Localization and Optimization of Debris Sonic Booms Based on Greedy Algorithm and TOA Technology

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Abstract. With the rapid development of aerospace technology, due to the complexity of Time synchronization and multi-parameter solution, the traditional Time of Arrival (TOA) method is inefficient and inaccurate in the case of multi-source, and more efficient and accurate algorithms are urgently needed to improve the positioning performance and reliability. Specifically, this study employs TOA technology to construct the equation and employs the least square method to transform it into an optimization problem, solving the location and time of a single debris sonic boom are obtained as (110.498°E,27.314°N,1153m) and 19.094s. Then, based on the constructed model, the location and time of the four groups of debris are obtained using the greedy algorithm, in which the time at (110.500°E, 27.950°N, 11528m) is 13.002s. Finally, by comparing the theoretical arrival time and actual arrival time of the debris sonic boom, the error of the results is within 6×10-4s, which proves the feasibility and accuracy of the method for accurately locating the location and time of multiple debris sonic booms, providing accurate tracking and positioning technology for rocket debris and sonic boom events, and further meeting the rapid needs of safety monitoring.

Keywords: Debris sonic boom, greedy algorithm, TOA technology, least square method.

1. Introduction

As aerospace technology progresses at a rapid pace, the debris that is created during the launch, operation, and return of spacecraft presents a potential hazard to the environment and safety. When this debris travels through the atmosphere at high speeds, it generates sonic booms, so it is important to accurately locate the source of the sonic boom (longitude, latitude, elevation) and the occurrence of time, to ensure the safety of spacecraft and ground facilities timely [1]. However, debris sonic boom localization faces the challenge of time synchronization of monitoring equipment and complex multi-parameter solutions. Currently, the majority of research predominantly depends on the traditional Time of Arrival (TOA) technology. While it can achieve relatively accurate positioning for a single sonic boom source, it commonly encounters issues of high computational complexity and low positioning accuracy when dealing with multiple debris sonic boom sources and time synchronization errors [2-3]. Furthermore, as the number of sonic boom sources rises, traditional methods struggle to fulfill the practical application requirements in terms of real-time capabilities and robustness [4]. Therefore, in view of these shortcomings, this study introduces a more efficient and accurate algorithm and establishes a set of accurate, fast and efficient landing site calculation model to improve the performance and reliability of sonic boom location [5].

This study employs TOA technology is used to establish the equation of location and time of the sonic boom point of the debris. Then, considering the error of equipment monitoring, the least square method is used to transform the solution equation into the optimization minimization problem to construct the optimization model. Accordingly, according to the coordinates of 7 monitoring equipment collected by the network and the relevant data for detecting the arrival time of debris sonic boom, 4 different combinations of monitoring equipment are arranged, corresponding target values are calculated for each group, and the equipment combination corresponding to the minimum value is found as the best monitoring equipment combination, and then the coordinates and time of the debris sonic boom of the can be solved. Then, considering the case of multiple debris sonic booms, it is difficult to determine the location and time of each debris sonic boom because the monitoring equipment will measure multiple groups of debris sonic boom arrival times. Therefore, based on the constructed model and the relevant data of the network, the greedy algorithm is adopted to obtain the coordinates and time of each debris sonic boom in turn, thus reducing the computational complexity and improving the positioning accuracy. Finally, the theoretical time of the sonic boom arriving at each monitoring equipment is calculated and compared with the real-time to verify the accuracy of the model algorithm.

2. 3D Localization Techniques for Debris Sonic Booms

2.1. Model building

This study data from http://www.m2ct.org/index.jsp. To achieve accurate positioning of the location (longitude, latitude, elevation) and time of the debris sonic boom, it depends on the location of the monitoring equipment and the arrival time of the debris sonic boom measured by them. Since there are 4 unknown parameters to be solved, theoretically 4 equations need to be established to solve 4 unknown parameters, so at least 4 monitoring equipment need to be arranged. TOA technology is a range-based positioning algorithm with relatively high positioning accuracy [6].

Next, a debris sonic boom location model is established by TOA technology. The steps are as follows:

First, assume the coordinates of the ith monitoring equipment are $(\lambda_i, \varphi_i,\ and\ z_i)$. Where λ_i is the longitude of the position of the ith monitoring equipment, φ_i is the latitude of the position of the ith monitoring equipment, and z_i is the elevation of the position of the ith monitoring equipment.

Next, the debris sonic boom location model is established by TOA technology. Further, assume the coordinates of the single debris sonic boom are (λ, φ, z), where, λ is the longitude of the position of the debris sonic boom, φ is the latitude of the position of the debris sonic boom, and z is the elevation of the position of the debris sonic boom.

In the positioning process, the monitoring device provides latitude and longitude information, which needs to be converted into two-dimensional XY coordinates for positioning, and finally the XY coordinates are reversely calculated as latitude and longitude [7]. Therefore, the standard coordinates of sonic boom points of monitoring equipment and debris can be obtained, as shown in Equation 1.

$$\begin{cases} x = \lambda \cdot dj \\ y = \varphi \cdot dw \end{cases} \quad \begin{cases} x_i = \lambda_i \cdot dj \\ y_i = \varphi_i \cdot dw \end{cases} \tag{1}$$

Where, $x, and\ y$ are the horizontal and vertical coordinates of the debris sonic boom respectively, $x_i,\ and\ y_i$ are the horizontal and vertical coordinates of the ith monitoring equipment respectively, dj represents the approximate distance value per longitude, which is 97.304 km, dw represents the approximate distance value per latitude, which is 111.263 km.

Then, according to the coordinate information, the distance d_i between the location of the debris sonic boom and the location of the monitoring equipment can be obtained, as shown in Equation 2.

$$d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} \tag{2}$$

Finally, according to the debris sonic boom time measured by the equipment, the distance d_i can also be calculated, as shown in Equation 3.

$$d_i = c(t_i - t) \tag{3}$$

To sum up, according to the distance obtained by the two methods, the equations of TOA technology for solving the coordinates and time of the debris sonic boom can be obtained, as shown in Equation 4.

$$\begin{cases} \sqrt{(x_1-x)^2+(y_1-y)^2+(z_1-z)^2}=c(t_1-t) \\ \sqrt{(x_2-x)^2+(y_2-y)^2+(z_2-z)^2}=c(t_2-t) \\ \sqrt{(x_3-x)^2+(y_3-y)^2+(z_3-z)^2}=c(t_3-t) \\ \sqrt{(x_4-x)^2+(y_4-y)^2+(z_4-z)^2}=c(t_4-t) \end{cases} \tag{4}$$

To synthesize the above equation, the total equation is shown in Equation 5.

$$\sqrt{(x_i-x)^2+(y_i-y)^2+(z_i-z)^2}=c(t_i-t) \tag{5}$$

Thus, a model has been established for the location of the debris sonic boom by the TOA technology.

2.2. Model solving

Relevant data required for model solving are shown in Table 1.

Table 1. The location of 7 equipment and the measured debris sonic boom arrival time

Equipment	Longitude(°E)	Latitude(°N)	Elevation(m)	Debris sonic boom arrival time (s)
A	110.241	27.204	824	100.767
B	110.780	27.456	727	112.220
C	110.712	27.785	742	188.020
D	110.251	27.825	850	258.985
E	110.524	27.617	786	118.443
F	110.467	27.921	678	266.871
G	110.047	27.121	575	163.024

Since there may be errors in the data monitored by the equipment, the solution of the equations may not be solved. Therefore, the study introduces the least square method to convert the solution equation into an optimal minimization problem. The least square method is a mathematical optimization algorithm, it uses the squared error minimization to find the best match for the data. The process of solving unknown variables by least square method is relatively simple, and the sum of squares of errors between the obtained results and the actual data is the minimum, it is used in many positioning algorithms [8-9].

For this study, the least square method can be used to make the difference between the two distance modes and calculate the sum of squares to achieve the transformation of the optimal minimum value problem. The corresponding objective function is shown in Equation 6.

$$\min f=\sum_{i=1}^{4}[\sqrt{(x_i-x)^2+(y_i-y)^2+(z_i-z)^2}-c(t_i-t)]^2 \tag{6}$$

Where, f is the objective function, and $x, y, and\ z$ are the decision variables of the optimization model.

Next, the constraint conditions are established, as shown in Equation 7.

$$x, y, z, t>0 \tag{7}$$

Due to the relevant data of 7 monitoring equipment, the location and time of the debris sonic boom can be solved by only 4 equipment according to the above analysis. Therefore, this study arranges 7 equipment into different combinations of 4 equipment to obtain a total of $35(C_7^4)$ combinations. The objective function of each combination is calculated, and the combination with the smallest objective function is regarded as the best monitoring equipment combination.

Then Using MATLAB's fmincon function to solve this problem, the results obtained are shown in Figure 1.

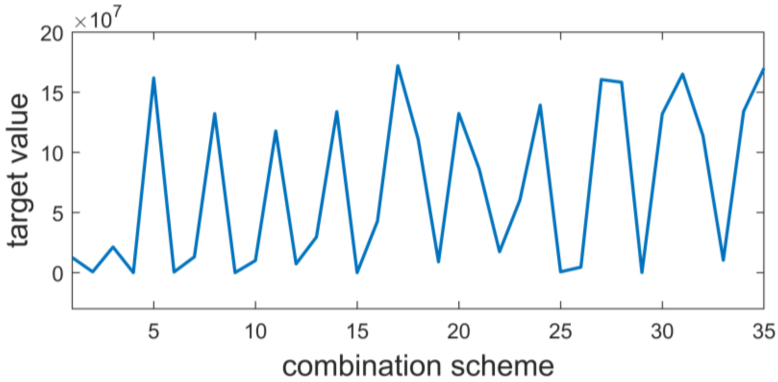

Figure 1. Target values for different combination

According to the figure above, it can be evidently found that the monitoring equipment combination corresponding to the minimum target value is specifically A, B, E, and G, and the coordinates and time of the debris sonic boom obtained from the monitoring data of this group are shown in Table 2.

Table 2. Location and time of the debris sonic boom

	Longitude (°E)	Latitude (°N)	Elevation (m)	Time (s)
Debris sonic boom	110.498	27.314	1153	19.094

Visualize the results, and the 3D figure obtained is shown in Figure 2.

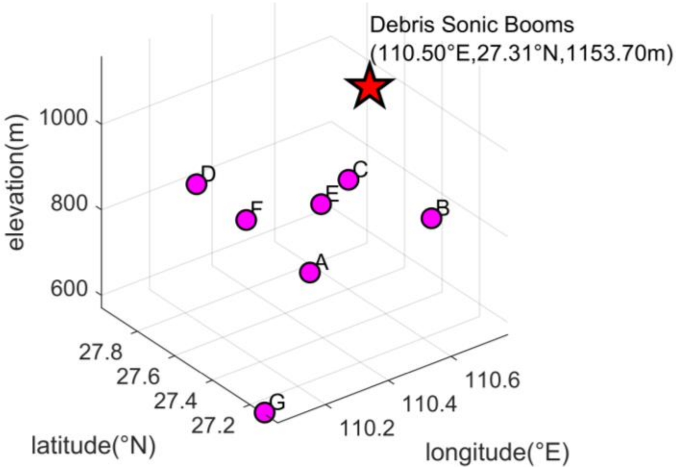

Figure 2. 3D coordinates of the single debris sonic boom

2.3. Error analysis

Given the above process, the location and time of the debris sonic boom are obtained, and these data are substituted into Equation 8 to obtain the theoretical sonic boom time of each monitoring equipment.

$$T_i = t + \frac{\sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}}{c}, i = 1,2,3,4 \tag{8}$$

Where T_i represents the theoretical time for the debris sonic boom to reach the ith monitoring equipment.

The results obtained are shown in Figure 3.

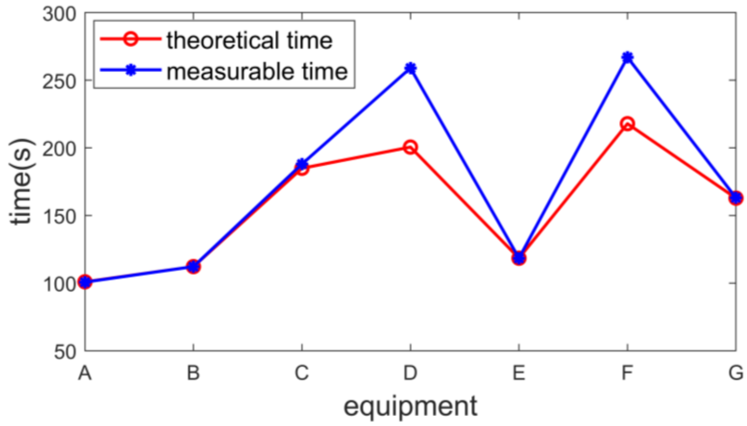

Figure 3. Comparison of the theoretical and measurable time of the debris sonic boom

By observing the figure above, it can be found that the theoretical debris sonic boom arrival time calculated by selecting four equipment A, B, E, and G is consistent with the measurable time, which verifies the rationality of the model and solution in this study. However, it is found that the time error on D and F equipment is relatively large, and the corresponding theoretical time is smaller than the measurable time. To analyze the reasons, the comparison chart of the debris sonic boom arrival time and time error measured by the equipment is drawn and sorted, as shown in Figure 4.

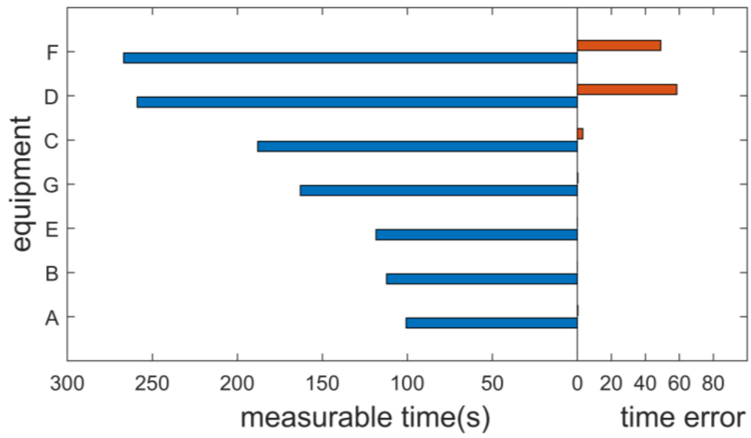

Figure 4. Comparison of the debris sonic boom arrival time and time error

The left side of the zero axis in Figure 4 shows the arrival time of the debris sonic boom measured by different equipment. The longer the time, the farther the monitoring equipment is from the location of the debris sonic boom. The right side of the zero axis is the time error (the value where the measurable time is less than the theoretical time).

It is found above that the measurable time of the two equipment D and F is greater than the theoretical time. The reason may be that the two pieces of equipment are far away from the debris sonic boom location, and the shock wave may encounter obstacles on the way to the two pieces of equipment, or other factors may cause the distance of the shock wave propagation to be longer, so the debris sonic boom measurable arrival time is larger than the theoretical arrival time.

Since the above process is only to solve the location and time of a single debris sonic boom, but in the actual process, there are often multiple debris sonic booms, in which case it is difficult to achieve the accurate location of sonic booms. Therefore, this study continues to study the location method when multiple debris sonic bursts occur. Since the above process is only to solve the location and time of a single debris sonic boom, but in the actual process, there are often multiple debris sonic booms, in which case it is difficult to achieve the accurate location of sonic booms. Therefore, this study continues to study the location method when multiple debris sonic bursts occur.

3. Multiple debris location and time measurement

3.1. Model building

Considering the actual situation, there will be multiple rocket debris at the same time, and the monitoring equipment may collect several sets of sonic boom data within the monitoring range, and these measurement data can't be determined from which debris sonic boom. Therefore, this study further studies how to accurately locate the location and time of multiple debris sound bursts.

First of all, based on the model constructed above, the equations to be solved are listed, as shown in Equation 9.

$$\sqrt{(x_i - X_j)^2 + (y_i - Y_j)^2 + (z_i - Z_j)^2} = c(t_{ij} - T_j) \tag{9}$$

Where, (X_j, Y_j, Z_j) and T_j are the standard coordinates and time of the jth debris sonic boom, (x_i, y_i, z_i) are the standard coordinates of the ith monitoring equipment, and t_{ij} is the jth debris sonic boom time measured by the ith monitoring equipment.

3.2. Model solving

The relevant data required for model solving are shown in Table 3.

Table 3. The location of 7 equipment and the measured debris sonic boom arrival time

Equipment	Longitude	Latitude	Elevation	Debris sonic boom arrival time			
A	110.241	27.204	824	100.767	164.229	214.850	270.065
B	110.783	27.456	727	92.453	112.220	169.362	196.583
C	110.762	27.785	742	75.560	110.696	156.936	188.020
D	110.251	28.025	850	94.653	141.409	196.517	258.985
E	110.524	27.617	786	78.600	86.216	118.443	126.669
F	110.467	28.081	678	67.274	166.270	175.482	266.871
G	110.047	27.521	575	103.738	163.024	206.789	210.306

For the equations established above, the least square method is also used to convert them into the problem of optimal minimization. Each debris solution has its corresponding objective function, and the expression of the jth objective function is shown in Equation 10.

$$F_j = \sum_{i=1}^{4} [\sqrt{(x_i - X_j)^2 + (y_i - Y_j)^2 + (z_i - Z_j)^2} - c(t_i - T_j)]^2,\ j = 1,2,3,4 \tag{10}$$

Next, it is necessary to solve the coordinates and time of the sonic boom of each debris, which is extremely difficult to solve because it is not possible to determine which debris corresponds to the four sonic boom arrival times measured by each monitoring equipment. Therefore, this study proposes a greedy algorithm to solve the problem. Greedy algorithm is a hierarchical processing method that can obtain the optimal solution in a certain metric sense, and the optimal selection is carried out step by step through dynamic analysis, that is, greedy strategy is a local optimal solution algorithm in a certain sense [10]. The algorithm steps are as follows:

First, the first sonic boom arrival time measured by the first monitoring equipment is fixed to certain debris, and the remaining three monitoring equipment are arranged and combined with the four groups of sonic boom arrival time measured by the remaining three monitoring equipment. Then, one of the permutation combinations is assumed to be that of this debris. Next, through the internal loop, the above steps are repeated to calculate the objective function values of all combinations, and the combination corresponding to the minimum value is found to be the debris sonic boom arrival time measured by four monitoring equipment, and the location and time of the debris are solved. Finally, for the remaining unconfirmed measurement data, the above steps are repeated through the external loop to obtain the location and time of the remaining debris sonic boom in turn.

Through the greedy algorithm steps above, this study uses MATLAB to solve the problem, and the debris situation corresponding to the measurement data of each monitoring equipment is obtained, as shown in Table 4, The location and time of the 4 debris sonic booms are shown in Table 5.

Table 4. Debris situation corresponding to the measurement data of each equipment

Equipment	Arrival time				From which debris			
A	100.767	164.229	214.850	270.065	1	2	3	4
B	92.453	112.220	169.362	196.583	3	1	2	4
C	75.560	110.696	156.936	188.020	3	4	2	1
D	94.653	141.409	196.517	258.985	4	2	3	1
E	78.600	86.216	118.443	126.669	3	2	1	4
F	67.274	166.270	175.482	266.871	4	2	3	1
G	103.738	163.024	206.789	210.306	2	1	4	3

Table 5. Location and time of the four debris sonic booms

Debris	Longitude (°)	Latitude (°)	Elevation (m)	Time (s)
1st debris	110.500	27.310	12514	12.000
2nd debris	110.230	27.650	11478	13.999
3rd debris	110.700	27.650	13468	15.000
4th debris	110.500	27.950	11528	13.002

Visualize the results obtained above, as shown in Figure 5.

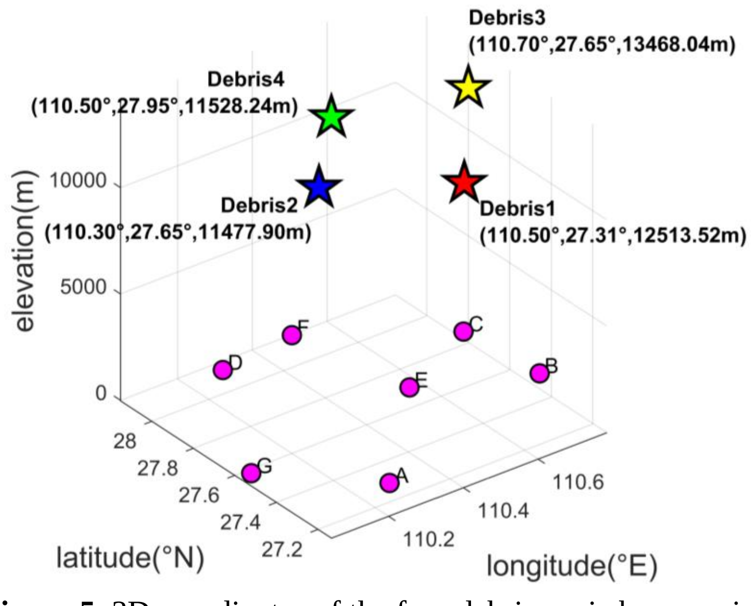

Figure 5. 3D coordinates of the four debris sonic boom points

3.3. Error analysis

According to Equation 8, the theoretical time for the arrival of the debris sonic boom at each monitoring equipment is calculated, and the time error obtained is compared with the actual measured time, as shown in Figure 6.

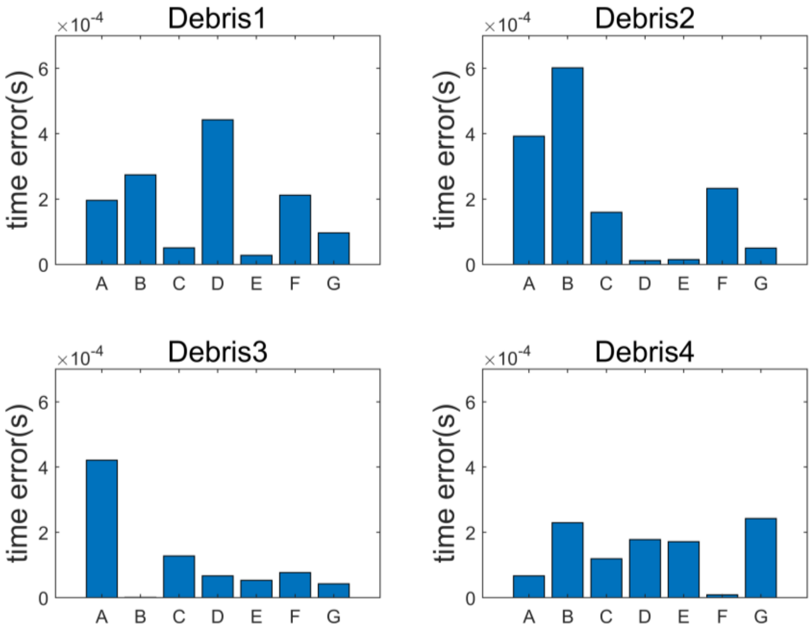

Figure 6. Time error diagram of the sonic boom of each debris

By analyzing the figure above, it is found that the difference between the actual arrival time of each debris measured by monitoring equipment and the theoretical arrival time is within 6×10^{-4}s, which indicates that the accuracy of the model established in this study is good, and the rationality and accuracy of the solution idea of the greedy algorithm is also verified.

4. Conclusion

In this study, the optimal model was constructed by combining TOA technology and the least square method. By arranging and combining the data of 7 monitoring equipment, the optimal 4 equipment combinations were selected, and the precise location and time of a single debris sonic boom were successfully determined. Further, based on the optimization model and greedy algorithm, the location and time of each debris sonic boom are determined, which effectively solves the problem of location and time confirmation of multiple debris sonic booms.

In this study, a research idea and framework are proposed to solve the problem that the traditional TOA technology makes it difficult to accurately measure the sonic boom location of multiple debris. The research confirms the feasibility of accurate location and time of multiple debris sonic booms by greedy algorithm, which provides an innovative technical means for accurate tracking and positioning of debris landing points and sonic boom events after rocket launch, and further meets the urgent demand for rapid response and accurate evaluation in the field of rocket launch safety monitoring. However, it is worth noting that there is still room for improvement in the error analysis of this study. At present, the error of monitoring equipment is mainly considered, but other sources of error have not been deeply explored. Therefore, future work can further analyze the influence of wind speed, temperature, humidity, and other factors on the measurement results, and further improve the accuracy of evaluation and model performance.

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