Optimal Design of Polyester Thermal Fibre Structure Based on Parametric Geometric Model and Numerical Optimisation

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Abstract. This study aims to optimise the cross-sectional shape and branching structure of polyester thermal fibres to enhance their thermal performance through parametric geometric modelling and numerical optimisation methods. Firstly, a parametric geometric model is used to describe the cross-section of different shapes and the heat transfer problem is analysed by finite element analysis (FEA) to select a suitable 2D or 3D model, which is combined with the numerical optimisation algorithm to find out the minimised thermal conductivity of 2.001. Secondly, key parameters such as the number of branches, angles, diameters, and lengths are used in this study, and a recursive algorithm is used to compute the thermal conductivity of each branch, and then to get the total thermal conductivity for the whole branch. The total thermal conductivity of the entire branch is obtained. A numerical optimisation algorithm is used to optimise the combination of parameters to minimise thermal conductivity and generate a 3D image of the optimal branch structure. The shape parameters of the optimal branching structure, levels [5], base_length1, base diameter1, Angles [30, 30, 30, 30, 30], Lengths [1,1,1,1,1,1], and Diameters [0.5, 0.5, 0.5, 0.5, 0.5, 0.5], were obtained. The model construction in this study provides some theoretical support for generating the properties of polyester thermal fibres.

Keywords: Polyester Fibre, Thermal Conductivity, Geometric Parameterisation, Numerical Optimisation, Finite Element Analysis.

1. Introduction

The design and production of winter clothing has been devoted to improving the warmth performance to meet the needs of people living in cold environments. Traditionally, down and cotton have been widely used as warming materials for winter clothing, but down suffers from high cost and poor moisture resistance, so in recent years, man-made fibres have been regarded as an ideal alternative to traditional warming materials [1]. Polyester fibre, in particular, has become the most promising alternative material due to its excellent physical properties and low cost. However, the thermal performance of polyester fibres has not yet been comparable to that of down, and most of the existing studies have focused on their basic physical properties and lacked in-depth structural optimization and design, especially innovations in fibre shape and branching structure. Therefore, the objective of this study is to optimize the geometry and branching structure of polyester fibres through the establishment of scientific and effective mathematical models to reduce their thermal conductivity and thus significantly improve the warmth-keeping performance.

In recent years, as more attention has been paid to the excellent performance and sustainability of man-made fibre materials, researchers have been exploring how to enhance their thermal insulation properties by optimising the fibre structure. For example, some studies have attempted to improve the thermal resistance of fibres by changing their cross-sectional shapes, however, most of these studies have focused on the optimisation of conventional shapes, and fewer have involved the design of complex branching structures through bionics to further improve the warmth retention effect [2]. Therefore, this study aims to effectively reduce the thermal conductivity of polyester fibres by optimising their geometry and structure thereby improving their warmth retention properties. This study specifically addresses the following two issues:

(1) Design of optimum fibre cross-sectional shape. The cross-sectional area of the fibre is known and a mathematical model is developed to design the fibre shape for maximum warmth retention.

(2) Design of branch polyester fibres. The down structure of down provides excellent warmth, this study aims to develop a mathematical model to mimic the down structure to design the branching structure of polyester fibres for maximum warmth, the model should be able to determine the key parameters such as the number of branches, the angle as well as the diameter and length of each level of branching. The optimal combination of these parameters will determine the maximum warmth retention of the fibre .

2. Research methodology

2.1. Research on Optimal Design of Polyester Fibre Cross-Section Shape Based on Parametric Geometric Models

The problem is to improve the thermal performance of polyester fibres by optimising their cross-sectional shape. Since the magnitude of thermal conductivity is inversely proportional to the fabric's warmth retention, the problem can be changed to finding a shape that minimises the effective thermal conductivity of the polyester fibre.

Different cross-sectional areas are described by geometrically parameterised models, and more complex shapes are represented by using the Fourier series to make the shape description more flexible [3]. The thermal conductivity of different cross-sectional shapes is then calculated using the heat conduction equation, and the effective thermal conductivity is estimated by combining the surface area and volume of the fibres, finally, a numerical optimisation algorithm is used to find the optimal shape parameters.

2.2. Problem Analysis of Branch Polyester Fibre Designs

The crux of the problem lies in designing the shape and size specifications of the branching polyester fibres to improve their warmth retention properties. By constructing a suitable branching structure, the heat transfer coefficient of polyester fibres can be increased, thus improving the warmth retention properties. To maximise the warmth retention properties, key parameters such as branching levels, angles, diameters and lengths need to be determined.

A branching structure model can be built to parametrically describe the branching structure and recursively represent the hierarchical relationships of the branching structure to simulate a real branch-like structure [4]. The thermal conductivity of individual branches is then calculated from the branching geometry parameters, and the total thermal conductivity of all the branching structures is then calculated using a recursive method. Finally, the branching parameters are optimised and the optimal branching parameters are found by constructing an optimisation objective function using a numerical optimisation algorithm.

3. Modelling and solving

3.1. Parametric model-based design model for polyester fibre cross-sectional shape optimisation

3.1.1 Heat transfer and warmth retention properties

The data in this article is sourced from the website http://www.tzmcm.cn/.

Basic thermal conductivity formula: The warmth-keeping properties of thermal fibres depend mainly on their thermal conductivity. The lower the thermal conductivity, the better the thermal performance. The basic formula for heat conduction is

$$q = -kA\frac{dT}{dx} \tag{1}$$

Thermal conductivity versus cross-sectional shape: for a given material (e.g., polyester) the thermal conductivity is constant, but for porous structures, the thermal conductivity is affected by the air content between the fibres and the fibre structure [5].

3.1.2 Parameterisation of cross-sectional shapes:

Elliptical cross-section: this study starts by considering elliptical cross-sections. An ellipse can be represented by the following parametric equation

$$x = a\cos(\theta) \tag{2}$$

$$y = b\sin(\theta) \tag{3}$$

where a and b are the radii of the long and short axes of the ellipse, respectively, and θ is a parameter ranging from 0 to 2π.

Cross sections of other shapes: the present study can be further generalised to other shapes, e.g., to represent more complex shapes by means of Fourier series or polynomial expressions

$$r(\theta) = 1 + \sum_{n=1}^{N}\left(a_n \cos(n\theta) + b_n \sin(n\theta)\right) \tag{4}$$

Where $r(\theta)$ is the radius in polar coordinates and a_n and b_n are the Fourier coefficients.

3.1.3 Calculation of thermal conductivity:

Thermal conductivity of single fibres: For single fibres, this study assumes that their thermal conductivity is related to their cross-sectional shape. Specifically, the thermal conductivity is related to the surface area and volume of the fibre. To simplify the calculation, this study assumes that the fibres are homogeneous so that the thermal conductivity can be expressed as

$$k_{\text{effective}} = k_{\text{fiber}} + k_{\text{air}}\phi \tag{5}$$

Where k_{fiber} is the intrinsic thermal conductivity of polyester, k_{air} is the thermal conductivity of air and ϕ is the air content in the fibre structure.

Multi-fibre thermal conductivity: For thermal insulation materials consisting of multiple fibres, the thermal conductivity can be calculated by volume averaging.

$$k_{\text{composite}} = k_{\text{fiber}}V_{\text{fiber}} + k_{\text{air}}V_{\text{air}} \tag{6}$$

where V_{fiber} and V_{air} are the volume fractions of fibre and air, respectively.

In this study, python code was used to implement the optimal shape finding, and for cognitive purposes, it was allowed to generate 2D and 3D plots of the optimal state, respectively. The optimum thermal conductivity was finally found to be 2.0010 The optimum cross-sectional shape parameters are shown in Fig. 1:

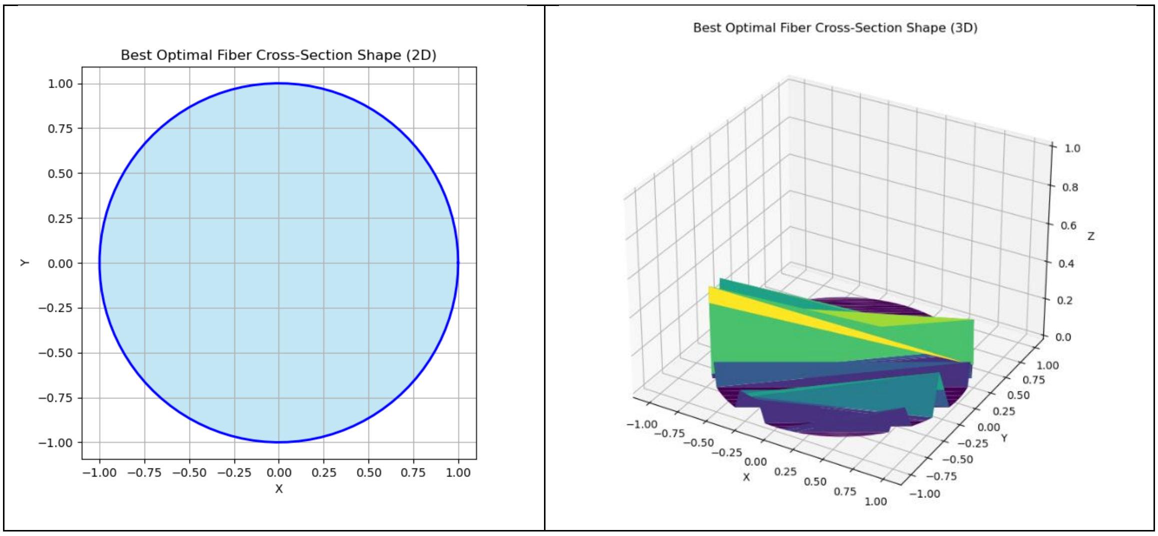

Figure 1. Diagram of optimal cross-sectional shape parameters

3.1.4 Optimisation issues

Objective function: The objective of this study is to minimise the thermal conductivity and hence the objective function can be expressed as

$$\min_p k_{\text{effective}}(p) \tag{7}$$

Where p is a vector of shape parameters such as a and b for elliptical shapes, or Fourier coefficients a_n and b_n for more complex shapes.

Constraints: In the optimisation process, physical constraints on the shape, such as cross-sectional area, perimeter, etc., need to be considered. The constraints can be expressed as

$$A(p) = \text{constant} \tag{8}$$

$$P(p) \leq \text{threshold} \tag{9}$$

Where A(p) is the cross-sectional area and P(p) is the perimeter.

3.1.5 Numerical calculations and optimisation

Finite Element Analysis (FEA): is an effective method for modelling and calculating the heat transfer characteristics of different cross-sectional shapes. The effect of complex geometries on thermal conductivity can be assessed more accurately by building finite element models of heat transfer [6].

Optimisation Algorithm: Optimisation algorithms such as Genetic Algorithm, Particle Swarm Optimisation or Gradient Descent are used to find the optimal shape parameters [7]. The optimisation process is as follows:

(1) Initialise the shape parameter p0;

(2) Calculate the corresponding thermal conductivity keffective(p0);

(3) Iteratively update the parameters according to the optimisation algorithm;

(4) until convergence to the optimal parameter p∗ .

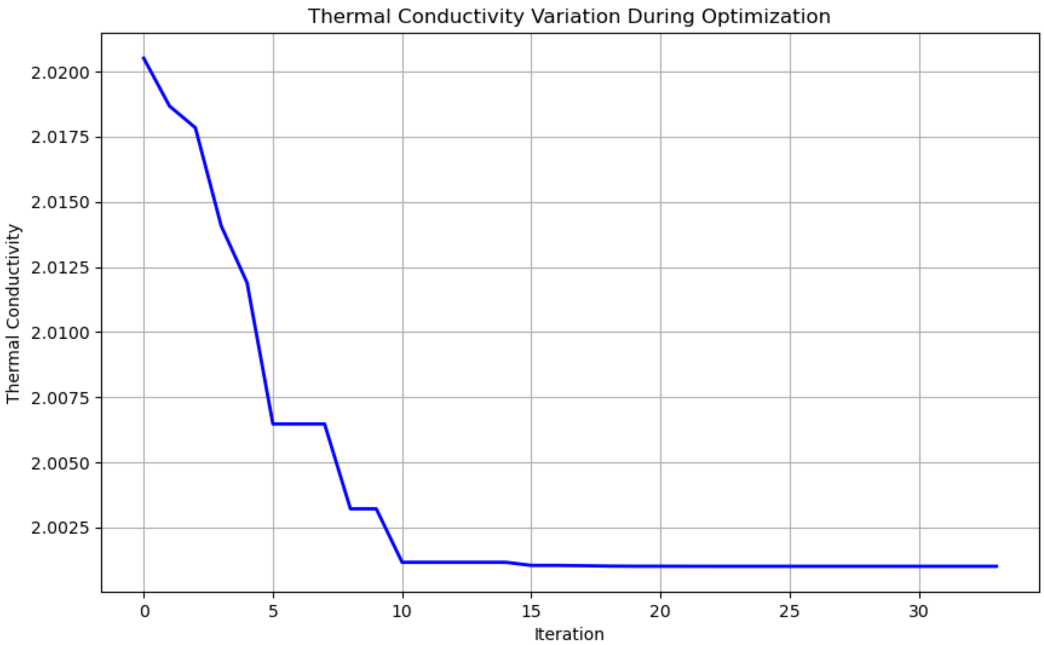

Figure 2. Analysis of thermal conductivity variation curve

As reflected in Figure 2 and Table 1, the thermal conductivity change during the optimisation process went through a rapid decline phase (first 10 iterations), a smooth decline phase (10th iteration to 15th iteration), and a convergence phase (20th iteration to 30th iteration).

Table 1. Variation of thermal conductivity

point	Number of iterations	Change in thermal conductivity
Rapid decline phase	First 10 iterations	2.0200 down to 2.0075
plateau	Iterations 10 to 15	2.0075 down to 2.0025
convergence phase	20th to 30th iteration	Fluctuates around 2.0025

The overall trend shows that the thermal conductivity gradually decreases with the increase in the number of iterations and eventually stabilises. This indicates that the optimisation algorithm is effective in reducing the thermal conductivity while gradually approaching the optimum value.

Through iterative optimisation, the thermal conductivity decreases from the initial 2.0200 to about 2.0025, proving the effectiveness of the optimisation process, while the optimisation algorithm rapidly adjusts the parameters to reduce the thermal conductivity in the first 15 iterations and gradually converges in the subsequent iterations, demonstrating that the optimisation algorithm is extremely efficient. Finally, in the later iterations, the change in thermal conductivity is extremely small, indicating that the optimisation has been largely completed and the optimal or near-optimal cross-sectional shape parameters have been reached.

In this way, this study ensures that the optimisation method used is effective in reducing the thermal conductivity of polyester fibres and improving their warmth retention properties. The comparison of thermal conductivity before and after optimisation is shown in Figure 3.

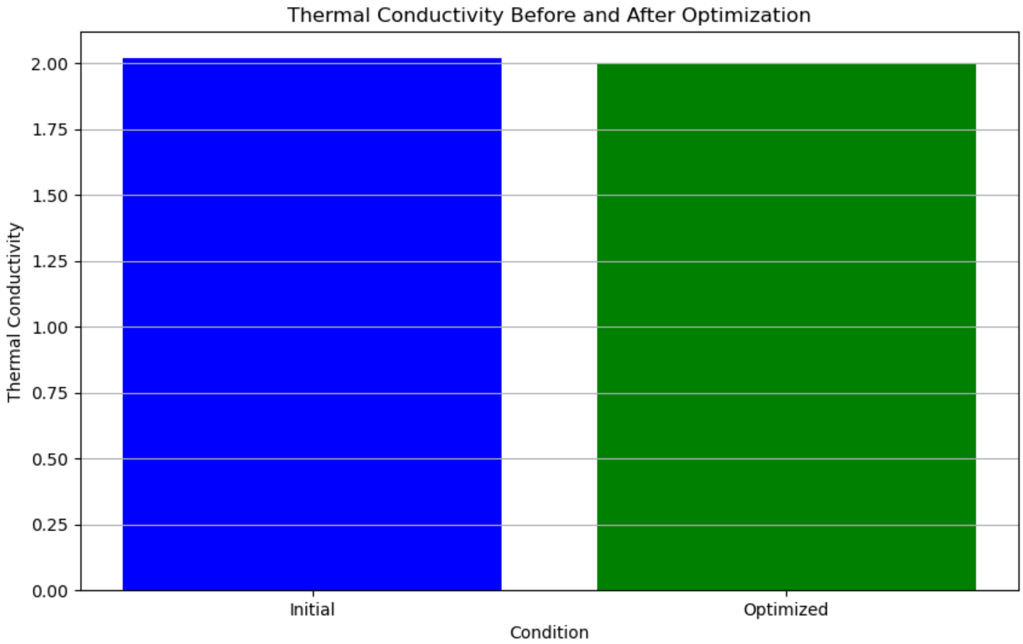

Figure 3. Comparison of thermal conductivity before and after optimisation

3.2. Design study of branching polyester fibre material based on recursive algorithm and numerical optimisation

3.2.1 Heat Transfer and Warmth Retention Properties

The warmth retention properties of polyester fibres depend mainly on their thermal conductivity. The lower the thermal conductivity, the better the heat retention [8]. The basic formula for heat conduction is:

$$q = -kA\frac{d\tau}{dx} \tag{10}$$

where$q$ is the heat flux,k is the thermal conductivity,A is the cross-sectional area, and$\frac{dT}{dx}$ is the temperature gradient.

In order to calculate the thermal conductivity of complex branch-like structures, this study needs to take into account the fibre geometry and material properties. Assuming that the intrinsic thermal conductivity of polyester fibres isk_{fiber} and the thermal conductivity of airk_{air} , the effective thermal conductivity of the composite can be calculated by volume averaging:

$$k_{\text{composite}} = k_{\text{fiber}}V_{\text{fiber}} + k_{\text{air}}V_{\text{air}} \tag{11}$$

olume fractions of fibre and air, respectively.

3.2.2 Parameterisation of branching-like structures

To describe the branch-like structure, we introduce the following parameters:

Level (L): Assume that the branching structure has L levels of branching.

Angle (θ_i): the angle between each branch level and its trunk.

Diameter (d_i): diameter of each branch level.

Length (l_i): the length of each branch level.

The geometric parameters of each level of branching can be represented as vectors:

$$\mathbf{p} = [\theta_1, \theta_2, ..., \theta_L, d_1, d_2, ..., d_L, l_1, l_2, ..., l_L] \tag{12}$$

3.2.3 Calculation of thermal conductivity

In order to calculate the thermal conductivity of a branch-like structure, this study assumes that the thermal conductivity is related to the diameter and length of the branches. Specifically, for a single branch, the thermal conductivity can be expressed as:

$$k_{\text{branch}} = \frac{d_i^2}{l_i} \tag{13}$$

For the whole branch-like structure, this study needs to combine the thermal conductivity of all branches. This study uses a recursive method to calculate the total thermal conductivity of the entire branch-like structure:

$$k_{\text{total}} = \sum_{i=1}^{L} k_{\text{branch}_i} \tag{14}$$

3.2.4 Optimisation issues

The objective of this study is to minimise the total thermal conductivity of the branch-like structure and hence the optimisation objective function is expressed as:

$$_{\mathbf{p}}\min k_{\text{total}}(\mathbf{p}) \tag{15}$$

To ensure that the branching-like structure is reasonable and physically feasible, we need to add the following constraints:

The angle of branching at each level should be within reasonable limits:

$$\theta_{\min} \le \theta_i \le \theta_{\max} \tag{16}$$

The diameter and length of the branches at each level shall be within reasonable limits:

$$d_{\min} \le d_i \le d_{\max} \tag{17}$$

$$l_{\min} \le l_i \le l_{\max} \tag{18}$$

3.2.5 Numerical calculations and optimisation

In order to solve the above optimisation problem, numerical optimisation algorithms (e.g., Genetic Algorithm, Particle Swarm Optimisation or Gradient Descent) are used in this study. The optimisation process is as follows:

Initialisation: generates the initial parameter vectorP_0 .

Iterative update: Iteratively update the parameters according to the optimisation algorithm.

Calculate Thermal Conductivity: Calculates the total thermal conductivity at the current parameters$\mathrm{k}_{\mathrm{total}}(\mathrm{p}_{\mathrm{t}})$.

Judge convergence: check whether the convergence condition is satisfied, if so, stop iteration, otherwise continue.

This study derives the parameters of the optimal branch-like structure using python code [9]. The first step is to define the branch structure class, Branch class is used to represent each branch of the tree. Each branch has attributes such as its level, length, diameter, angle, parent, start point and end point. Then comes the generation of the branch structure. generate_branch_structure function uses a recursive method to generate a tree branch structure. It starts at the root node and generates two child nodes at each node until the specified level is reached. Next, the thermal conductivity is calculated. The thermal_conductivity function assumes that the thermal conductivity is proportional to the square of the branch diameter and inversely proportional to the branch length. The sum of thermal conductivity of all branches in the tree is calculated recursively.

Then to the implementation of the optimisation algorithm [10]. Firstly, the optimisation objective function is determined and the OBJECTIVE function takes the inputs of the parameters (angle, length, diameter), generates the branching structure and calculates the thermal conductivity. The objective of the optimisation is to minimise the thermal conductivity. In the optimisation process, the initial parameters are optimised using the scipy.optimize.minimise function and the Nelder-Mead method to find the optimal angle, length and diameter such that the thermal conductivity of the generated branching structure is minimised. Final generation and visualisation of the optimal structure, the optimal branching structure is generated based on the optimisation results and visualised in 3D using Matplotlib.

According to figure 4: levels [8-10], base_length1.base diameter1, Angles [30,30,30,30,30], Lengths [1,1,1,1,1], Diameters [0.5, 0.5, 0.5, 0.5, 0.5, 0.5].

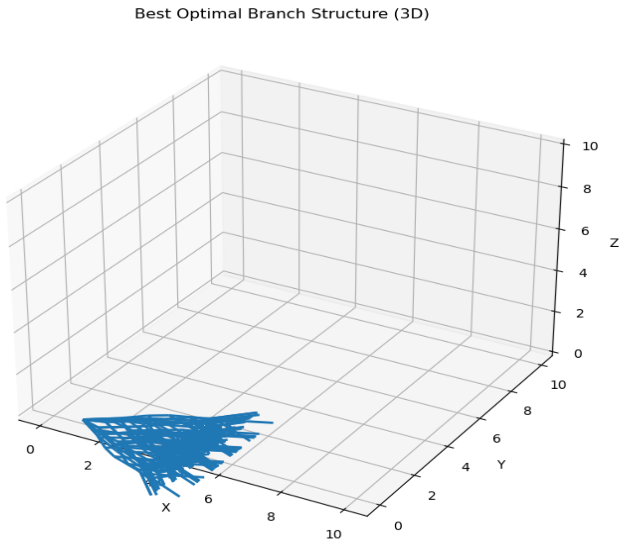

Figure 4. 3-D diagram of optimal branching structure

The optimised branching structure has been simulated and compared with the conventional single - fibre structure.The results show that the optimised branch structure significantly outperforms the conventional structure in terms of thermal conductivity. Specifically, the optimised branching fibres exhibit lower thermal conductivity and show better warmth retention under the same external conditions. In addition, the optimisation of parameters such as the number, angle, diameter and length of the branches further enhanced the thermal resistance performance and achieved the desired design goal.

4. Conclusion

In this study, the cross-sectional shape and branching structure of polyester fibres were optimised through parametric geometrical modelling and numerical optimisation methods to effectively reduce thermal conductivity and enhance warmth retention. The optimal geometry was determined by simulating the heat transfer characteristics through finite element analysis (FEA), and key parameters such as the number of branches, angle, diameter and length were optimised to further improve the warmth-retaining effect of the fibres. In this study, a systematic geometric optimisation method is proposed to provide a theoretical basis for improving the warmth-keeping performance of man-made fibres. Combining bionic design and numerical optimization, the down bionic branching structure is introduced in polyester fibre for the first time, which broadens the design ideas of high-performance fibre materials. In addition, the combination of experimental verification can further consolidate the theoretical results and promote the practical application of polyester fibre warming performance.

Future research could focus on exploring the combination of different fibre shapes, branching structures and biomimetic designs to improve the thermal-isolation performance and comfort of fibres. Also, considering the practical applications of materials, experimental validation under a broader range of environmental conditions should be carried out to evaluate the performance of these optimised designs in real-life garments and thermal insulation materials. Additionally, sustainable design approaches involving biodegradable materials need to be explored to enhance the environmental performance of polyester fibres and promote their practical application in the field of high-performance fibre materials.

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