Quantitative Analysis and Application of The Bench Dragon's Movement on An Archimedean Spiral

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Abstract. To optimize the kinematic parameters of the "Bench Dragon" in Chinese folk culture, it is requisite to establish a mathematical model to conduct real-time simulation of the motion state of the Bench Dragon when it moves along the Archimedean Spiral. At present, there are relatively few related studies on such issues. Thus, in this paper, by utilizing the properties of the Archimedean Spiral and differential equations in the polar coordinate system, a mathematical motion model of the "Bench Dragon" is established. The model is solved by employing the Runge-Kutta Method and the Finite Difference Method, obtaining the velocity and position of the "Bench Dragon" when it moves along the trajectory of the Archimedean Spiral. Simultaneously, this model is also applicable to deducing the real-time motion state of other chain-like objects moving along the Archimedean Spiral, and holds significant guiding significance for the design and operation of objects involving chains or link structures.

Keywords: Archimedean Spiral, Differential Equation, Finite Difference Methods, Runge-Kutta.

1. Introduction

The "Bench Dragon" is a traditional folk activity in the regions of Zhejiang and Fujian in China, and is highly favored by the local residents. Participants link dozens to hundreds of wooden benches end to end, forming a long "Chinese dragon". Guided by the dragon head, the dragon body meanders and progresses in a sinuous manner. When studying the relevant parameters of the Bench Dragon, the impact of its movement along the Archimedean Spiral on the overall performance effect has emerged as an issue worthy of in-depth exploration. During the performance, understanding the speed and position of the Bench Dragon's movement can avoid collisions among the dragon head, the dragon body and the dragon tail, which is of great significance for ensuring the safe conduct of the activity.Although predecessors have carried out rather elaborate research on the development of the "Bench Dragon", such as Chuanwen Liu of Ningbo University who discussed the inheritance and development of the Pujiang Bench Dragon from the perspective of cultural confidence in his article [1], the mathematical modeling analysis regarding its specific movement laws remains in the nascent stage. Hence, this paper endeavors to introduce the concept of mathematical modeling to deeply dissect the movement characteristics of the "Bench Dragon", particularly its behavioral patterns when it moves along a special path like the Archimedean Spiral [2-3]. Based on the Differential Equation [4-5] model, combined with the Finite Difference [6-8] methods and Runge-Kutta [9-10] to recursively solve for the movement speed and position, a mathematical model of the movement of the "Bench Dragon" has been established. This mathematical model can be utilized for the rapid prediction of the kinematic parameters of the Bench Dragon when it moves along the Archimedean

Spiral. This not only contributes to enhancing the quality of performances in practical activities but also offers theoretical foundations and technical support for the design of other similar structural objects, such as chains and link structures. This paper is divided into five chapters to progressively introduce the thought process for solving the problem. The first chapter is the Introduction, presenting the background and research objective of the problem. The second chapter is the Relevant Theories, introducing the theoretical knowledge and solution algorithms utilized in this paper. The third chapter is the Experiment, detailing the specific research ideas and elaborate process of this paper. The fourth chapter is the Results, presenting the results obtained from this research. The fifth chapter is the Conclusions, providing a detailed summary of this paper and the outlook for the practicality of this mathematical model.

2. Relevant Theories

2.1. Archimedean Spiral

The Archimedean spiral is the locus generated when a point moves uniformly away from a fixed point while rotating around the fixed point at a constant angular velocity. The form defined in polar coordinates is:

$$r = a + k\theta \tag{1}$$

Among them, r represents the distance from the origin to a certain point on the curve, θ is the angle of the point and the polar axis, known as the polar angle. a,b are constants, which determine the shape and size of the spiral.

2.2. Differential Equation

An equation involving the derivative of an unknown function is a differential equation, such as $\frac{dy}{dx} = 2x$. Its definitional formula is:

$$f(x, y, y', y^n, \cdots, y^{(n)}) = 0 \tag{2}$$

This paper pertains to nonlinear differential equations. The Runge-Kutta method is applicable for solving nonlinear ordinary differential equations. This approach is mainly utilized when the derivative of the equation and the initial value information are known, and it is employed in MATLAB simulation, circumventing the complex process of solving differential equations.

2.3. Finite Difference Methods

Finite Difference Methods are fundamental tools in numerical analysis for approximating the derivatives of functions and solving differential equations. According to the direction and form of the difference, finite differences mainly consist of forward finite difference, backward finite difference, and central finite difference.

Forward finite difference refers to the difference between the current point and its subsequent point. For equidistant nodes $x_0, x_1, x_2, \cdots, x_n$ and step size $h = x_{i+1} - x_i$, the forward finite difference is defined as:

$$\Delta f(x_i) = f(x_{i+1}) - f(x_i) \tag{3}$$

Backward finite difference denotes the difference between the current point and its predecessor point. For equidistant nodes $x_0, x_1, x_2, \cdots, x_n$ and step size $h = x_i - x_{i-1}$, the backward finite difference is defined as:

$$\Delta f(x_i) = f(x_i) - f(x_{i-1}) \tag{4}$$

Central Finite Difference employs the function values of the current point and the points preceding and succeeding it for differentiation. For equidistant nodes $x_0, x_1, x_2, \cdots, x_n$ and step size $h = x_{i+1} - x_i = x_i - x_{i-1}$, the Central Finite Difference is defined as:

$$\Delta f(x_i) = f(x_i) - f(x_{i-1}) \tag{5}$$

3. Experiment

Based on the data from www.mcm.edu.cn, a certain bench dragon consists of 223 bench sections. The first section functions as the dragon head, the subsequent 221 sections constitute the dragon body, and the final section forms the dragon tail. The length of the board of the dragon head is 341 cm, while the lengths of the boards of both the dragon body and the dragon tail are 220 cm. The width of all the benches is uniformly 30 cm. Each bench is equipped with two holes, with the diameter of the holes being 5.5 cm. The center of the holes is 27.5 cm away from the nearest board end (see Figures 1 and 2). Adjacent benches are connected via handles (see Figure 3).

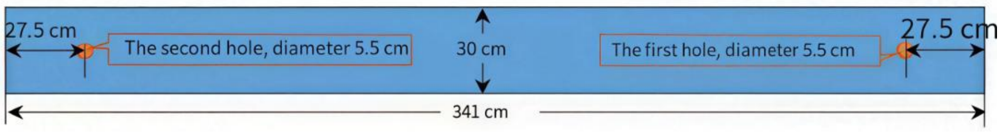

Figure 1 Top View of the Dragon Head

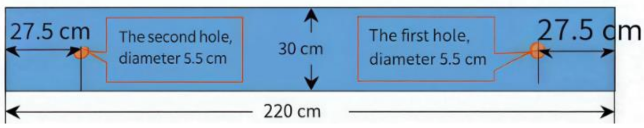

Figure 2 Top View of the Dragon's Body and Tail

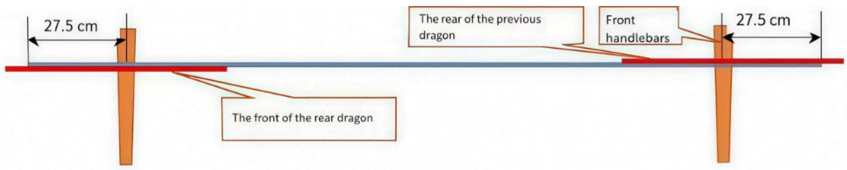

Figure 3 Front View of the Bench

The movement trajectories of each handle of the bench dragon are all in accordance with the Archimedean spiral. The positions and velocities of each handle can be deduced from the adjacent handles. To simplify the model, in this paper, the movement model of the front handle of the dragon head along the spiral is first established, and then the positions and velocities of other handles are deduced from the movement model of the dragon head.

3.1. The position model of the handlebar in front of the dragon head

From the Archimedean Spiral, the spiral equation can be obtained:

$$r = k\theta \tag{6}$$

Wherein $k = \dfrac{0.55}{2\pi}$, and te pitch of the spiral is $0.55m$.

In polar coordinate, the formula of arc length concerning angle is known as:

$$ds = -\sqrt{r^2 + \left(\frac{dr}{dt}\right)^2}\, d\theta \tag{7}$$

Wherein, s represents the arc length traversed by the faucet head. Additionally, the relationship between the angle and time during the movement of the faucet head should also be determined. Supposing that the velocity of the front handlebar of the faucet head remains at $1m/s$, it follows that:

$$\frac{ds}{dt}=1 \tag{8}$$

Combining Equations (6), (7), and (8) together, the relationship between the angle and time of the movement of the front handlebar of the faucet can be represented as:

$$\frac{d\theta}{dt}=-\frac{1}{k\sqrt{\theta^2+1}} \tag{9}$$

In conclusion, the model of the movement position of the front handlebar of the dragon head is as follows:

$$\begin{cases} r=k\theta \\ \dfrac{ds}{dt}=1 \\ ds=-\sqrt{r^2+\left(\dfrac{dr}{dt}\right)^2}\,d\theta \\ \dfrac{d\theta}{dt}=-\dfrac{1}{k\sqrt{\theta^2+1}} \end{cases} \tag{10}$$

3.2. The position model of the dragon body and tail

Suppose the distance between each handle is D. Then the positions of the other handles can be calculated based on the obtained position of the dragon head through the following manner:

$$\left(x_{m,n}{}^2-x_{m,n+1}{}^2\right)+\left(y_{m,n}{}^2-y_{m,n+1}{}^2\right)=D^2 \tag{11}$$

Herein, m stands for time and n represents the bench number (with the leading bench numbered as 1 and arranged successively). For example, $x_{m,n}$ is the coordinate of the n th handle at the m th second.

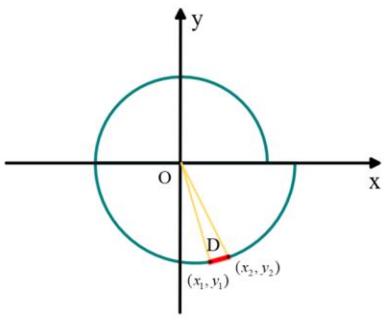

Figure 4: Schematic Diagram of the Positions between Adjacent Front and Back Handles

As shown in Figure 4, taking the position of the known handle as the center and the distance D between the two handles as the radius to draw a circle, numerous intersection points are obtained. Among them, only point a is the required handle point. To guarantee the correctness of the position information of the previous handle obtained, certain constraints need to be imposed on the above equation. First of all, it is necessary to ensure that the previous handle is located at the rear of the current handle, that is, the polar angle of the previous handle should be greater than that of the known handle:

$$\theta_{m,n+1} > \theta_{m,n} \tag{12}$$

Then, it is also necessary to ensure that the previous handle and the known handle are in the same circle. It is sufficient to have the difference between their polar radii less than half of the pitch:

$$| r_{m,n+1} - r_{m,n} | < \frac{d}{2} \tag{13}$$

Herein, d is the pitch of the helix.

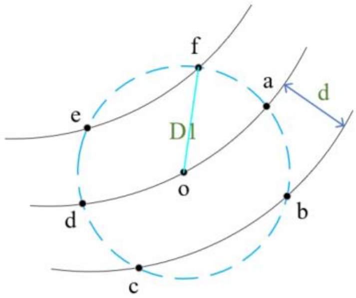

Figure 5: The Diagram Illustrating Multiple Solutions for the Position of the Handlebar

In conclusion, the models for the positions of the dragon's body and tail are as follows:

$$\begin{cases} \left(x_{m,n}{}^2 - x_{m,n+1}{}^2\right) + \left(y_{m,n}{}^2 + y_{m,n+1}{}^2\right) = D^2 \\ r_{m,n} = k\theta_{m,n} \\ x_{m,n} = r_{m,n}\cos\theta_{m,n} \\ y_{m,n} = r_{m,n}\cos\theta_{m,n} \\ x_{m,n+1} = r_{m,n+1}\cos\theta_{m,n+1} \\ y_{m,n+1} = r_{m,n+1}\cos\theta_{m,n+1} \\ \theta_{m,n+1} > \theta_{m,n} \\ | r_{m,n+1} - r_{m,n} | < \frac{d}{2} \end{cases} \tag{14}$$

3.3. The speed model of the Handlebar

Based on the analysis presented above, the velocity of each handlebar can be derived by differentiating the arc length s with respect to time t, namely:

$$\frac{ds}{dt} = v \tag{15}$$

Given the relationship between the infinitesimal arc length and the infinitesimal angle, dividing both sides of the equation by dt simultaneously leads to the model of the velocity of each handlebar:

$$\frac{ds}{dt} = -\sqrt{r^2 + \left(\frac{dr}{dt}\right)^2}\,\frac{d\theta}{dt} \tag{16}$$

That is:

$$v = -\sqrt{r^2 + \left(\frac{dr}{dt}\right)^2}\,\frac{d\theta}{dt} \tag{17}$$

For any bench at the intermediate moment of time, through the previous solution, we have known its polar angle at this moment and at the moments within the time step before and after this moment, namely. Thus, the central difference method is employed to solve the differential equation, and based on Equation 17, we obtain:

$$v_{m,n} = -k\sqrt{1+\theta_{m,n}^2}\,\frac{\theta_{m+1,n} - \theta_{m-1,n}}{2\Delta t}, 1 < m < 300, \tag{18}$$

Where $v_{m,n}$ represents the movement velocity of the handlebar of the nth bench at the mth time step. $\theta_{end,n}$ represents the polar angle of the polar coordinate of the handlebar of the nth bench at the mth time step.

For any bench at the initial time, as there is no previous time node at this moment, namely only two consecutive time nodes $\theta_{1,n}, \theta_{2,n}$, the backward difference is therefore employed to solve the differential equation:

$$v_{1,n} = -k\sqrt{1+\theta_{1,n}^2}\,\frac{\theta_{2,n} - \theta_{1,n}}{\Delta t}, \tag{19}$$

Where $v_{end,n}$ is the movement velocity of the nth bench handlebar at the last time step, and $\theta_{end,n}$ is the polar angle of the polar coordinate of the nth bench handlebar at the last time step.

For any bench at the time corresponding to the final step size, since there is no subsequent time node at this instant, that is, only two consecutive time nodes, $\theta_{end,n}, \theta_{end-1,n}$, the forward difference method is utilized to solve the differential equation:

$$v_{end,n} = -k\sqrt{1+\theta_{end,n}^2}\,\frac{\theta_{end,n} - \theta_{end-1,n}}{\Delta t}, \tag{20}$$

Where $v_{end,n}$ represents the movement velocity of the handlebar of the nth bench at the last time step, and $\theta_{end,n}$ represents the polar angle of the polar coordinate of the handlebar of the nth bench at the last time step.

4. Results

The key to solving the problem of the movement of the "Bench Dragon" along the Archimedean Spiral lies in deducing the movement of the subsequent benches based on the movement relationship between the previous bench and the next one. Moreover, the movement of the previous benches can be recursively obtained from the movement of the bench at the dragon head.

It is hypothesized that the dragon head was moving along the Archimedean Spiral with a greater polar radius prior to entering point A, and its position began to be recorded only upon entering point A. Consequently, the spiral was appropriately expanded before point A. Based on the aforementioned formula, the position coordinates of the dragon head at each time step were solved. The coordinates for each second from 1 to 300 seconds were extracted to draw the trajectory diagram as shown in Figure 6:

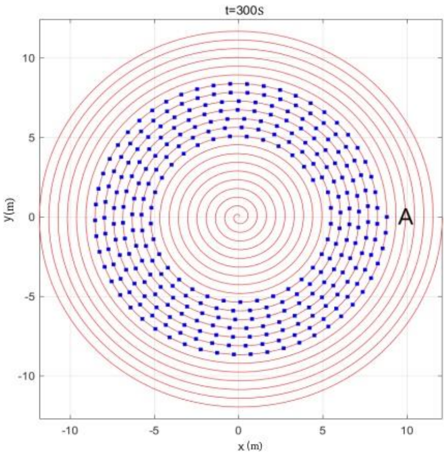

Figure 6: Trajectory Diagram of the Coordinates of the Front Handle of the Dragon Head

The movement of the dragon body and tail is solved by means of recursion. The position of the handle immediately behind the head stool, as well as the positions of each handle of the stools of the dragon body and tail at each time point, are determined. Every time node is traversed, and a loop calculation is carried out for each row to obtain the position information of each handle at this time. Some of the resulting partial results are presented in Table 1:

Table 1: Coordinates of the Dragon Dance Team

	0s	60s	120s	180s	240s	300s
dragon head x(m)	8.800000	5.799209	-4.084888	-2.963608	2.594492	4.420274
dragon head y(m)	0	-5.771092	-6.304479	6.094781	-5.356744	2.320429
The first dragon body x(m)	8.363824	7.456758	-1.445473	-5.237117	4.821220	2.459488
The first dragon body y(m)	2.826544	-3.440399	-7.405882	4.359628	-3.561951	4.402476
The 51st dragon body x(m)	-9.518732	-8.686317	-5.543149	2.890456	5.980010	-6.301346
The 51st dragon body y(m)	1.341137	2.540108	6.377946	7.249289	-3.827760	0.465829
The 101st dragon body y(m)	2.913983	5.687115	5.361938	1.898794	-4.917373	-6.237723
The 101st dragon body x(m)	-9.918311	-8.001384	-7.557638	-8.471614	-6.379873	3.936007
The 151st dragon body x(m)	10.861726	6.682312	2.388757	1.005155	2.965380	7.040740
The 151st dragon body y(m)	1.828753	8.134544	9.727411	9.424751	8.399720	4.393014
The 201st dragon body x(m)	4.555102	-6.619663	-10.62721	-9.287720	-7.457153	-7.458662
The 201st dragon body y(m)	10.725118	9.025570	1.359848	-4.246672	-6.180724	-5.263385
Dragon's tail x(m)	-5.305444	7.364557	10.974348	7.383896	3.241053	1.785032
Dragon's tail y(m)	-10.676584	-8.797992	0.843473	7.492370	9.469336	9.301164

Based on the previously acquired position information, the portion of the Archimedean spiral prior to point A was likewise appropriately expanded. The visualization motion model diagram of the

"Bench Dragon" at the 300th second was constructed, as depicted in Figure 7. It can be discerned from the figure that at the 300th second, the "Bench Dragon" had not completely entered point A.

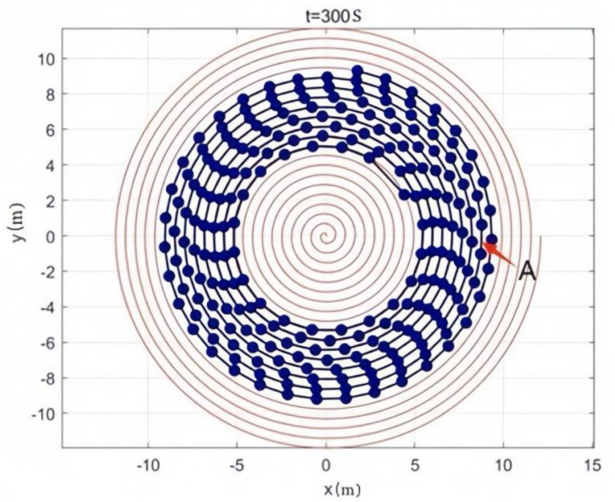

Figure 7: Visualization Motion Model Diagram of the "Bench Dragon" at the 300th Second

The movement velocity of any handle of the bench at any given moment is determined by the Finite Difference Method. To guarantee the accuracy of the calculation, the obtained result needs to be verified. The specific approach is to calculate the movement velocity of the handle in front of the faucet at any given moment using the Finite Difference Method and determine whether its velocity is within the acceptable error range and equal to 1 m/s.

By solving with Matlab, the time step was set at 0.1 s. The velocity results of the faucet at 0 s, 60 s, 120 s, 180 s, 240 s, and 300 s, the first dragon body, the 51st dragon body, the 101st dragon body, the 151st dragon body, the 201st dragon body, and the rear part of the dragon tail in the extracted results are presented in Table 2 as shown.

Table 2: Results Table of the Speeds of the Dragon Dance Team

	0s	60s	120s	180s	240s	300s
dragon head(m/s)	1.000057	1.000000	1.000000	0.999999	0.999998	0.999825
The first dragon body(m/s)	1.000027	0.999961	0.999945	0.999916	0.999857	0.999538
The 51st dragon body(m/s)	0.999789	0.999662	0.999538	0.999330	0.998939	0.997957
The 101st dragon body(m/s)	0.999616	0.999453	0.999269	0.998970	0.998434	0.997223
The 151st dragon body(m/s)	0.999484	0.999180	0.999077	0.998727	0.998113	0.996800
The 201st dragon body(m/s)	0.999380	0.999136	0.998934	0.998551	0.997892	0.996524
Dragon's tail(m/s)	0.999341	1.000000	0.998882	0.998488	0.997815	0.996431

5. Conclusions

This paper conducts an in-depth exploration of the mathematical motion model of the folk activity "Bench Dragon", and undertakes a detailed study on its movement characteristics along the trajectory of the Archimedean Spiral. This work pertains to the scientific and quantitative analysis of traditional folk culture. By introducing mathematical tools such as Differential Equations, Finite Difference Methods, and the Runge-Kutta Method, a dynamic model capable of accurately depicting the position and velocity variation rules of each part of the "Bench Dragon" has been established. To ensure the validity and accuracy of the model, a considerable number of numerical calculations were carried out. Through adjusting parameters and continuously optimizing the algorithm, stable and reliable simulation results were ultimately obtained. It is hoped that this mathematical model can not only be applied to entertainment activities such as "Bench Dragon" and small train performances in shopping malls, but also be utilized to calculate the motion states of trains, mine carts, and even microscopic particles when they move along the Archimedean Spiral trajectory. In terms of future development directions, with the continuous advancement of computer technology and numerical simulation methods, the existing model can be further refined to accommodate more complex motion scenarios. For instance, combining machine learning algorithms to achieve intelligent optimization of path selection; or exploring the motion characteristics under other types of spiral paths.

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