Quantitative Analysis Of 'Bench Dragon' Motion Characteristics Based on The Euclidean Spiral Equation

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Abstract. The purpose of this paper is to quantitatively analyze the movement characteristics of the traditional folklore activity “Bench Dragon” through mathematical methods. An ideal mathematical model for this problem has not been proposed in the existing literature. In this paper, the isometric solenoidal equation is used to describe the trajectory of the “bench dragon”, and the coordinate recurrence relationship between the parts of the dragon team is deduced by combining the distance formula and kinematics principle with the associated equations. Further, the angular recurrence relationship between the parts of the dragon team is obtained by solving the triangle problem. Based on these kinematic relationships, this paper constructs a model of the corresponding differential equations and integrates them to derive a functional expression for the angle as a function of time. Finally, the recursive relationship between the speeds of the parts of the Dragons is obtained by taking the first order derivative of the angle with respect to time. The above study provides a reliable mathematical model and solution method for the quantitative analysis of the motion characteristics of the “Bench Drago”.

Keywords: Isometric Solenoid Equation, Kinematics Equation, Differential Equation, Cosine Theorem, Recurrence Relation.

1. Introduction

The “bench dragon” is one of the traditional folklores in southern Fujian. The purpose of this paper is to explore the variation rules of the positions and speeds of various parts of the dragon body under specific conditions, so as to provide diversified solutions for the design of the “Bench Dragon”. Liu (2024), in view of the difficulties faced by the inheritance of “Bench Dragon”, proposed that the protection and attention to the inheritors should be strengthened and a special Bench Dragon organization should be established [1]. Luo (2022) pointed out that although the status of “Bench Dragon” as a folk culture is declining, it still has a strong social cohesion [2]. Chen (2024) discusses the change and reconstruction of traditional folk rituals in cultural performances in response to the above problems [3]. Hu (2024) proposed that the public's cultural enthusiasm should be stimulated by enhancing local cultural confidence as an internal drive [4]. Li (2024), in response to the above problems, proposed that innovations should be made in terms of equipment, costumes and music [5]. The main contributions of this paper include: 1) describing the route of the bench dragon team as an isometric solenoid and giving the form of its equation; 2) proposing the recursive equations for the position and speed of the handles before and after each section of the dragon body; and 3) quantitatively analyzing for the first time the position and speed of the bench dragon team as it advances with a mathematical model.

In this paper, the parametric equations of the motion trajectory of the bench dragon team are firstly derived, and the distance relationship equations between the handles are given. Subsequently, based on the principle of kinematics, the recursive relational equation of the team's position and velocity is established to determine the position and velocity situation of any part of the dragon team at any moment.

This paper is structured as follows:The first part is the introduction, which presents the background, current status and contribution points of the study. The second part is the related theory, which describes the general form of the equidistance solenoidal equation, the derivation of differential equations for curvilinear motion, the recursive idea and the cosine theorem. The third part is the design and analysis of the experiment, which describes in detail the idea and process of the experiment. The fourth part is the conclusion of the experiment, which summarizes the results of the research and shows the results of the model run. The fifth part is the summary, which describes the significance of the study and the future outlook.

2. Related Theories

An equidistant solenoid is a mathematical curve characterized by the fact that the distance from each point on the solenoid to a fixed point is proportional to the angle that has been turned along the solenoid from a given starting point. An isometric solenoid can be thought of as a curve in a polar coordinate system in which the radius$\boldsymbol{\rho}$grows linearly at some fixed rate as the angle increases. The polar coordinate parametric equation for an isometric solenoid can be generally expressed as:

$$\rho = a + b\theta \tag{1}$$

Included among these,$\boldsymbol{\rho}$ is the radius at angle $\boldsymbol{\theta}$,$\boldsymbol{a}$ is the initial radius (the radius at $\boldsymbol{\theta}= 0$),$\boldsymbol{b}$ is a constant related to the sparsity of the solenoids,$\boldsymbol{b} = \frac{\boldsymbol{p}}{\boldsymbol{2\pi}}$,$p$ is the pitch,$\boldsymbol{\theta}$ denotes the angle.

Linear velocity, angular velocity and radius of motion are related by the following equation:

$$v = \omega r \tag{2}$$

Included among these,the angular velocity $\boldsymbol{\omega} = \frac{\boldsymbol{d\theta}}{\boldsymbol{dt}}$,At this point, equation (2) is transformed into equation (3).

$$vdt = rd\theta \tag{3}$$

After determining the upper and lower limits on both sides of the equation and integrating the time and angle separately, the expression for the angle with respect to time can be obtained.

Recursive thinking is a way of thinking about solving a problem by breaking it down into smaller sub-problems. Recursion is often used in algorithm design, especially when dealing with problems with self-similar structures or overlapping subproblems. Recursive methods can effectively reduce the computational complexity and improve the efficiency of program execution, especially common in dynamic programming and partitioning algorithms.

Recursive problems are often described by recurrence relations. This relationship defines how a problem is derived from the solutions of its subproblems. Usually expressed as:

$$T(n)=f(T(n-1), T(n-2) \ldots) \tag{4}$$

Included among these,$T(n)$ denotes the solution when the problem size is n, and f is a recursive function that defines how to derive the solution of the original problem from the solution of a subproblem of smaller size. The recursive process must have a base case, i.e., the smallest size or simplest case of the problem, where the solution can be obtained directly without further decomposition. The termination condition of the recursion is the key to preventing infinite recursion and ensuring that each recursion approaches the base case. Recursive algorithms are usually recursive implementations, whereas iteration is performed through loops. Recursion and iteration can be transformed into each other when dealing with certain problems.

The cosine theorem is an important theorem in trigonometry that describes the relationship between the lengths of the three sides of any triangle and one of the interior angles. In ΔABC, the cosine theorem can be expressed as equation (5).

$$cosA = \frac{b^2+c^2-a^2}{2bc} \tag{5}$$

Included among these,$BC=a$,$AC=b$,$AB=c$.

3. Experments

This paper focuses on the change law of the overall position and speed of the “Bench Dragon” in the process of advancing, as well as the key factors affecting the advancement of the team. A recursive relation describing the state of each handle at a given moment is constructed by introducing the formula for the distance between the front and rear handles and the solenoidal equation for an equidistant solenoid. Combined with the kinematic equations related to the physical quantities of velocity and angular velocity, this paper further deduces the position and velocity state of the front handle of the dragon head, so that the position and velocity information of the whole dragon dance team at each moment can be calculated and obtained by recursive method. The flowchart is shown in Figure 1 .

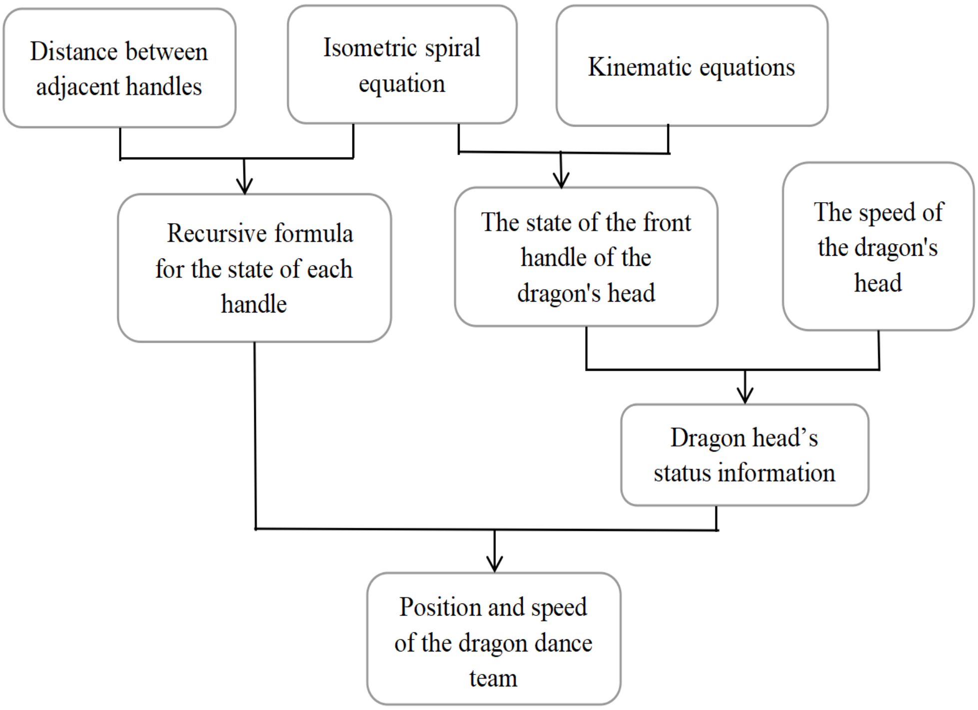

Figure 1. Experimental flow chart

The data for this study is sourced from: https://www.mcm.edu.cn/. The movement trajectory of the bench dragon team follows an equidistant spiral with an initial radius of 0 and a pitch of 0.55m [6][7]. Its expression in polar coordinates is as follows:

$$\rho = \frac{0.55}{2\pi}\theta \tag{6}$$

Its parametric equation in the Cartesian coordinate system can be expressed as:

$$\begin{cases} x = \rho cos\theta \\ y = \rho sin\theta \end{cases} \tag{7}$$

Take any section of the dragon body, are satisfied that the section of the dragon body between the position of the front handle (x_i, y_i) and the position of the rear handle (x_{i+1}, y_{i+1}), to meet the distance equation:

$$D(i, i+1) = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \tag{8}$$

Included among these, the distance between front and rear handles of faucet $D(0,1) = 2.86m$,Distance between front and rear handles of other parts of the Dragons $D(i, i+1) = 1.65m$.

The solution to the equation obtained by combining the equation for the distance between two points with the equation for the equidistance solenoid is not unique. Because the solution of the system of equations formed by two implicit function equations is equivalent to the intersection of these two curves, so make a graph of the intersection of the curves represented by the two equations, as shown in Figure 2.

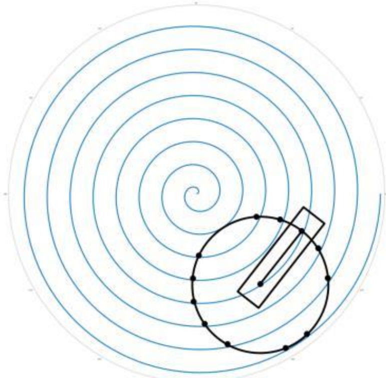

Figure 2. Schematic diagram of curve intersection

Each intersection of the black circle with the solenoid corresponds to a solution of the equation. The angle of the rear handle should be greater than the angle of the front handle because the Bench Dragons are coiled gradually along the screw line from the outside to the inside. In addition, considering that the team advances in a circle along the spiral, the smallest of all feasible solutions that are larger than the angle of the front handle should be chosen. As a result, it is only necessary to determine the initial position parameters of the faucet handles and substitute them into the equation for iteration, so that the specific position coordinates of all handles can be calculated.

In the initial state, the faucet is located in the outermost circle of the isometric solenoid, and it is stipulated that the initial angle $\theta_0 = 16 \times 2\pi$. The linear velocity of motion of the front handle of the faucet $v_0 = 1.0m/s$. Combined with the kinematic equation $v_0 = \omega r$,the following differential equation can be derived.

$$\frac{d\theta}{dt} = \frac{v_0}{\rho} \tag{9}$$

Integrating this differential equation,one can solve for the angle θat each moment in time [8-9]:

$$\int_0^t v_0 \, dt = -\int_{\theta_0(0)}^{\theta_0(t)} \rho \, d\theta \tag{10}$$

Substituting into the solenoidal equation, it can be deduced that:

$$\theta_0(t)^2 = \theta_0(0)^2 - \frac{4\pi t}{p} \tag{11}$$

The above model can determine the angle of the front handle of the faucet. Substitute the angle parameter into the parametric equation of the screw thread, you can find the coordinates of the position of the faucet before the handle at each moment. Further, using the recursive [10] relation, the coordinates of the spatial positions of the handles of the whole dragon dance team at different moments can be calculated.

As can be seen from Fig. 3, the screw angle coefficients of the front and rear handles are$\theta_i$andθ_{i+1}respectively, which can be introduced according to the cosine theorem:

$$\Delta\theta = arccos\frac{\rho^2(\theta_i)+\rho^2(\theta_{i+1})-D(i,i+1)^2}{2\rho(\theta_i)\rho(\theta_{i+1})} \tag{12}$$

Equation (12) into $\theta_{i+1} = \theta_i + \Delta\theta$, can be in the tap before the handle $\theta_0(t)$ is known in the case of all the handles behind the angle, and then into the parametric equation of the screw thread can be obtained handle the corresponding angle of the equation on the time, and then on the time of the first-order derivatives of the speed relationship can be obtained, it can be deduced that.

$$\begin{cases} dx_i(t) = \left[\rho\big(\theta_i(t)\big)cos\theta_i(t)\right]' dt \\ dy_i(t) = \left[\rho\big(\theta_i(t)\big)sin\theta_i(t)\right]' dt \end{cases} \tag{13}$$

To summarize, by using the recurrence relation of the angle parameter, we can solve the magnitude of the velocity of the handle of the whole dragon dance team at different moments.

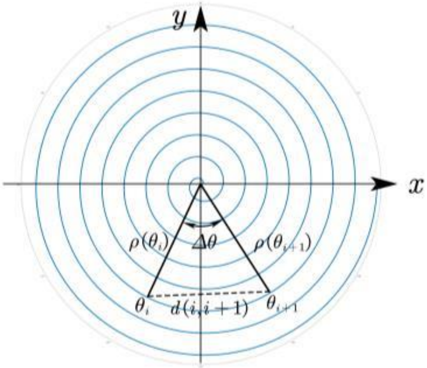

Figure 3. Geometric relationships using the cosine theorem

4. Results

Based on the above modeling and programming, the coordinates and speed data of each part of the dragon team can be obtained, and the morphology of the whole team after the dragon team advances for 300*s* is plotted (Fig. 4), as well as the curve of the speed of some parts of the dragon team with the change of time (Fig. 5).

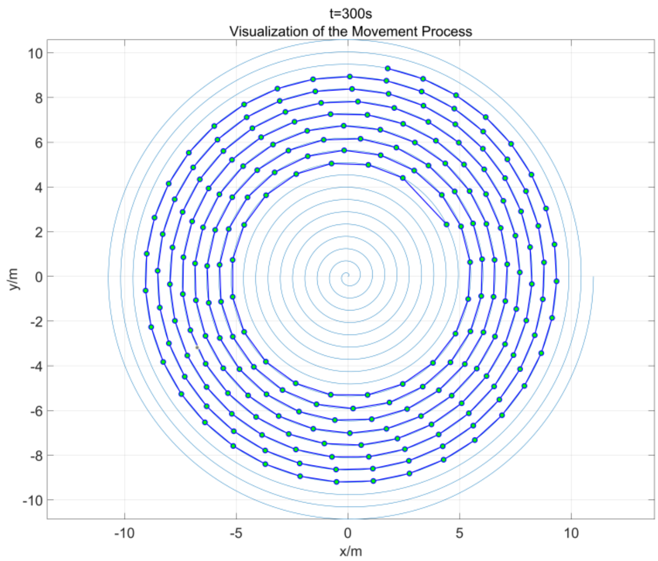

Figure 4. Visualization of team morphology

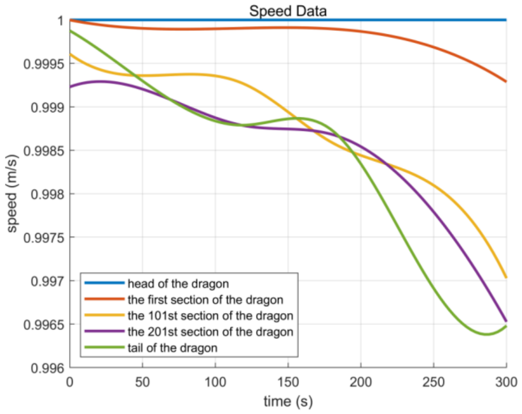

Figure 5. Visualization of partial handle speeds

The position information of the faucet at some moments is shown in Table 1 (Here the angle range is taken as -180°~180°).

Table 1. Location information for Dragon's Head

	0*s*	60*s*	120*s*	180*s*	240*s*	300*s*
Polar radius(*m*)	8.8000	8.1815	7.5122	6.7771	5.9520	4.9923
Polar angle (°)	0.0000	-14.2796	-39.1332	36.9021	-20.4219	8.8163

The velocity data (*m/s*) of some parts of the Bench Dragons at some moments are shown in Table 2. Included among these,v_i represents the speed of the dragon body in section *i*.

Table 2. Dragons speed information

	0*s*	60*s*	120*s*	180*s*	240*s*	300*s*
v_{head}	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
v_1	0.999999	0.999897	0.999905	0.999897	0.999737	0.999287
v_{101}	0.999608	0.999362	0.999261	0.998590	0.998192	0.997028
v_{201}	0.999229	0.999144	0.998782	0.998683	0.997983	0.996526
v_{tail}	0.999876	0.999181	0.998789	0.998732	0.997182	0.996481

Through the above research and modeling, a method was proposed to solve the state information of the Bench Dragons, successfully enabling the quantitative analysis of their motion state.

5. Conclusions

In this paper, a quantitative analysis is conducted on the movement characteristics of bench dragon, a traditional folk activity. A model is established that can determine the position and speed of any part of the bench dragon team at any time by depicting its running trajectory as an equidistant solenoidal equation, coupled with the physics motion formula and a series of mathematical methods. Subsequently, the running results of this model are visualized through programming technology, thereby addressing the lack of research on the movement state of "bench dragon" in existing literature. The model can be used to solve the position and velocity of any part at any time during the running of "Bench Dragon". The experimental process of this paper can provide new research ideas and methods for the academic community, which is conducive to promoting the creative transformation and innovative development of traditional folk culture. In the future, with the advancement of big data, artificial intelligence, visual processing, and machine learning technologies, real-time fitting of the movement trajectory of the bench dragon team will be achievable. This will enable more accurate calculations of the team's position, speed, and other state data during its movement. Utilizing these scientific technologies, it is also possible to accurately and flexibly design the performance strategy of the dragon dance team according to the actual situation, such as designing the trajectory of the disk in and out, calculating the total running time, adjusting the length and width of the benches and the height of the personnel, and so on, together with the appropriate costumes, which is conducive to the creation of a better visual effect.

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