Optimization of crop planting strategies: an integrated scheme of linear programming and intelligent optimization algorithms

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Abstract. To address the challenge of developing optimal planting strategies for multiple crops under the constraints of diverse greenhouse conditions and various types of cultivated land, a crop planting strategy based on intelligent optimization algorithms is proposed. First, assuming stable crop production and sales across years, a linear programming model is formulated to account for diversified planting risks. Uncertainty factors are incorporated into the decision-making process through interpolation techniques, with the dual objectives of maximizing net income and minimizing overproduction losses. Optimization and solution of the model are achieved using a combination of a greedy algorithm and a genetic algorithm, enhanced by the Pearson correlation coefficient. Sensitivity analysis is conducted to effectively evaluate the robustness and adaptability of the proposed planting strategy under different scenarios. Furthermore, the scope of the study is extended to scenarios involving year-on-year increases in crop sales, with additional considerations given to crop uncertainty and planting risks.

Keywords: Crop strategy, Price elasticity coefficient, Linear programming, Greedy strategy, Genetic algorithm.

1. Introduction

In the context of limited arable land resources in rural areas of our country, optimizing crop planting strategies has become a critical requirement for enhancing agricultural production efficiency. Such optimization should aim not only to maximize economic profits but also to address issues arising from soil degradation caused by continuous cropping. By mitigating planting risks associated with uncertain factors, these strategies can improve overall efficiency and foster the sustainable development of rural economies.

Currently, research on optimizing crop planting strategies has yielded certain phased achievements [1]. For instance, optimization models based on mathematical programming have been developed, dividing objectives into multiple stages and employing dynamic programming to derive optimal planting plans that achieve overall system optimization [2]. In scenarios involving multi-objective intelligent optimization algorithms, planting structure schemes tailored to different target scenarios have been proposed [3]. However, while a foundational body of work exists, current research has not fully adapted to advancements in modern planting techniques, nor has it comprehensively accounted for the myriad constraints faced in agricultural production. The genetic algorithm, particularly when integrated with greedy strategies, is a powerful numerical method for solving linear and nonlinear problems [4-7]. It efficiently identifies global optimal solutions and is particularly well-suited to large-scale optimization problems. Despite its potential, the application of genetic algorithms in optimizing crop planting strategies remains underexplored, highlighting a significant gap in the literature that warrants further investigation.

Building on this analysis, this study aims to optimize the use of limited arable land resources by developing crop planting strategies that maximize economic benefits while minimizing risk losses caused by climate variability and other unforeseeable factors. The proposed approach incorporates key parameters such as expected sales volumes, planting costs, yield per mu, and crop sales prices. The study formulates an optimal planting plan that accounts for the time-varying nature of these parameters, employing linear programming and other advanced optimization techniques to determine the ideal crop combinations and planting areas. This ensures the efficient allocation of resources while satisfying market demand. By establishing corresponding mathematical models, this study seeks to enable decision-makers to develop scientifically sound planting arrangements and provide actionable guidance for sustainable agricultural development in the region.

2. Algorithm description

2.1. Linear programming

A linear programming problem involves determining the optimal solution that maximizes a linear objective function while adhering to a set of linear equality or inequality constraints [8]. In essence, it seeks to optimize the given objective within a feasible region defined by these constraints.

$$\max\ z=\sum_{j=1}^{n} c_j x_j \ \ \text{s.t.} \begin{cases} \sum_{j=1}^{n} a_{ij} x_j = b_i & i=1,2,3,\ldots,m \\ x_j \geq 0 & j=1,2,3,\ldots,n \end{cases} \tag{1}$$

2.2. Greedy algorithm

The greedy algorithm is an iterative method for constructing solutions, designed to select the locally optimal choice at each step with the goal of ultimately achieving a globally optimal solution [9]. The core concept of this algorithm lies in employing a local optimization strategy to progressively solve the problem, without revisiting or modifying decisions made in previous steps.

Given a definite problem instance (X, F, C), where X represents all possible solution sets, F represents feasible solution sets, and $F \in X$, $C: X \rightarrow R$ is the objective function. The greedy strategy is to construct gradually in an increasing order. In the i-th step, the element $x_i \in F_i$ is selected, so that

$$C(S_i \cup \{x_i\}) = \min_{x_i \in F_i} C(S_i \cup \{x_i\}) \tag{2}$$

This represents the partial solution selected in the previous steps and is the currently available local solution set.

2.3. Genetic algorithm

The core concept of a genetic algorithm is to leverage population-based evolution to search for the optimal solution to a given problem [10]. Inspired by the principles of natural selection and genetic variation in biological evolution, the algorithm iteratively evolves a population of candidate solutions (referred to as individuals). Each individual, represented in an encoded format, corresponds to a potential solution, and its quality is assessed using a fitness function. The genetic algorithm process is shown in Figure 1. The general process can be outlined as follows:

(1) Initialization: Define the solution space S and the fitness function $f(x)$, and randomly generate an initial population $P_0 = \{x_i | \ x_i \in S\}$.

(2) Fitness: Calculate the selection probability p_i according to the fitness function $f(x_i)$, Among them:

$$p_i = \frac{f(x_i)}{\sum_{j=1}^{n} f(x_j)} \tag{3}$$

(3) Crossover: For the selected individuals, select the crossover point and generate offspring:

$$x_{new} = S_{1:k} \cup y_{k+1:n} \tag{4}$$

(4) Mutation: For the gene bit, randomly flip with the mutation probability P_m, that is:

$$x_{ij} \leftarrow 1 - x_{ij} \tag{5}$$

(5) Population Update: Replace a subset of existing individuals in the population with newly generated ones to introduce diversity and improve potential solutions.

(6) Termination Condition: Conclude the algorithm when either the maximum number of generations is reached or the fitness value meets the predefined threshold.

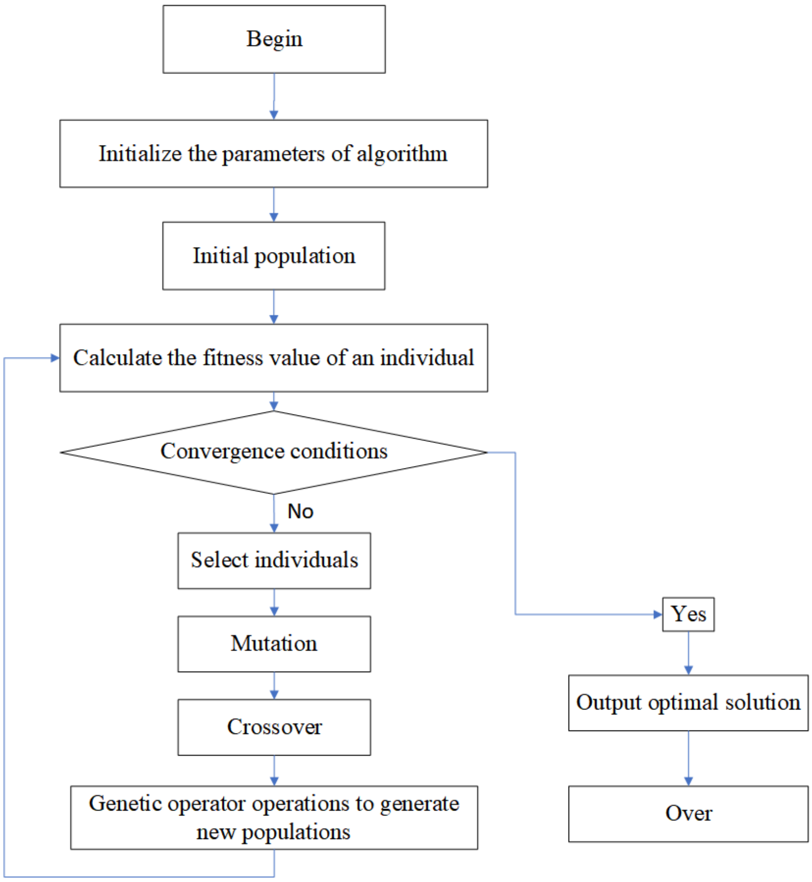

Figure 1. Genetic Algorithm Process

3. Crop planting strategy model

In constructing mathematical models for crop planting strategies, two primary objective functions are introduced: maximizing net income and minimizing overproduction losses. Maximizing net income involves optimizing crop combinations and planting areas to achieve the highest total income over the entire planting cycle, accounting for the difference between sales revenue and planting costs. Minimizing overproduction losses focuses on crops with predefined expected sales volumes. When actual production exceeds these expectations, adjustments to the planting plan are necessary to reduce overproduction. This helps to minimize economic losses caused by an inability to sell surplus produce at normal prices, thereby mitigating risks associated with delayed sales or additional costs from discounted sales.

This section begins by formulating model constraints for determining the optimal crop planting strategy under constant production conditions. It then incorporates uncertainties related to expected sales volumes, yield per mu, planting costs, and crop sales prices, as well as potential planting risks, to establish a comprehensive framework for identifying the optimal planting plan.

3.1. The optimal planting model under steady production

Assuming the presence of diverse crop types, with the expected sales volume, planting cost, yield per mu, and sales price of each crop remaining stable across seasons, it is further presumed that all crops planted in a given season are sold within that season. However, if the total yield of a particular crop exceeds its expected sales volume in any season, the surplus cannot be sold at regular market conditions. Therefore, it is essential to conduct a comprehensive evaluation of factors such as land parcel concentration, area, and crop sales volume. The constraints imposed by these factors are modeled as follows:

Land Area Constraint:

$$\sum_{j\in J}\sum_{t\in T} x_{ijt} \leq A_i \quad (\forall i\in I) \tag{6}$$

Among them: x_{ijt} represents the area of crop j planted on plot i in season t, and A_i represents the area of plot i.

Sales volume constraint:

$$\sum_{j\in J}\sum_{t\in T} x_{ijt} * Y_{jt} \leq P_j \quad (\forall j\in J) \tag{7}$$

Among them: Y_{jt} represents the yield per mu of crop j in the planting season t, and P_j represents the expected sales volume of crop j.

Repetitive planting restrictions:

$$x_{ijt} + x_{ij,\,t+1} \leq A_i \quad (\forall i\in I\ \forall j\in J\ \forall t\in T) \tag{8}$$

Among them: $x_{ij,t+1}$ represents the area of crop j planted on plot i in the season $t + 1$.

Requirements for legume rotation:

$$\sum_{j\in \text{legume crop}}\sum_{t\in T} x_{ijt} \geq \frac{A_i}{3} \tag{9}$$

Requirements for Land Concentration and Area:

$$x_{ijt} \geq \varepsilon \quad (\forall i\in I\ \forall j\in J\ \forall t\in T)\ (\forall \varepsilon>0) \tag{10}$$

Among them: $\forall \varepsilon > 0$ represents the minimum planting area.

In summary, the objective function of maximizing net profit can be expressed as:

$$\max Z = \sum_{j\in J}\sum_{t\in T}\sum_{i\in I}\left(x_{ijt} * Y_{jt} * S_{jt} - x_{ijt} * C_{jt}\right) \tag{11}$$

Among them: Z represents net income, S_{jt} represents the area of crop j in planting season t, C_{jt} represents the planting cost of crop j in planting season t, and $\sum_{j\in J}\sum_{t\in T}\sum_{i\in I}\left(x_{ijt} * Y_{jt} * S_{jt} - x_{ijt} * C_{jt}\right)$ represents the sum of total production value minus total planting costs for all crops across all seasons and all plots, i.e., the total net income for all crops.

Two models are constructed to address the surplus exceeding the expected sales volume: the "unsold waste" model and the "discounted sales" model. These can be formulated as follows:

3.1.1. Unsold model

Two models are constructed to address the surplus exceeding the expected sales volume: the "unsold waste" model and the "discounted sales" model. These can be formulated as follows:

Assuming that S_j^+ represents the excess portion of crop planting each year, the loss due to overproduction is as follows:

$$\min \acute{Z} = \sum_{j\in J} S_j^+ \tag{12}$$

Among them: $\acute{Z}$ represents the loss due to overproduction, and $\sum_{j\in J} S_j^+$ represents the total sum of the excess portions of all crops planted each year.

Due to the fact that the excess portion, which is unmarketable, is not considered for asset liquidation, let P_j denote the upper limit of the anticipated sales volume for crop j, namely:

$$\sum_{t\in T}\sum_{i\in I} x_{ijt} * Y_{jt} \leq P_j \quad (\forall j\in J) \tag{13}$$

Among them: $\sum_{t\in T}\sum_{i\in I} x_{ijt} * Y_{jt} \leq P_j\ (\forall j \in J)$ represents the total production of crop j across all seasons and all plots.

3.1.2. Price reduction model

For the surplus exceeding the expected sales volume, the price will be reduced by 50% of the original sales price. The total price reduction is calculated as follows: first, determine the total sales before the price reduction by multiplying the yield per mu by the planting area. Then, subtract the revenue corresponding to the expected sales volume. Finally, calculate the excess revenue by multiplying the reduced price (50% of the original price) by the planting area of the crop for the season, resulting in the following expression:

$$R_{Reduce}=\sum_{j\in J}\sum_{t\in T}\sum_{i\in I}\left(x_{ijt}Y_{jt}-P_j\right)*S_{jt}*50\% \tag{14}$$

Among them: R_{Reduce} represents the amount of price reduction. The formula for calculating net profit is expressed as follows:

$$\pi_{ji}=\left(S_{jt}*Y_{jt}\right)-C_{jt} \tag{15}$$

Among them: π_{ji} represents the net profit per mu for crop j on plot i.

3.2. Optimal planting model under uncertain risk constraints

Building on the previous considerations, and further accounting for the uncertainty in factors such as expected sales volume, yield per mu, planting costs, sales prices, and potential planting risks, it is assumed that the expected sales volume of crops remains relatively stable year after year. Based on this assumption, the following additional constraints are introduced:

$$P_{jy}=P_{j(y-1)}*\left(1+r_j\right)\ \forall j\in J \tag{16}$$

Among them: P_{jy} represents the expected sales volume of crop j in year j, and r_j represents the rate of change for crop j in year y. Considering constraints such as changes in yield per mu, planting costs, and sales prices, for changes in yield per mu, the following additional constraint conditions are added:

$$Y_{jty}=Y_{jt(y-1)}*\left(1+\delta_t\right) \tag{17}$$

Among them: Y_{jty} represents the yield per mu of crop j in season t of year j. $\&_t$ denotes the rate of change in yield per mu, considering an annual variation threshold of less than 10%,$\&_t \in [-0.1,0.1]$. Taking into account the corresponding annual percentage increases in planting costs and sales prices, an exponential model is established as follows:

$$C_{jty}=C_{jt(y-1)*}\left(1+0.05\right)^{\left(y-2023\right)} \tag{18}$$

$$S_{jty}=S_{jt(y-1)}*\left(1+g_j\right)^{\left(y-2023\right)} \tag{19}$$

Among them: g_j refers to a coefficient where it is 0 for grain crops, 0.05 for vegetable crops, within the range of $[-0.05, 0]$ for edible fungi, and -0.05 for morels. The following analysis is conducted to maximize net income. Firstly, a recursive model is established:

$$Z_y=\alpha_y Z_{y-1}+\sum_{j\in J}\sum_{t\in T}\sum_{i\in I}\left(\frac{x_{ijt,y}*Y_{ijt,y}*S_{jt,y}}{1+\log\left(1+C_{jt,y}\right)}-x_{ijt,y}*C_{jt,y}\right)-S^+{}_j\left(y\right)*S_{jt,y}\# \tag{20}$$

Among them: $-S_j^+(y)*S_{jt,y}$ represents the loss due to overproduction. Minimizing the loss due to overproduction is as follows:

$$\hat{Z}_y=\beta_y*\hat{Z}_{y-1}+\sum_{j\in J}S_j{}^+\left(y\right)*S_{jt,y} \tag{21}$$

Among them: β_y represents the growth rate of overproduction loss in the y-th year compared to the previous year, $\sum_{j\in J}S_j{}^+(y)*S_{jt,y}$ represents the loss due to overproduction.

4. Case solution

The case study presented in this article utilizes the 2023 statistical data on rural crop planting in the mountainous areas of North China, along with publicly available datasets on existing farmland and basic crop conditions in rural regions. These data serve as the training set for implementing problem-solving using a genetic algorithm in a Python environment. The relevant initial conditions are set as follows:

The total cultivated land area is 1,200 acres, encompassing four types of land: flat dry land, terraced land, hillside land, and irrigated land. It is assumed that flat dry land, terraced land, and hillside land are suitable for planting one season of grain crops per year, while irrigated land can support either one season of rice or two seasons of vegetables per year. Additionally, there are 20 vegetable greenhouses, consisting of 16 ordinary greenhouses and 4 smart greenhouses, each with a cultivated area of 0.6 acres.

The following assumptions are made:

(1) Crops cannot be planted continuously on the same plot, as this would result in reduced yield.

(2) The soil enriched with root bacteria from leguminous crops benefits the growth of other crops. Therefore, leguminous crops should be planted at least once in each plot within a three-year cycle.

(3) The planting areas for each crop and season should not be overly dispersed, and the planting area in any single plot (including greenhouses) should not be too small.

4.1. Optimal Crop Strategy Solution

The problem-solving process was implemented in a Python environment, and the results are presented in Table.1. As shown in the table, with an increasing number of iterations, the objective function value of the candidate solutions steadily improves, converging toward the global optimal solution.

Table 1. Result of Algorithm solutions

Number	Candidate solution	Objective function value
100	X1, X2, X3,...X390	F (X1, X2, X3,...X390) = 3.103e+06
300	X1', X2', X3',...X390'	F (X1', X2', X3',...X390') =3.784e+06
1000	X1 (1000), X2 (1000),...X390 (1000)	F (X1(1000), X2 (1000),...X390 (1000)) =4.894e+06
...	...	...
5000	X1 (5000), X2 (5000), X390 (5000)	F (X1 (5000), X2 (5000),...X390 (5000)) =6.844e+06

Among them: X1 represents the quantity of soybeans planted in plot A1, X2 represents the quantity of black beans planted in plot A2, X3 represents the quantity of adzuki beans planted in plot X3, ……, X390 represents the quantity of barley planted in plot C6. Based on the above allocation and constraint conditions, an iterative study is conducted in this paper. When the iteration reaches 5000 times, the predicted profit result is shown in Figure 2.

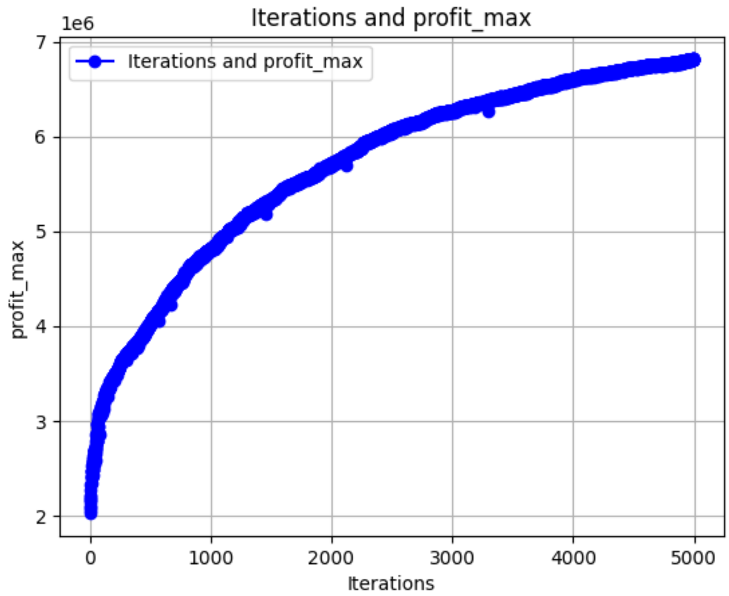

Figure 2. Profit Prediction Results Using Genetic Algorithm

Using the model, the net profit for each acre of land planted with various crops can be calculated, as depicted in Figure 3.

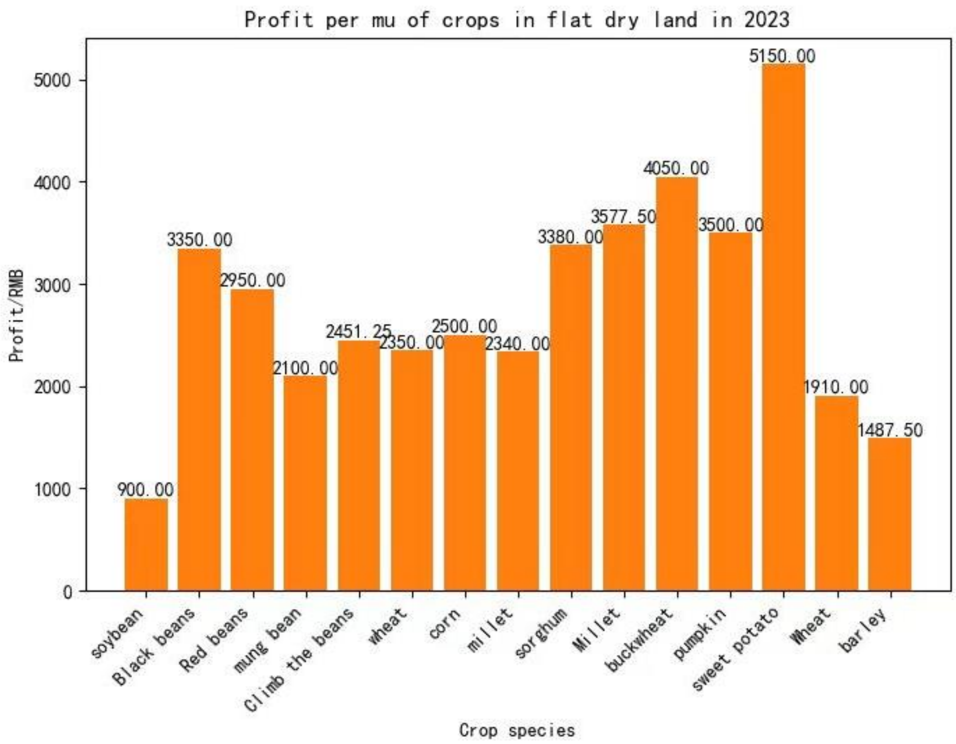

Figure 3. Single acre profit of crops in flat and arid land

As illustrated in Figure 4, the profit per acre for sweet potatoes is the highest among the crops grown on flat dry land, terraced fields, and hillside plots. Given the optimal crop yield performance on these plots, it is concluded that planting sweet potatoes as the primary crop will maximize the land's high-yield potential and economic benefits. This strategy ensures higher unit area profits while maintaining high yields, thereby improving overall planting efficiency. Similarly, the planting scenario for flat dry land in 2024 can be determined, as shown in Table 2.

Table 2. Planting situation of different crops in terraced fields

Crop	Single acre profit	Allocate land parcels	Allocate acres
Climbing beans	2451.25	B10	23.75
Wheat	2350.00	B2, B4, B5, B8, B11, B14	2, 40, 28, 25, 44, 60, 14.55
Millet	3577.50	B1, B7, B9, B12, B13	60, 2.75, 50, 45, 35
Mung bean	2100.00	B2, B6, B10, B14	1.9, 86, 1.25, 5.45
Naked oats	1910.0	B7	33.25
Barley	1487.5	B7	19

Among them:B_i represents plot type B. Using the same method as above, the net profit per mu for different crops in plot type C is obtained similarly, as shown in Figure 4.

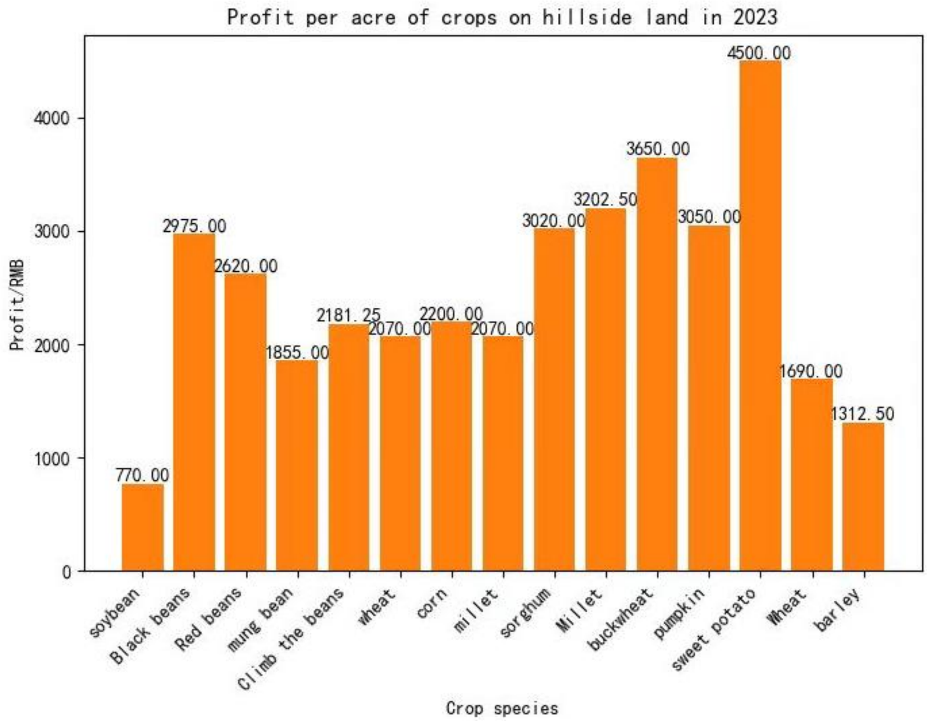

Figure 4. Single acre profit of different crops on hillside land

For any excess production, it will be sold at a 50% discount based on previous years' sales prices. To further analyze this, profit trends and 50% discount profit trends for different crops across the three land types are depicted in Figure 5.

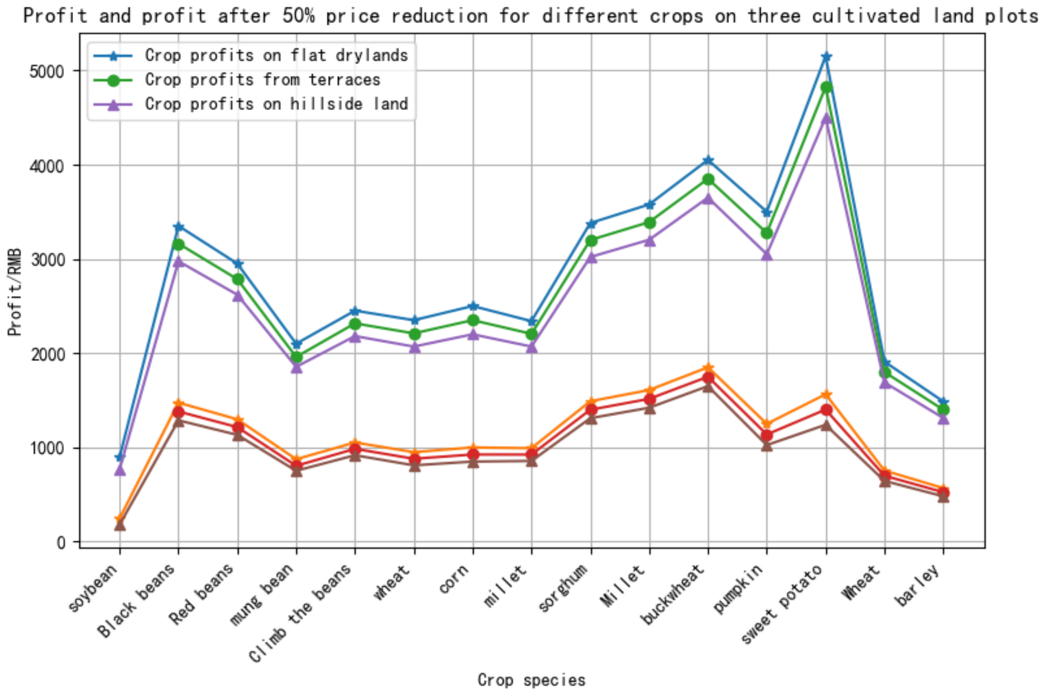

Figure 5. Three Different Crops on Cultivated Land and Profit after a 50% Price Reduction

Upon analysis, it is observed that certain crops planted across the three types of farmlands still yield higher net profits even after the price reduction. Consequently, crops with lower original profits can be excluded in favor of those with higher profitability. After a thorough evaluation, we have decided to exclude three crops with lower net profits—namely, oats, barley, and soybeans. Based on this selection, the optimized planting strategy for flat land, terraced fields, and hillside land is derived, as shown in Table 3.

Table 3. Planting situation on flat land, terraced fields, and hillside land

Plot	Original crop	Area/acre	New crops
A1	Wheat	80	Corn
A2	Corn	55	Millet
……	……	……	……
C5	Wheat	27	Sorghum

In the case of irrigated land, some crops that have experienced a price reduction still yield higher net profits compared to others. After further analysis, it was found that crops such as rice offer relatively low profits, while eggplants generate higher profits. As a result, eggplants have been selected to replace tomatoes, adzuki beans, potatoes, and rice. Using the same calculation methodology as described previously, the original profits and the profits after a 50% price reduction for different crops in both ordinary and smart greenhouses are depicted in Figure 6.

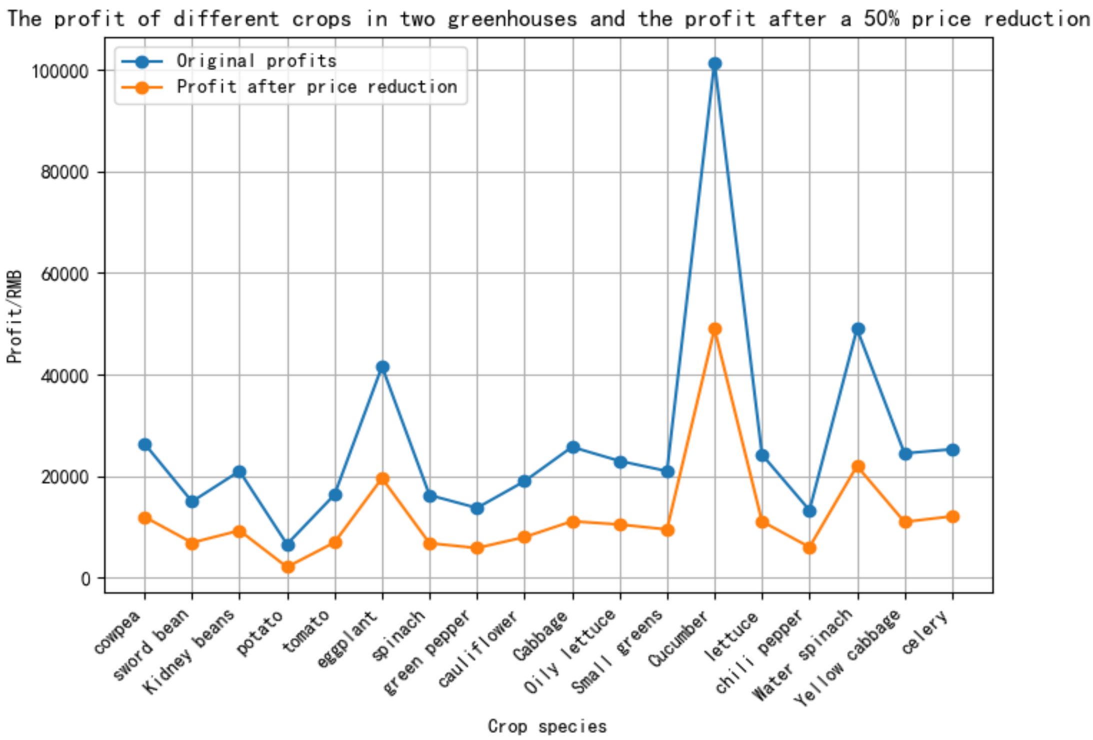

Figure 6. Profit and 50% price reduction profit for different crops in the first season of two greenhouses

Based on Figure 6, this paper decides to plant eggplants in plot E_1 and green peppers in all the other plots. This allows us to initially obtain the optimal planting plan for the three plots of farmland.

4.2. Optimal Planting Strategy under Risk Constraints

The results of the solution under risk constraints based on a unified dataset are shown in Table 4.

Table 4. Planting situation of flat dry land, hillside land, and terraced fields

Crop	Net profit	Area/acre
Sweet potato	5050.0	16.2
Buckwheat	4032.5	13.5
……	……	……
Soybean	880.0	142.5

For the first season in the greenhouse, the analysis indicates that planting 0.87 acres of cucumber, 0.3 acres of water spinach, 5.67 acres of eggplant, and 5.16 acres of cowpea yields the optimal planting configuration. For irrigated land, the analysis suggests allocating 5.96 acres to cowpeas and 1.09 acres to cabbage.

5. Conclusion

This study proposes a strategy for optimizing crop planting under constraints involving multiple greenhouse areas and types of cultivated land, utilizing an intelligent optimization algorithm. Assuming stable crop production and sales over time, a linear programming model is constructed, incorporating diversified planting risks. Uncertainty factors are integrated into the decision model via interpolation. Optimization solutions are derived using both the greedy algorithm and the genetic algorithm, while sensitivity analysis is conducted to evaluate the robustness and adaptability of the strategy across various scenarios. Furthermore, the study expands to account for the annual growth of crop sales and delves deeper into the coupling of crop complementarity in cultivation, offering theoretical support for informed planting decisions.

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