Research on the Motion of Bench 'Dragon Based' on Isometric Spiral Model

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Abstract. "Bench Dragon" is a traditional local folk activity in Zhejiang and Fujian regions of China. When the activity is held, hundreds of benches are connected end to end and rotated, which is visually spectacular and rhythmical. This paper aims to use mathematical methods to reveal the internal operation law of "Bench dragon", find out the possible collision problems of "Bench dragon", establish a simulation model, predict the possible collision time and the position and speed of each part at the collision moment. At present, there is no complete and reliable research in this field. Firstly, the formula for calculating the position and velocity of each part in the process of "Benching dragon" movement is summarized, and the collision time is obtained by combining the collision model, and the position and velocity of each part at the collision moment is deduced. In order to verify the reliability of the model established in this paper, a set of initial data is brought in for simulation output, and the results show that the reliability of the collision model established in this paper is good.

Keywords: Collision model, Spiral motion, Bench dragon.

1. Introduction

The movement trajectory and dynamic changes of the bench dragon are the key factors, which affect the fluency and enjoyment of the whole performance. To ensure the normal operation of the bench dragon to avoid collision. The researchers built a mathematical model to analyze the movement patterns of Benbensaurus. By establishing the Archimedes spiral motion model, we can simulate and predict the dynamic movement of Benzosaurus and avoid collision[1]. Several problems were considered in the study, including the position and speed of the bench dragon at different time points, the critical time position speed of the collision between the bench, and the scientific and effective analysis of the motion process of the bench dragon.

Archimedes spiral is widely used in mechanical, electronic and antenna design, especially in high speed, high frequency and high efficiency systems, showing its unique advantages[2]. Zhang et al. 's research review focuses on the application potential of Archimedes' spiral in mechanical transmission systems and its contribution to improving motion efficiency. They analyzed the dynamic characteristics of spiral motion in detail and proposed that the application of Archimedes spiral in mechanical facilities can effectively improve the stability and performance of the system [3].

The social force model (DC-SFM) based on individual perception dynamic grouping is discussed in this paper. This model combines the agent and the traditional social force model. By introducing the personality perception range and dynamic grouping strategy, Su Tenghui improves the realism and computational efficiency of crowd simulation, and improves the simulation accuracy of motion model [4].

RVO algorithm focuses on the collision avoidance problem in large crowd simulation, and discusses the key role of RVO algorithm and its optimization method in the realization of crowd simulation. Through the cross application of linear programming and computer graphics, Xie Kaili simulated the model motion process and studied to provide effective path planning and collision avoidance solutions [5].

Ziqi Wu proposes a hybrid simulation framework that combines fluid dynamics and agent simulation to improve the real-time and authenticity of large-scale crowd simulation. Through the innovative group comprehensive force calculation method and the multi-detail level virtual node acceleration algorithm, the calculation consumption is reduced and the analysis accuracy is improved [6].

Together, these studies demonstrate the importance of Archimedes' spirals in a variety of applications, particularly their unique advantages in improving system efficiency, optimizing design, and responding to changes in the external environment.

In this paper, a collision model is established for the first time based on the collision time, position and velocity of each part of "Bench Dragon" by using spiral motion analysis method and isometric spiral motion principle.

In order to better introduce the collision model established in this paper, this paper is divided into five chapters: Chapter 1 introduction introduces the research background and contribution of this paper; The second chapter is about the theory of spiral motion and introduces the basic knowledge used in the process of establishing the model. The third chapter is the experimental part, which introduces the idea, the overall process design and the detailed process of the experiment. Chapter four introduces the conclusion of this paper. Chapter five summarizes the work and results of this paper on the "bench dragon" problem, as well as the prospect of the model.

2. Correlation theory

2.1. Spiral motion model

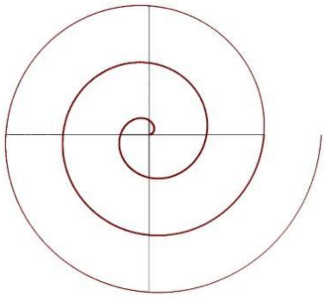

Figure 1. Archimedes helix

According to Figure 1, the polar equation of the Archimedes spiral is:

$$r = a + b\theta \tag{1}$$

Where and are real numbers. At that time, it is the distance from the starting point to the polar origin. Is the value corresponding to each increase in unit Angle of the helix. At that time, it is equivalent to rotating the spiral, and the parameter controls the distance between the two adjacent curves.

The plane Cartesian equation for the Archimedes spiral is [7]:

$$\begin{aligned} x &= (\alpha + \beta\theta)\cos(\theta) \\ y &= (\alpha + \beta\theta)\sin(\theta) \end{aligned} \tag{2}$$

The general method of transformation from Cartesian to polar coordinates is:

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ \theta &= \arctan\left(\frac{y}{x}\right), x \neq 0 \end{aligned} \tag{3}$$

The general method of transformation from polar to Cartesian coordinates is:

$$\begin{aligned} x &= r\cos(\theta) \\ y &= r\sin(\theta) \end{aligned} \tag{4}$$

The Archimedes helix formula can be expressed by the specified radius, circular velocity, and linear motion velocity, and the formula is:

$$\rho = \mathrm{r} \times \left(\frac{\omega}{v} \times \theta + 1 \right) \tag{5}$$

According to this formula [8], when the circular velocity and linear velocity are doubled at the same time, the shape of the Archimedes helix will not change, therefore, the Archimedes helix belongs to the equal velocity ratio helix, and because it is expanded at equal distances in each rotation cycle, it can also be called the isometric helix.

2.2. Collision detection model

Detection of collision between two figures can be transformed into detection of coincidence between two figures, and two-dimensional figures can be converted into one-dimensional points, so the final construction of a coordinate system can simplify the collision problem between two figures to whether the points formed by the two figures have the same coordinates, if there are two figures on the same coordinates of points, then a collision occurs between the two figures, and otherwise there is no collision between the two figures.

3. Experiment

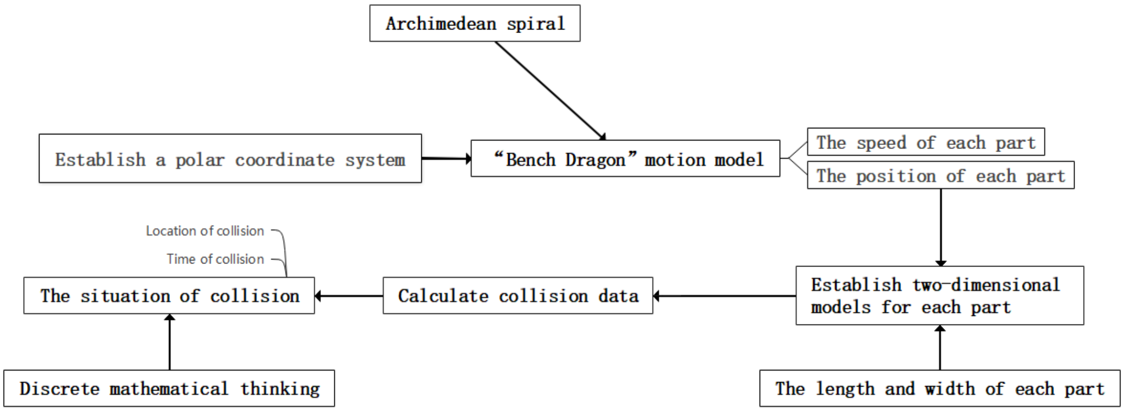

Figure 2. Mind map

According to Figure 2: In this experiment, the polar coordinate system is first established. According to the characteristics of Archimedes' spiral motion on the polar coordinate system, the "Bench Dragon" motion model is constructed, including the motion speed and real-time position of each part of "Benching Dragon". According to the position data of each part of "Bench Dragon", the two-dimensional model of each part is established, and the collision detection model is applied to obtain the time and position of collision.

3.1. Construct the "bench dragon" movement model

The "Bench Dragon" motion can be approximated to the Archimedes spiral motion, which can be regarded as the motion of two degrees of freedom on the plane. In order to describe and understand the motion of the model, the following coordinate system can be established [9], as shown in Figure 3.

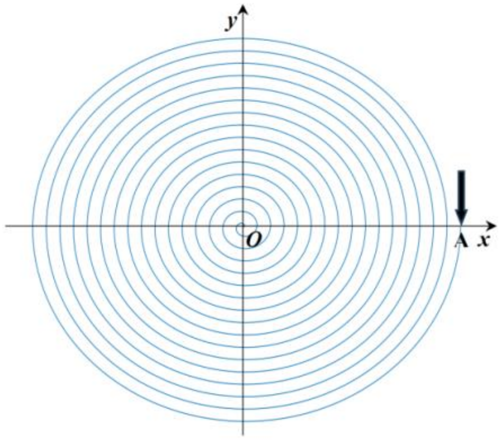

Figure 3. "Bench Dragon" moving system

Make the center point of the spiral motion of the "bench dragon" o to ensure consistency and accuracy of all motion descriptions. The starting point A is located on the X-axis, ensuring that the starting position of the "bench dragon" motion is aligned with the coordinate system, and the Archimedes spiral motion is made in a clockwise direction.

The differential arc length ds of "Bench dragon" is satisfied:

$$\mathrm{ds}=\sqrt{\left[\mathrm{r}^2+\left(\frac{\mathrm{dr}}{\mathrm{d}\theta}\right)^2\right]}\mathrm{d}\theta=\sqrt{\mathrm{k}^2\times\theta^2+\mathrm{k}^2}\,\mathrm{d}\theta=\mathrm{k}\sqrt{1+\theta^2}\,\mathrm{d}\theta \tag{6}$$

The equation system a can be obtained by equivalent substitution:

$$\begin{cases}\dfrac{\mathrm{d}\theta}{\mathrm{dt}}=\dfrac{-1}{\mathrm{k}\times\sqrt{1+\theta^2}}=-\dfrac{2\pi}{\mathrm{d}\sqrt{1+\theta^2}}\\[2ex] \theta\Big|_{\mathrm{t}=0}=32\pi\end{cases} \tag{7}$$

Different values are brought in, and the corresponding values are calculated using Runge Kutta formula, and the positions of different time and different parts are calculated according to the values.

$$\mathrm{x_i}=\mathrm{r}\cos\theta=\mathrm{k}\theta\times\cos\theta \tag{8}$$

$$\mathrm{y_i}=\mathrm{r}\sin\theta=\mathrm{k}\theta\times\sin\theta \tag{9}$$

$$\mathrm{v}=\frac{\mathrm{ds}}{\mathrm{dt}}=\mathrm{k}\sqrt{1+\theta^2}\,\frac{\mathrm{d}\theta}{\mathrm{dt}} \tag{10}$$

The known velocity relation is:

$$\mathrm{v}=-\mathrm{k}\times\sqrt{1+\theta^2}\,\frac{\mathrm{d}\theta}{\mathrm{dt}} \tag{11}$$

The forward difference method is used to find the velocity of the first time node:

$$\mathrm{v}=-\mathrm{k}\times\sqrt{1+\theta_1^2}\times\frac{\theta_2-\theta_1}{\Delta\mathrm{t}} \tag{12}$$

Using backward difference method to find the last time node velocity:

$$\mathrm{v}=-\mathrm{k}\times\sqrt{1+\theta_L^2}\times\frac{\theta_L-\theta_{L-1}}{\Delta\mathrm{t}} \tag{13}$$

Using the central difference method to find the intermediate time node velocity:

$$v = -k \times \sqrt{1+\theta_i^2} \times \frac{\theta_{i+1} - \theta_{i-1}}{2\Delta t} \tag{14}$$

3.2. Computational collision problem

When establishing the two-dimensional model, this paper selects two bench and establishes the corresponding geometric model [10]. The Cartesian coordinate system is established with P as the origin and the line where P1 'P2 is located as the X-axis.

Bench P1 rotates clockwise so that point P1 falls on the line P1 'P2.

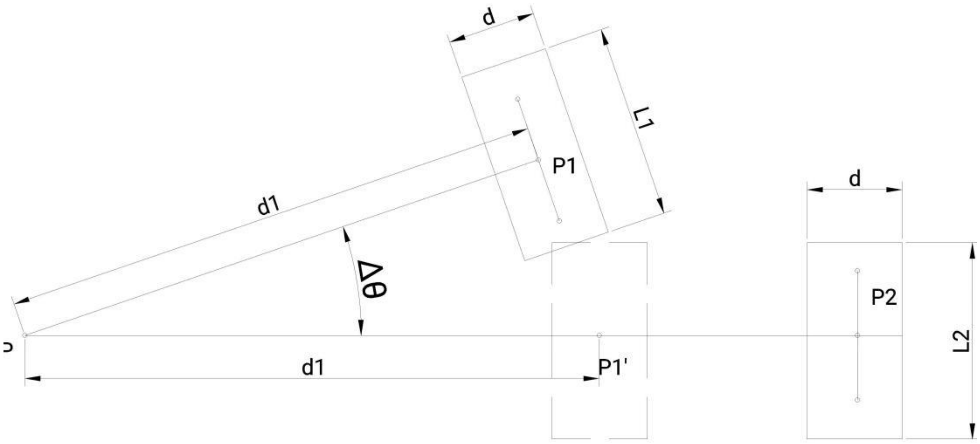

Figure 4. Bench collision detection model

According to Figure 4, all points in the L2 region of the bench meet the conditions:

$$\begin{cases} \left| x - x_{p2} \right| \le \dfrac{d}{2} \\ \left| y \right| \le \dfrac{L_2}{2} \end{cases} \tag{15}$$

The L1 bench is centered at point p and rotates $\Delta\theta$ clockwise so that p' is on the X-axis.

Then all points in the bench area L_1' meet the conditions:

$$\begin{cases} \left| x - d_1 \le \dfrac{d}{2} \right| \\ \left| y \right| \le \dfrac{L_1}{2} \end{cases} \tag{16}$$

Scatter the bench L_1' into a $m \times n$ coordinate point, again with p as the center of the circle, rotate $\Delta\theta$ counterclockwise, and restore the discrete point to the L_1 bench position, that is $(x, y) \times T\begin{pmatrix} \cos\Delta\theta - \sin\Delta\theta \\ \sin\Delta\theta \cos\Delta\theta \end{pmatrix}$ If it meets the conditions for points in the L_2 bench region, a collision occurs.

Taking the faucet as an example, the bench that may collide with it should meet the following conditions:

$$\theta \le \theta_0 + 2\pi \tag{17}$$

Where θ_0 is the position of the handle in front of the tap.

3.3. Model checking

In order to verify the reliability and accuracy of the model established in this paper, a typical "bench dragon" scene in Zhejiang and Fujian regions was selected: the bench dragon consisted of 223 benches, of which the first bench was the dragon head, 221 sections were the dragon body and the last section was the dragon tail. The board length of the head is 341 cm, the board length of the body and tail of the dragon is 220 cm, and the board width of all the benches is 30 cm. There are two holes in each bench, the aperture (diameter of the hole) is 5.5 cm, and the center of the hole is 27.5 cm from the nearest plate head. The two adjacent benches are connected by handles. The dragon dance team rotates clockwise along an isometric spiral with a pitch of 55cm, and the center of each handle is located on the spiral. The speed of the front handle of the faucet is always maintained at 1 m/s.

Through the above experiments, the collision time and the position and speed of each part of the "bench dragon" at the collision time are obtained, as shown below.

4. Results

Through the construction of the movement model of "Bench dragon" and the calculation of the collision problem, the collision time of "Bench dragon" and the position and speed of each part at the collision time are obtained, and the schematic diagram of the point position of each part during the collision of "Bench dragon" is drawn, as shown in Figure 4.

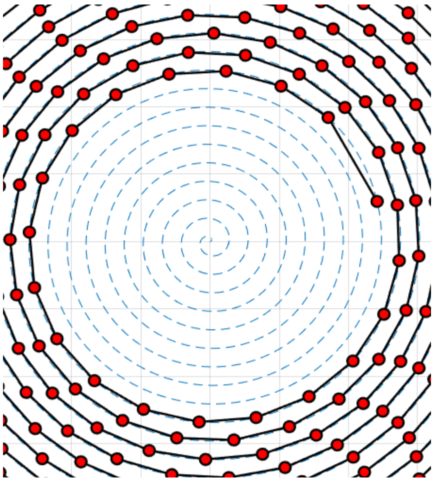

Figure 5. Schematic diagram of bench position

As can be seen from Figure. 5, during the movement of "bench dragon", the possibility of collision between various parts comes from the fact that the distance between the inner ring and the outer ring in the process of disk entry is too small, so the key to avoid collision is to increase the pitch of the disk entry spiral.

According to the above data, the "bench dragon" model was established to calculate the collision time and the position and velocity of each part at the collision time. The obtained data are shown in Table 1.

Table 1. Position and speed of each part of "Bench dragon" at the moment of collision

	Horizontal coordinate x(m)	Vertical coordinate y(m)	Speed (m/s)
tap	1.60786	1.60034	0.99579
Section 1 dragon body	-1.21645	2.05072	0.98762
Section 21 Dragon Body	1.50258	3.04706	0.97896
Section 41 Dragon Body	-3.69343	-1.92967	0.97624
Section 61 Dragon Body	0.08128	-4.81329	0.97491
Section 81 Dragon Body	1.24756	-5.23641	0.97412
Section 101 Dragon Body	-1.04298	-5.80397	0.97360
Section 121 Dragon Body	-5.57041	-3.08847	0.97322
Section 141 Dragon Body	-4.95445	4.67050	0.97294
Section 161 Dragon Body	4.94177	5.26586	0.97273
Section 181 Dragon Body	4.06331	-6.43652	0.97255
Section 201 Dragon Body	-7.95019	-0.72221	0.97241
Section 221 Dragon Body	4.50688	7.01442	0.97229
tail	3.04529	7.78008	0.97229

According to Table 1, the positions and speeds of each part of "Bench dragon" at the time of collision can be obtained. It is found that the closer the "bench dragon" is to the tail of the dragon, the slower the speed is, and the collision place is located at the head. Therefore, it is necessary to focus on controlling the speed of the head to better avoid collision.

5. Conclusion

This paper, aiming at the possible collision risk in the traditional folk activity of "bench dragon", systematically propose and establish a mathematical model based on Archi $r = a + b\theta$ medes spiral motion and collision detection. Through simulation experiments, the research successfully predicted the specific positions and velocities of each component of "Bench Dragon" at different time nodes, and identified possible collision points. The experimental results proved the reliability and effectiveness of the model in this paper. The research not only provides scientific support for the safety and enjoyment of Bench dragon activities, but also fills the research gap in this field, reflecting the value of combining cultural inheritance with modern science and technology. In the future, the research plans to further extend this model to other traditional folk activities, such as lion dance, dragon dance and other performances with complex movement characteristics, to evaluate its wide applicability.

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