Research on dynamic path simulation and collision detection based on the isometric spiral motion model

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Abstract. Aiming at the traditional folk cultural activity "Bench Dragon" in Zhejiang and Fujian regions of China, this paper establishes an isometric spiral motion model and conducts an in-depth study on its dynamic path simulation and collision detection in the process of spiral entry. Firstly, the spiral motion model of the dragon dance team is established, and the positions and velocities of each part of the dragon dance team at different moments are accurately calculated by means of parametric equations and iterative algorithms. Secondly, the collision detection algorithm is established. The motion state of the dragon dance team is monitored in real time, and the relative speed and relative position changes between adjacent benches are calculated to determine whether there is a collision, so as to determine the termination moment of the dragon dance team's disc entry and give the speed and position information of the dragon dance team at this time. The experimental results show that the method proposed in this paper can effectively simulate the spiral motion trajectory of the dragon dance team and accurately detect the collision between the benches, so as to provide theoretical basis and technical support for the safety of the dragon dance team.

Keywords: Isometric Spiral, Dynamic Path Simulation, Iterative Algorithm, Collision Detection.

1. Introduction

"Bench Dragon" is a traditional local folk culture activity in Zhejiang and Fujian regions of China. It is deeply loved by people for its unique performance and exquisite skills. People put at least dozens, sometimes hundreds of benches end to end, forming a winding bench dragon. When the dragon is coiled, the dragon head is in the front, the dragon body and the dragon tail hover together, and the whole is in the shape of a disc. Generally speaking, under the premise that the dragon dance team can freely enter and disc out, the smaller the area required by the coiling dragon and the faster the traveling speed, the better the viewing.

In this paper, after simplifying the model of this folk skill, it can be concluded that the dragon dance team carries out the movement of disc entry and disc exit along the equidistant spiral. In the process of spiral entry, due to the limitation of space size and bench length, in order to avoid collision, the dragon dance team will need to terminate the disc entry movement at some point, which puts forward higher requirements for the movement time and path control of the dragon dance team. However, the existing research mainly focuses on the artistic and technical aspects of dragon dance, and the research on its motion trajectory and collision detection is less. This paper aims to establish spiral motion model of dragon dance team, simulate and design collision detection algorithm by using parametric equation and iterative algorithm, so as to provide theoretical basis and technical support for the safety of dragon dance team performance. In the study of this paper, the bench dragon consists of 223 benches, of which the first is the head, the back is 221 sections of the body, and the last section is the tail. The board length of the head is 341 cm, the board length of the body and tail of the dragon is 220 cm, and the board width of all the benches is 30 cm. Each bench has two holes with a diameter of 5.5 cm, and the center of the hole is 27.5 cm from the nearest plate head (Refer to Figures 1 and 2). Two adjacent benches are connected by a handle (Refer to Figure 3).

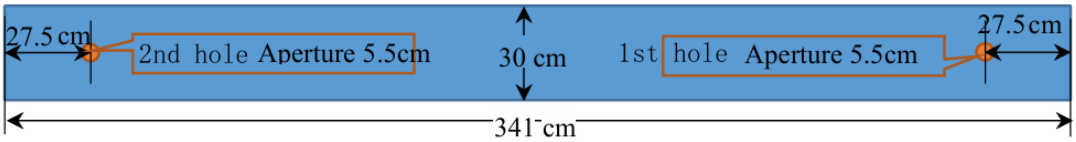

Figure 1. Top view of the dragon head

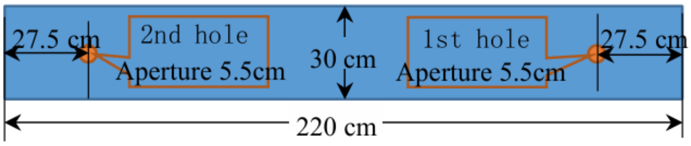

Figure 2. Top view of the dragon body and tail

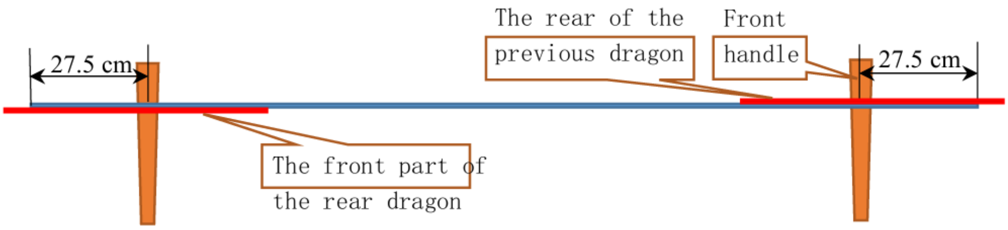

Figure 3. A front view of the bench

This paper focuses on solving the following two questions:

Question 1: The dragon dance team entered clockwise along the isometric spiral with A pitch of 55cm, and the speed of the front handle of the faucet is always maintained at 1m/s. The faucet is initially located at point A of the 16th circle of the spiral (Refer to Figure 4). Figure out the position and speed of the whole dragon dance team per second from the initial moment to 300s, and give the position and speed of the front handle and the rear handle of the dragon head, the dragon body and the dragon tail at a specific moment in the paper.

Question 2: The dragon dance team will disc in along the spiral set in Question 1, determine the termination time of the dragon dance team's disc in, so that there is no collision between the benches, and give the position and speed of the dragon dance team at this time, and give the position and speed of the relevant handles at this time in the paper.

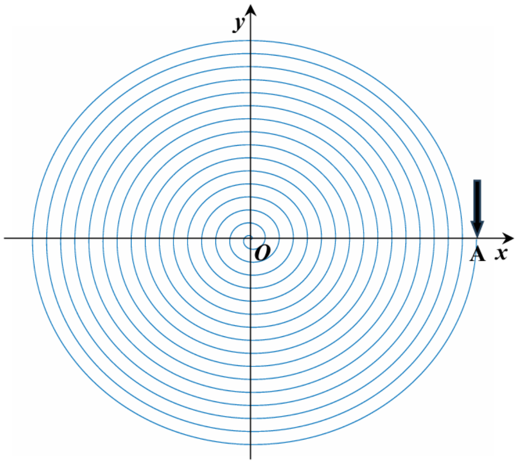

Figure 4. Disk entry spiral diagram

2. Data and research methods

The physical parameters of the Bench Dragon and the pitch of the motion curve in Figure 1 to Figure 4 in this paper are approximate values simulated after field visits to this folk activity. These data will be used to solve the model in this paper.

The main steps of the research method in this paper are as follows: (1) Firstly, parametric equation modeling is used to establish a parametric equation of the position of the handle in front of the dragon head changing with time according to the movement law and geometric constraints of the dragon dance team [1], and the law of the position and speed of each handle in the dragon body and tail changing with time is derived. (2) Iterative algorithm. Iterative algorithm is used to solve the parametric equation [2], and the position and speed of each part of the dragon dance team at different moments are obtained. (3) Collision detection algorithm. The collision detection algorithm is designed to calculate the relative speed and position change between adjacent benches to determine whether collision occurs and determine the end time of the dragon dance team. (4) Data analysis and visualization. The simulation results are analyzed, and the movement trajectory and collision detection results of the dragon dance team are displayed by visualization tools [3-4].

3. The establishment and solution of the model

3.1. Model assumption

(1) Assume that the posture of all benches remains unchanged during the movement of the dragon dance team, that is, there will be no rotation or tilt.

(2) Assume that the movement of the dragon dance team is limited only by the given conditions, without considering the influence of other factors not mentioned.

(3) It is assumed that the shape and parameters of the spiral remain stable and will not change when the dragon dance team coils in and out of the spiral.

(4) It is assumed that the friction between the parts of the dragon dance team is negligible and will not affect the movement of the parts.

(5) Assume that in the process of turning around, the connection of each arc is smooth, and there will be no sudden turning or shaking.

(6) Assume that the movement of the dragon dance team is continuous throughout the whole process, and there will be no instant pauses or jumps.

(7) It is assumed that the dragon dance team will not interact with the surrounding environment in any form during its movement, except for contact with the boundary of the turning space.

3.2. Symbol specification

The units used in this paper are shown in Table 1:

Table 1. Symbol specification

Mathematical Symbol	Paraphrase	Units
p	Pitch of the spiral	m
b	Spiral parameter	m/rad
r	Spiral pole diameter	m
v	Velocity of motion	m/s
θ	The polar Angle of the spiral	rad
l	The distance between the front and back handles of a bench	m
d_1	Distance between the center of the handle and the nearest board head	m
d_2	Half the width of the board	m

3.3. The establishment and solution of the model for the Question 1

3.3.1. Data preprocessing

In this model, the parameters of the spiral structure and the physical structure of the dragon are first defined and initialized. The pitch of the spiral structure is set to $p_1 = 0.55m$, the initial speed is

$v_0 = 1m/s$, the simulation time range is 0 to 300 seconds, the time interval is 1 second, and the position of the handle is set to 0, 60, 120, 180, 240, 300 seconds.

In addition, the physical structural parameters of the dragon include the board length of the dragon head is $D_0 = 3.41m$, the board length of the dragon body and tail is $D_1 = 2.20m$, and the distance between the center of the handle and the nearest board head is $d_1 = 0.275m$.

Therefore, the distance between the front and back handles of the dragon head is $L_0 = 3.41 - 0.275 \times 2 = 2.86m$, and the distance between the front and rear handles of the dragon bodies and the tail is $L_1 = 2.20 - 0.275 \times 2 = 1.65m$. The initial Angle θ_0 is set to 16 turns and converted to radians as: $\theta_0 = 16x2\pi = 32\pi$. These parameters are used to adjust the actual position of each dragon segment.

The time step is set from 0 to 300 seconds, and the space segment index i is from 0 to 223, representing the segments of the dragon from the head to the tail.

3.3.2. Establishment of the spiral motion model

(1) Determine the center position of the front handle of the faucet

From the known pitch $p_1 = 0.55m$ and the starting point A$(8.8, 0)$, this paper can establish the polar coordinate equation corresponding to the disk entry spiral:

$$r(\theta) = b_1\theta \tag{1}$$

The calculation of spiral motion parameter pitch b_1 is based on the following formula:

$$b_1 = \frac{p_1}{2\pi} = \frac{0.55}{2\pi} \approx 0.0875m/rad \tag{2}$$

Thus, there is

$$r(\theta) = \frac{p_1}{2\pi}\theta = \frac{0.55}{2\pi}\theta \tag{3}$$

The parameter b_1 is used to calculate the position, speed and other variables of the head and the dragon body.

Since the traveling speed of the handle in front of the tap along the disk into the spiral is constant $v_0 = 1m/s$, from the initial time $t = 0s$ to $t = t_0 \in [0,300]$, the path length of the handle in front of the tap is $v_0 t_0$, and the Angle of the pole Angle changes from $\theta = 32\pi$ to $\theta = \theta_0$. According to the integral formula of arc length of curve, the following relation can be obtained:

$$v_0 t_0 = \int_\alpha^\beta \sqrt{r^2(\theta) + (r'(\theta))^2} d\theta \tag{4}$$

From the known t_0, the corresponding θ_0 can be solved, and then through the transformation formula of polar coordinates and cartesian coordinates:

$$\begin{cases} x_0 = r(\theta_0) \cos\theta_0 \\ y_0 = r(\theta_0) \sin\theta_0 \end{cases} \tag{5}$$

Get the central coordinates of the front handle (x_0, y_0) at $t = t_0$

(2) Establish the position iteration formula

After obtaining the coordinates of the center of the handle in front of the faucet at each moment, this paper establishes a position iteration formula to calculate the coordinates of each handle at each moment [5-7]. The establishment process is as follows:

Assume that at a certain moment, the center distance of the front and back handles of a certain bench is t, the center position coordinates of the front handles are (x_1, y_1), the pole Angle is $\theta = \theta_1$, and the pole diameter is $r = r_1$. Suppose that the coordinate of the center position of the handle behind the bench at this time is (x_2, y_2), and the pole Angle is $\theta = \theta_2$. According to the spiral equation, its pole diameter satisfies the equation:

$$r_2 = r_1 + \frac{p_1}{2\pi}(\theta_1 - \theta_2) \tag{6}$$

In the triangle shown in Figure 5, the following formula can be obtained through law of cosines:

$$r_1{}^2 + r_2{}^2 - 2r_1r_2\cos(\theta_2 - \theta_1) = l^2 \tag{7}$$

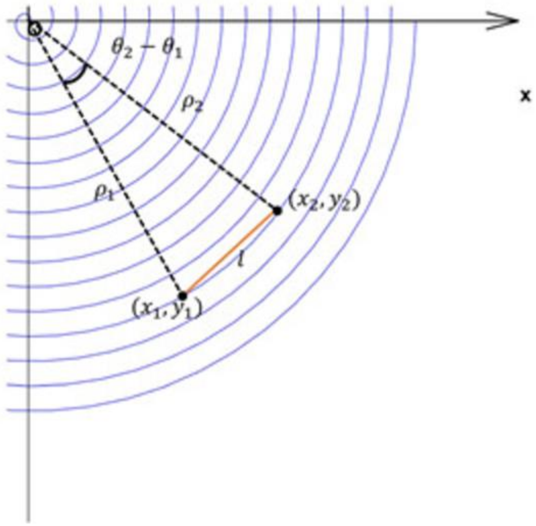

Figure 5. Diagram of the relative position of the front and rear handles

Simultaneous equation (4) and (5), adopt the method of dichotomy for zero solution, and then through coordinate transformation formula:

$$\begin{cases} x_2 = r_2\cos\theta_2 \\ y_2 = r_2\sin\theta_2 \end{cases} \tag{8}$$

Get the coordinates of the position of the handle behind the bench (x_2, y_2).

Through the center position coordinates of the front handle of the dragon head at every moment, this paper can solve the position of the handle of the whole dragon dance team at every moment by using the iterative formula.

(3) Solution of velocity

For the convenience of the model study, this paper approximates that the velocity of each handle meets $v = 1m/s$.

3.3.3. Model calculation results

The results are shown in Table 2 and Table 3:

Table 2. Position results

	0 s	60 s	120 s	180 s	240 s	300 s
$i = 0$ X(m)	8.8	5.799209	-4.084887	-2.963609	2.594494	4.420274
$i = 0$ Y(m)	0	-5.771092	-6.304479	6.094780	-5.356743	2.320429
$i = 1$ X(m)	8.363824	7.456758	-1.445473	-5.237118	4.821221	2.459489
$i = 1$ Y(m)	2.826544	-3.440399	-7.405883	4.359627	-3.561949	4.402476
$i = 51$ X (m)	-9.518732	-8.686317	-5.543149	2.890455	5.980011	-6.301346
$i = 51$ Y(m)	1.341137	2.540108	6.377946	7.249289	-3.827758	0.465829
$i = 101$ X(m)	2.913983	5.687116	5.361939	1.898795	-4.917371	-6.237722
$i = 101$ Y (m)	-9.918311	-8.001384	-7.557638	-8.471614	-6.379874	3.936008
$i = 151$ X (m)	10.861726	6.682312	2.388757	1.005154	2.965378	7.040740
$i = 151$ Y (m)	1.828753	8.134544	9.727411	9.424751	8.399721	4.393013
$i = 201$ X (m)	4.555102	-6.619664	-10.627210	-9.287720	-7.457151	-7.458662
$i = 201$ Y (m)	10.725118	9.025570	1.359848	-4.246673	-6.180726	-5.263384
$i = 223$ X (m)	-5.305444	7.364557	10.9743489	7.383895	3.241051	1.785033
$i = 223$ Y (m)	-10.676584	-8.797992	0.843473	7.492371	9.469336	9.301164

Table 3. Velocity results

	0 s	60 s	120 s	180 s	240 s	300 s
The dragon head (m/s)	1	1	1	1	1	1
The first dragon body (m/s)	1	1	1	1	1	1
Section 51 Dragon Body (m/s)	1	1	1	1	1	1
Section 101 Dragon Body (m/s)	1	1	1	1	1	1
Section 151 Dragon Body (m/s)	1	1	1	1	1	1
Section 201 Dragon Body (m/s)	1	1	1	1	1	1
Dragon tail (rear) (m/s)	1	1	1	1	1	1

3.4. The establishment and solution of the model for the Question 2

3.4.1. Data preprocessing

In this model, a series of constants and initial conditions are first defined and initialized, which are used to describe the spiral motion characteristics of the dragon and its physical structure. As the dragon dance team continues to reel in along the spiral set in problem 1, the parameters used in Problem 1 above continue to be used.

The pitch is set to $p_1 = 0.55m$, indicating the vertical height of each circle of the spiral.

The initial speed is $v_0 = 1m/s$, indicating the movement speed of the faucet.

The distance between the center of the handle and the nearest plate head is $d_1 = 0.275m$, which is the distance threshold for collision detection.

The half-plate width is $d_2 = 0.15m$, which is the distance threshold for collision detection.

The initial conditions include:

The initial Angle θ_0 is set to 16 turns and converted to radians as: $\theta_0 = 16x2\pi = 32\pi$

Calculation of pitch parameter b:

$$b_1 = \frac{p_1}{2\pi} = \frac{0.55}{2\pi} \approx 0.0875m/rad \tag{9}$$

The physical structural parameters of dragons include:

The distance between the front and back handles of the faucet is $L_0 = 2.86m$

The distance between the front and rear handles of the dragon body and tail is $L_1 = 1.65m$

The time step is set from 0 to 300 seconds, and the space segment index i is from 0 to 223, representing the segments of the dragon from the head to the tail.

3.4.2. Establishment of the spiral motion model

This model is designed to simulate the movement of the dragon on the spiral trajectory, and calculate the position and speed of the dragon head and each segment through the iteration of time steps. The establishment of the model is divided into the following steps:

(1) Determination of the position of the external four corner points of the dragon head and the first section

For any bench, note that the distance between the center of the handle and the nearest board head is $d_1 = 0.275m$, half the width of the board is $d_2 = 0.15m$, the line connecting the center of the front and back handles is l_1, the line where the outer side of the board is a_3, the line where the center of the front and back handles and the nearest outer corner are a_2, a_4. According to the symmetry, the Angle between l_1 and a_2, a_4 is γ, as shown in Figure 6:

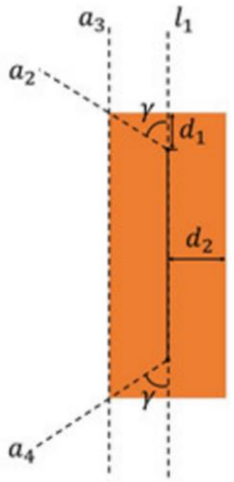

Figure 6. Diagram of the bench

The position of the external corner point A_1 in front of the faucet is determined as follows:

Assume that the central position coordinate of the handle in front of the tap at time t is (x_1, y_1), and the pole Angle $\theta = \theta_1$. According to the position iteration formula of problem 1, the position coordinates of the center of the handle behind the faucet are obtained as (x_2, y_2), and then the analytic formula of the line where the center of the two handles of the faucet is located is obtained.

$$l_1: y - y_1 = k_1(x - x_1) \qquad k_1 = \frac{y_1 - y_2}{x_1 - x_2} \tag{10}$$

Since a_2 is obtained by l_1 rotating γ counterclockwise around the center of the front handle, according to the formula of line rotation Angle, the analytical formula of a_2 is obtained as follows:

$$a_2: y - y_1 = k_2(x - x_1), \quad k_2 = \frac{\tan\gamma + k_1}{k_1 \tan\gamma - 1}, \ \tan\gamma = \frac{d_2}{d_1} \tag{11}$$

Since a_3 is obtained by translating d_2 from l_1 away from the central origin, the analytic expression for a_3 is given:

$$a_3: y = k_1 x + b \tag{12}$$

Where b meets the conditions:

$$\frac{|b - y_1 + k_1 x_1|}{\sqrt{1 + {k_1}^2}} = d_2 \ \ and \ |b| > |y_1 - k_1 x_2| \tag{13}$$

By combining a_2 and a_3, the coordinates of A_1 can be solved:

$$(x_{A1}, y_{A1}) = \left(\frac{y_1 - k_2 x_1}{k_1 - k_2}, \frac{k_1 y_1 - k_1 k_2 x_1 - k_2 b}{k_1 - k_2}\right) \tag{14}$$

Similarly, the coordinates of the outer corner A_2 behind the dragon at time t, the outer corner A_3 in front of the first dragon body, and the outer corner A_4 behind the first dragon body can be determined.

(2) The judgment of the collision of four corners

In step (1), the coordinates of the four corner points A_1, A_2, A_3 and A_4 are obtained by using the coordinates of the faucet front handle (x_1, y_1) determined at time t.

At the same time, the center coordinates (x_i, y_i) of each front handle of the polar Angle $\theta \in [\theta_1 + \frac{3\pi}{2}, \theta_1 + \frac{5\pi}{2}]$ at time t can be obtained through the results obtained from Question 1 (i means that the front handle belongs to the bench i, and the faucet is the first bench). Let the set of i satisfying that the front handle coordinates (x_i, y_i) fall within the required polar Angle range be I. By the formula for a line between two points, the analytical formula of the line l_i where the center line of the front and back handles of the bench i is located can be solved:

$$l_i = y - y_i = \frac{y_i - y_{i+1}}{x_i - x_{i+1}}(x - x_i) \tag{15}$$

After the analytic formula of corner coordinates and line l_i is obtained, the distance between corner A_j and line l_i is d_{ij}, that is:

$$d_{ij} = \frac{\left|\frac{y_i - y_{i+1}}{x_i - x_{i+1}}(x_{Ai} - x_i) - y_{Ai} + y_i\right|}{\sqrt{\left(\frac{y_i - y_{i+1}}{x_i - x_{i+1}}\right)^2 + 1}} \tag{16}$$

For whether collision occurs, this paper gives the following judgment criteria:

$$\begin{cases} \forall i, j(i \in I, j = 1,2,3,4), d_{ij} > d_2 \quad Then\ no\ collision\ occurs\ at\ time\ t \\ \exists i, j(i \in I, j = 1,2,3,4) \ \ lets\ to\ d_{ij} \leq d_2 \ \ Then\ a\ collision\ occurs\ at\ time\ t \end{cases} \tag{17}$$

From this, it is possible to determine the end time of the dragon dance team's disc entry [8-10].

3.4.3. Model calculation results

After substituting the data, this paper obtains through the program that the termination time is 412.473894s, and the collision point is the left front corner of the faucet. The position and speed results are shown in Table 4 and Table 5 below:

Table 4. Position results

Position result	End time 412.473894s
$i = 0$ X(m)	1.209931
$i = 0$ Y(m)	1.942784
$i = 1$ X(m)	-1.643792
$i = 1$ Y(m)	1.753399
$i = 51$ X(m)	1.281201
$i = 51$ Y(m)	4.326588
$i = 101$ X(m)	-0.536245
$i = 101$ Y(m)	-5.880138
$i = 151$ X(m)	0.968841
$i = 151$ Y(m)	-6.957479
$i = 201$ X(m)	-7.893161
$i = 201$ Y(m)	-1.230764
$i = 223$ X(m)	0.956216
$i = 223$ Y(m)	8.322736

Table 5. Velocity results

Velocity result	End time 412.473894s
$i = 0$ (m/s)	1.000000
$i = 1$ (m/s)	1.000000
$i = 51$ (m/s)	1.000000
$i = 101$ (m/s)	1.000000
$i = 151$ (m/s)	1.000000
$i = 201$ (m/s)	1.000000
$i = 223$ (m/s)	1.000000

4. Conclusion

After in-depth exploration of the dynamic path simulation and collision detection of the isometric spiral motion model of "Bench Dragon" in the spiral entry process, this paper draws the following conclusions: Firstly, the spiral motion model of the dragon dance team is successfully constructed in this paper. By using parametric equation and iterative algorithm, the position and speed of each part of the dragon dance team at different time points can be accurately calculated, which lays a solid foundation for the subsequent dynamic path simulation. Secondly, an efficient collision detection algorithm is designed and implemented in this paper. The algorithm can monitor the motion state of the dragon dance team in real time, and determine whether the collision occurs timely and accurately by accurately calculating the relative speed and relative position changes between adjacent benches. This algorithm not only determines the end time of the dragon dance team, but also provides the speed and position information of the dragon dance team, which provides a strong guarantee for the safety of the performance. The experimental results show that the method proposed in this paper can not only effectively simulate the spiral motion trajectory of the dragon dance team, but also accurately warn the potential collision risk at the critical moment, so as to ensure the safety and fluency of the whole performance process. Therefore, the research results of this paper provide a solid theoretical basis and technical support for the safe performance of the dragon dance team, which has important practical significance and application value.

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