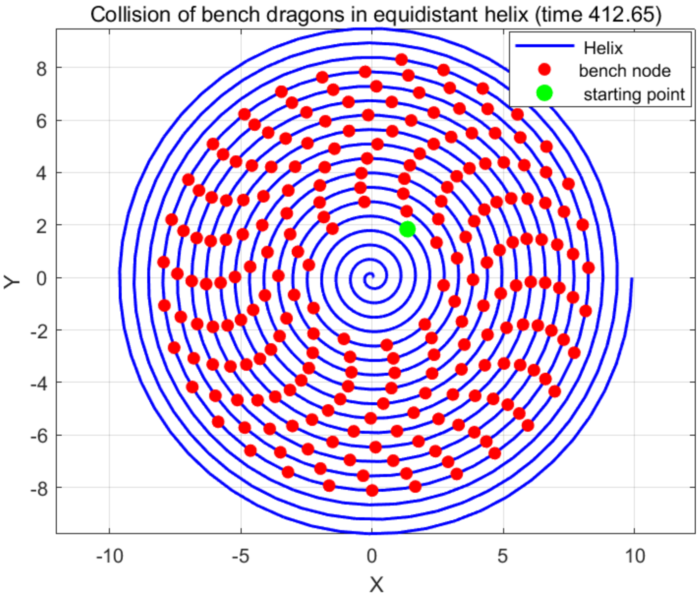

Figure 5. The trajectory diagram of the bench dragon movement

5. Conclusions

In this paper, a mathematical model is developed and an optimization algorithm is applied to simulate and analyze the problem of collision prevention and maximum safe coiling range during the coiling of bench dragons along an isometric screw thread.The analytical solution of the helix arc length in polar coordinates is calculated by integration, which improves the accuracy of position and velocity calculation; the particle swarm algorithm is used in combination with the dichotomous method to solve the complex planning problem, which enhances the algorithm's efficiency and extensibility; and the collision determination logic is simplified by geometric relations, which reduces the computational volume and verifies the reliability and efficiency of the model. In the future, the model is expected to be extended to the dynamics research of other traditional folk activities, providing scientific support for the protection and inheritance of intangible cultural heritage.

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