Research on Collision Avoidance and Path Planning of Bench Dragon Dance Based on Spiral Model

Mingqi Zhang *

School of cyber science and technology, Shandong University, Qingdao, China, 266237

* Corresponding Author Email: zhangmingqi701@126.com

Abstract. The bench dragon performance is an important intangible cultural heritage of China. It is not merely a form of entertainment but also a significant way to express deep familial affection, family ties, ancestral heritage, and geographical connections. These elements are woven into China's long-standing five-thousand-year cultural history. By participating in and watching these performances, communities strengthen their connection to cultural traditions and ancestral roots. The article specifically studies the optimization of route design, turning movements, and speed control of dragon dance teams during festive events. A spiral curve model is introduced, which micro-segments time and checks the conditions for collision and turning at each time slice, addressing challenges such as collision avoidance and path planning in the bench dragon dance" performances. It identifies the factors leading to collisions and the minimum turning distance required. The research on path planning and collision avoidance has promising prospects today. The article aims to make the dance arrangements more reasonable while preserving the customary significance, enhancing the safety and efficiency of these cultural activities.

Keywords: Bench Dragon, Archimedean Spiral, Speed Control.

1. Introduction

Bench dragon dance is an important activity in the traditional Chinese folk festival culture and an important intangible cultural heritage of my country. In large traditional festivals such as the Lunar New Year and the Lantern Festival every year, dragon dance performances are often used to show auspiciousness and harmony, which contains profound folk beliefs and cultural traditions. Therefore, the supply and demand scale of bench dragon dance performances are constantly expanding. During the performance, the dragon dance team uses the tightly combined dragon body to coil in, turn around and coil out along a specific path, showing complex fancy movements, which are highly ornamental. The complex performance movements require reasonable path planning and turning design, which are key factors affecting the performance effect.

In previous studies, the main research direction is to explore the strategies for the development and inheritance of the “bench dragon dance” culture ecology from the perspectives of natural environment, social environment and ethnic members, including analyzing the impact of various factors on cultural inheritance and development, and proposing natural geographical strategies, scientific and technological strategies, industrial economic strategies, organizational structure strategies, and value concept strategies to promote the sustainable development of the bench dragon dance culture [1-3]. Bench dragon dance as a cultural feature of folk sports is another common research direction. Previous studies have covered the construction of collective memory in the practice of Wolong rituals [2], the religious significance of bench dragon [4], and the relationship between bench dragon and crafts in Pujiang area.

In previous studies, culture was the main research direction, and there were few discussions based on mathematical models. At the same time, the previous applications of the Archimedean spiral were primarily in the research of Archimedean spiral antennas [5-6] and Archimedean spiral coils [7-8]. In this paper, we will establish an Archimedean spiral model to study the path planning and speed control of bench dragon dance. This paper aims at the path optimization, turnaround design and travel speed control of dragon dance teams in festival activities. Based on the idea of motion control, by determining multiple key indicators such as pitch and turnaround space, a spiral curve model is established, and Python is used for simulation solution and analysis. The model is close to application needs and real life, and makes full use of methods such as combination of numbers and shapes, iteration method, differential method, Python visualization, etc., and tries to accurately analyze the collision avoidance and path planning problems in the performance of dragon dance teams as much as possible under the premise of proposing some basic assumptions.

2. Application of the Archimedean Spiral Model

2.1. Calculation of Collision Conditions Based on the Movement Model of a Fixed-Length Line Segment on an Archimedean Spiral

When a moving point moves at a uniform speed along the polar diameter in polar coordinates, and the polar diameter rotates at a uniform angular velocity at the same time, the trajectory of the moving point is the Archimedean spiral [9-10]. The initial point A of the polar diameter is on the polar axis, and the initial radius, that is, the distance from the point to the pole, is a. Starting from point A to any point P (r, θ) on the curve, its polar coordinate equation is

$$r = a + \frac{p}{2\pi}\theta \tag{1}$$

Where p is the pitch, i.e. the distance between two adjacent lines [11-12].

The data below comes from the paper "A therizinosauroid dinosaur with integumentary structures from China" written by Xing Xu, Zhi-lu Tang and Xiao-lin Wang [13]. The bench dragon consists of 223 benches, of which the first section is the dragon head, the next 221 sections are the dragon body, and the last section is the dragon tail. The length of the dragon head is 341 cm, the length of the dragon body and the dragon tail is 220 cm, and the width of all benches is 30 cm. There are two holes on each bench, with a hole diameter (hole diameter) of 5.5 cm, and the center of the hole is 27.5 cm away from the nearest board head (see Figures 1 and 2). In the two adjacent benches, the rear hole of the front bench and the front hole of the rear bench remain coincident during movement and are connected by a handle.

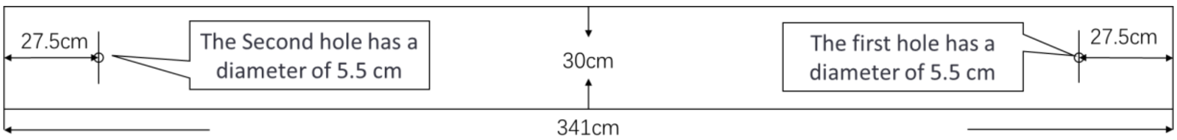


Figure 1. The top view of the dragon head

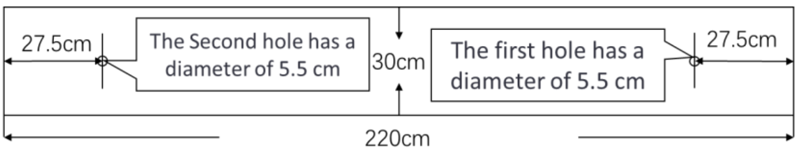
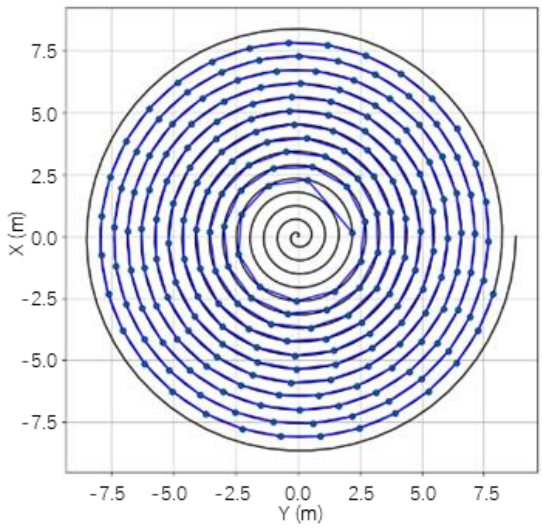


Figure 2. The top view of the dragon body and tail

First, the clockwise direction is defined as the positive direction of the angle, and the winding trajectory of the bench dragon is regarded as an Archimedean spiral. The trajectory has a total of 16 turns, and the pitch is -0.55m. The initial radius can be obtained as 8.8m, and during the winding process, the handle entering the path curve is always on the spiral. The bench dragon starts to wind clockwise from A (8.8, 0) and ends when the curve passes through the origin. The speed of the front handle of the faucet is always maintained at 1m/s. Its movement can be regarded as several fixed-length line segments moving on the spiral, where the endpoints of the line segments are all on the spiral, as shown in Figure 3.



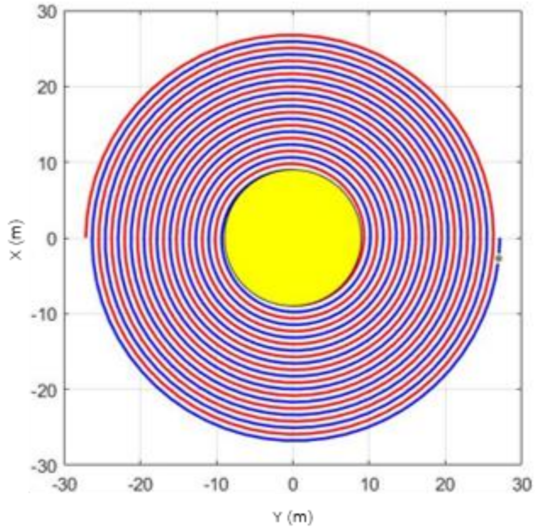


Figure 3. Coiling process of the bench dragon **Figure 4.** Turnaround area of the bench dragon

Given the actual size of the bench, when the bench dragon coils for a while, the dragon head may collide with a part of the dragon body. This model will calculate the specific moment when the bench dragon collides under the conditions of a given pitch and initial radius.

2.2. U-turn path simulation based on four end-to-end Archimedean spirals

The actual performance of the bench dragon includes winding in, turning around and winding out. The specific path is shown in Figure 4, where the yellow circle is the turning area, the blue spiral is the winding in path, and the red spiral is the winding out path.

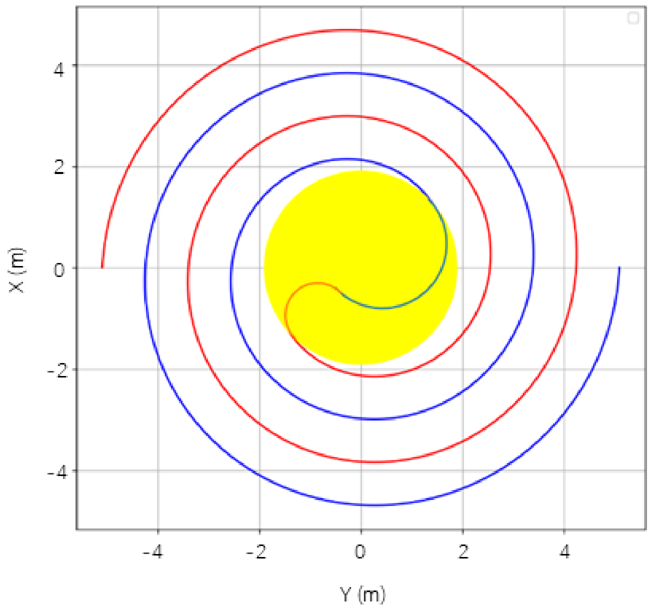


Figure 5. Local paths for spiral in, turnaround, and spiral out

During the performance, the pitch of the winding spiral is 1.7 m, the winding spiral is symmetrical with the winding spiral about the center of the spiral, the U-turn space is a circular area with a diameter of 9 m and the center of the spiral. The U-turn path is an S-shaped curve formed by two tangent arcs. The radius of the first arc is twice that of the second arc, and it is tangent to both the winding spiral and the winding spiral.

The model will find the factors that restrict the U-turn and thus find the shortest U-turn path. That is to say, after entering the U-turn area, it is not necessary to start the U-turn directly, but to continue winding for a period of time before making a U-turn. Therefore, the radius of the real U-turn area will be less than 9m, and the local path is shown in Figure 5.

3. Results

3.1. Analysis of collision conditions of bench dragon

Collision is the phenomenon that dots at two different rectangles replaced two different benches overlap. In this question collision concretely presents in Figure 4, which means the outer front vertex of a certain board overlaps with a point on the inner long side of a certain plate on the outer ring.

For length of dragon head longer than length of dragon body, the collision must occur when the first board collides with one of the outer boards.

For a given Archimedean spiral as (1), where a represents 8.8m, p equals to -0.55. The expression for the trajectory length of the dragon head is

$$s = \int_0^{\theta} \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta \tag{2}$$

Taking (1) to (2), we can get

$$s = \int_0^{\theta} \sqrt{\left(\frac{p}{2\pi}\right)^2 + \left(a + \frac{p}{2\pi}\theta\right)^2}\, d\theta \tag{3}$$

According to Newton-Raphson method [14], Firstly estimate an approximate value of the solution and set it as θ_0, then use the iterative method to continuously update θ until an approximate exact solution that is infinitely close to the root of the equation can be obtained.

The conversion relationship between polar coordinates and Cartesian coordinates is as follows.

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \tag{4}$$

Substituting (1) into (4) we get

$$\begin{cases} x = \left(a + \dfrac{p}{2\pi}\theta\right)\cos\theta \\ y = \left(a + \dfrac{p}{2\pi}\theta\right)\sin\theta \end{cases} \tag{5}$$

The parametric equation of the faucet trajectory equation can be obtained, so the coordinates of the faucet front handle(x_{head}, y_{head})are

$$\left(\left(a + \frac{p}{2\pi}\theta\right)\cos\theta\ ,\ \left(a + \frac{p}{2\pi}\theta\right)\sin\theta\right) \tag{6}$$

Next, we use the iteration method to find the coordinates of each handle: if we know the coordinates of the front handle, then we use the coordinates of the front handle as the center and the distance between two adjacent handles as the radius to make a circle that intersects the spiral. When the angle between the line connecting the front handle and the origin of the coordinates and the line connecting the intersection point and the origin of the coordinates does not exceed π, we have determined the position of the next handle and can get its coordinates. We already know the coordinates of the front handle of the dragon head, so we can deduce the coordinates of the front handle of the first section of the dragon body, and by analogy, we can get the coordinates of all the handles.

Assuming that the dragon head has collided with a section of the dragon body, the coordinates of the front and rear handles of the dragon head at all times can be calculated. From Figure 1, assuming that the coordinates of the front handle of the dragon head are (x_i, y_i), the coordinates of the rear handle are (x_j, y_j), and $i \neq j$, a straight line is determined between the two points, and the general formula of the l_1 equation can be obtained as:

$$\left(y_i - y_j\right)x - \left(x_i - x_i\right)y + x_i y_j - x_i y_i = 0 \tag{7}$$

Since l_1 is parallel to l_2, we use the method of undetermined coefficients to set the constant term of the general form of the l_2 equation as C, and use the two-line distance formula to solve C, and we can get the equation of the line l_2:

$$d_1 = \frac{\left|x_i y_j - x_j y_i - C\right|}{\sqrt{\left(y_i - y_j\right)^2 + \left(x_i - x_i\right)^2}} \tag{8}$$

Draw a perpendicular line through the point (x_i, y_i) about l_1, denoted as l_3, and we can get the general equation as follows:

$$\left(x_i - x_i\right)x - \left(y_i - y_j\right)y + x_i^2 + y_i^2 - x_i x_j - y_i y_j = 0 \tag{9}$$

Since l_3 is parallel to l_4, the constant term of the general formula of the l_4 equation is set to M using the method of undetermined coefficients. Similarly, the distance formula between two lines is used to solve M, and the equation of the line l_4, is obtained:

$$d_2 = \frac{\left|x_i^2 + y_i^2 - x_i x_j - y_i y_j - M\right|}{\sqrt{\left(y_i - y_j\right)^2 + \left(x_i - x_i\right)^2}} \tag{10}$$

The coordinates of the collision point can be obtained by solving the equations for l_2, and l_4,.

The polar angle of the collision point is θ_{CP}, and the dragon head is located on the track of one pitch unit inside the collided dragon body. Then the polar angle values of the front and rear handles on the corresponding section of the collision dragon body should meet the following conditions:

$$\theta_1 \leq \theta_{\text{CP}} - 2\pi \leq \theta_2 \tag{11}$$

Iteratively examine the relationship between the polar angle of the front and rear handles of each board and the polar angle of the collision point. When equation (11) is satisfied for the first time, we can determine which section of the dragon body the dragon head collides with. The coordinates of the front and rear handles of the colliding dragon body can uniquely determine a straight line. The equation of the straight line where the expected collision point is located at the edge of the dragon body can be uniquely determined from the known equations. At the same time, we can determine the coordinates of the expected collision point of the dragon head. The polar point forms a straight line with this point, and the intersection of the two straight lines is the expected collision point of the dragon body. When the absolute value of the ordinate of the expected collision point of the dragon body is equal to the absolute value of the ordinate of the expected collision point of the dragon head, it is the end time of the dragon dance team's coiling.

Figure 7 shows the specific state of the bench dragon when it collides. In order to clearly and intuitively reflect the collision scene, we only draw the specific shapes of the two colliding boards, of which the green rectangle is the dragon head and the blue rectangle is the eighth dragon body; the rest of the boards are replaced by the front and rear handles and the lines between them, and the green dots represent each handle. After calculation, the dragon head and the eighth dragon body collided at 414.6 seconds.

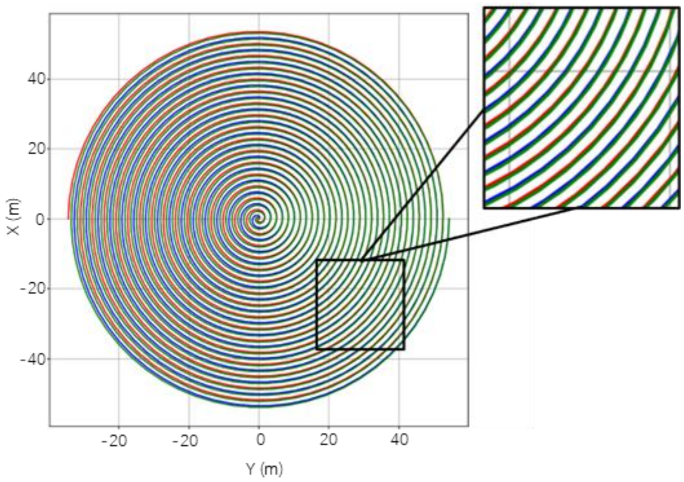


Figure 6. Comparison of the bench dragon routes

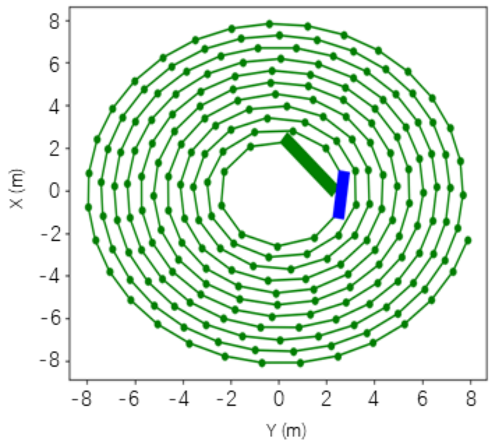


Figure 7. Bench dragon collision

3.2. Calculation of the shortest U-turn route for the bench dragon

Analyze the factors that may cause the inability to turn around. First, consider whether a collision will occur when the faucet is outside the turning space. From previous calculations, we can know that when the pitch is 0.55m, the faucet can reach the 12th circle. Therefore, when the pitch is 1.7m, the probability of collision is extremely low. Even when the faucet walks out of the turning area, the two adjacent arcs are 0.85m apart, and the probability of collision is still low.

In Figure 6, the blue track is the entry track of the bench dragon, the red track is the exit track of the bench dragon, and the green track is the 32-turn Archimedean spiral with a pitch of 0.85m. In the right half of the figure, the route of the bench dragon and the 32-turn Archimedean spiral with a pitch of 0.85m are almost partially overlapped, and the route of the bench dragon is centrally symmetrical, so the Archimedean spiral with a pitch of 0.85m can be used instead to examine its collision situation. Due to the widening of the pitch, it is initially speculated that no collision will occur.

Then, considering that the bench dragon is a festival custom, it should have a good meaning, so each handle should not retreat. The limit radius of the rear handle that does not retreat when the faucet passes a certain position on the arc is calculated to obtain the shortest U-turn path.

After obtaining the shortest U-turn path, the position of the rear handle of the faucet at each time and the distance from the origin are examined to verify that the bench dragon does not retreat. For the convenience of calculation, the two semicircles are approximated as Archimedean spirals with their centers at the intersection of the original two semicircles, and the maximum polar angle does not exceed $\pi/2$. The calculation error is within 0.7s and only occurs in the U-turn area. In actual situations, the path of the spiral is not much different from the path of the semicircle, so it is acceptable.

First, we examine whether a collision will occur. Calculations show that for a 32-turn Archimedean spiral with a pitch of 0.85m: the faucet collides with the first board; a collision occurs at 2727.40 seconds; the distance between the faucet and the board at the time of collision is 1.40. Therefore, it is concluded that no collision actually occurred.

Then, we examine the limit conditions for the bench dragon not to retreat. As can be seen from Figure 6, the most likely position for retreat is near the intersection of the two arcs at both ends. At this time, the rear handle of the faucet is outside the U-turn range, so the directions of the front handle and the rear handle are roughly opposite, so it the data below comes from the paper "A therizinosauroid dinosaur with integumentary structures from China" written by Xing Xu, Zhi-lu Tang and Xiao-lin Wang. is possible that the front handle moves forward and the rear handle is forced to move backward to ensure that the distance between the front and rear handles remains unchanged.

Therefore, we examine the two quarter-circle arcs connected in the two semicircular arcs, take $2\pi/7200$ as the step length, judge whether there is a retreat phenomenon on the arc, and find the shortest radius of the U-turn area where there is no retreat phenomenon.

As shown in Figure 8, the front handle of the faucet in the red line segment moves forward 2π/7200 to obtain the blue line segment, but the rear handle of the faucet moves backward.

The minimum radius of the U-turn area is calculated to be 1.9m.

Taking 1.9m as the radius of the U-turn area, continue to calculate the position and speed information of each handle at each moment.

Since in the previous calculation, the input of any point coordinates (x, y), the initial radius a and pitch p of the Archimedean spiral with the spiral center at the origin and the initial position on the x-axis, the lower limit m of the polar diameter of the target point and the distance *dist*, the output point coordinates (x_1, y_1) with the smallest polar diameter on the spiral but greater than m and a fixed distance *dist* from (x_1, y_1) are achieved.

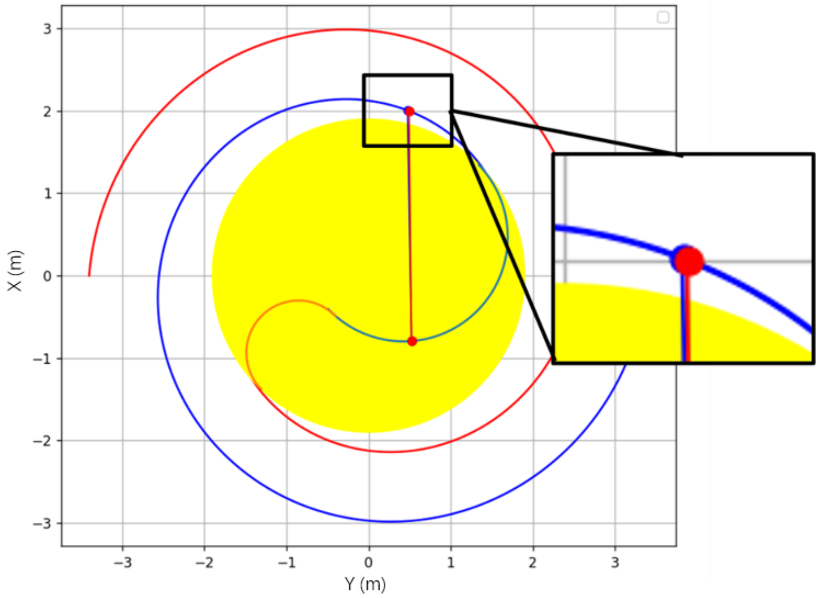


Figure 8. The retraction of the head of the bench dragon

Through the transformation of translation and rotation matrices, the above function can be applied to any Archimedean spiral. The rotation matrix is

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{12}$$

Therefore, for the convenience of calculation, we try to approximate the two semicircular arcs as two Archimedean spirals.

As shown in Figure 9, the polar diameter of the large spiral is twice that of the small spiral, and the maximum polar angle does not exceed π/2. The centers of the two spirals are located at the intersection of the original two semicircular arcs. By rotating, the two ends of the arc can be connected to the ends of the spirals entering and exiting the U-turn area.

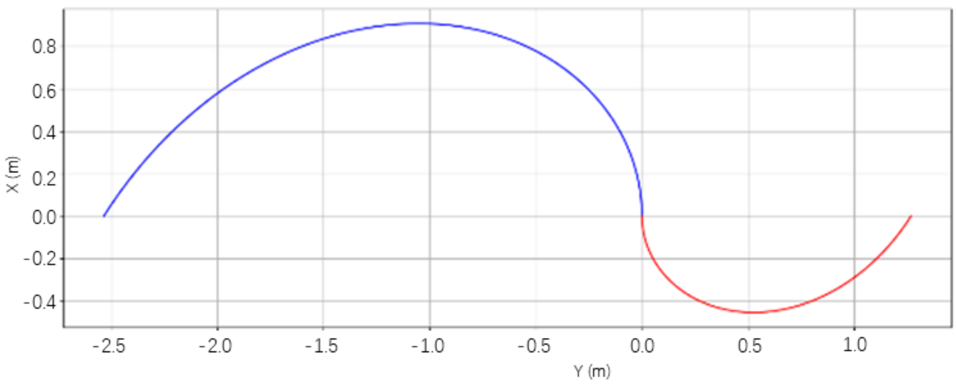


Figure 9. Approximate Shape of U-turn Trajectory

After calculation, the error of approximating the arc to the Archimedean spiral is -0.94m, which is acceptable.

Try to consider more situations and design a function to use the previous coordinate as a condition to deduce the next coordinate. The function considers the following 8 cases:

(1) The previous point is on the spiral entering the U-turn area, and the next point must be on the spiral entering the U-turn area

(2) The previous point is on the large spiral in the U-turn area, and the next point is on the spiral entering the U-turn area

(3) The previous point is on the large spiral in the U-turn area, and the next point is also on the large spiral in the U-turn area

(4) The previous point is on the small spiral in the U-turn area, and the next point is on the spiral entering the U-turn area

(5) The previous point is on the small spiral in the U-turn area, and the next point is on the large spiral in the U-turn area

(6) The previous point is on the spiral leaving the U-turn area, and the next point is also on the spiral leaving the U-turn area

(7) The previous point is on the spiral leaving the U-turn area, and the next point is on the small spiral in the U-turn area

(8) The first point is on the spiral line that leaves the U-turn area, and the second point is on the large spiral line in the U-turn area.

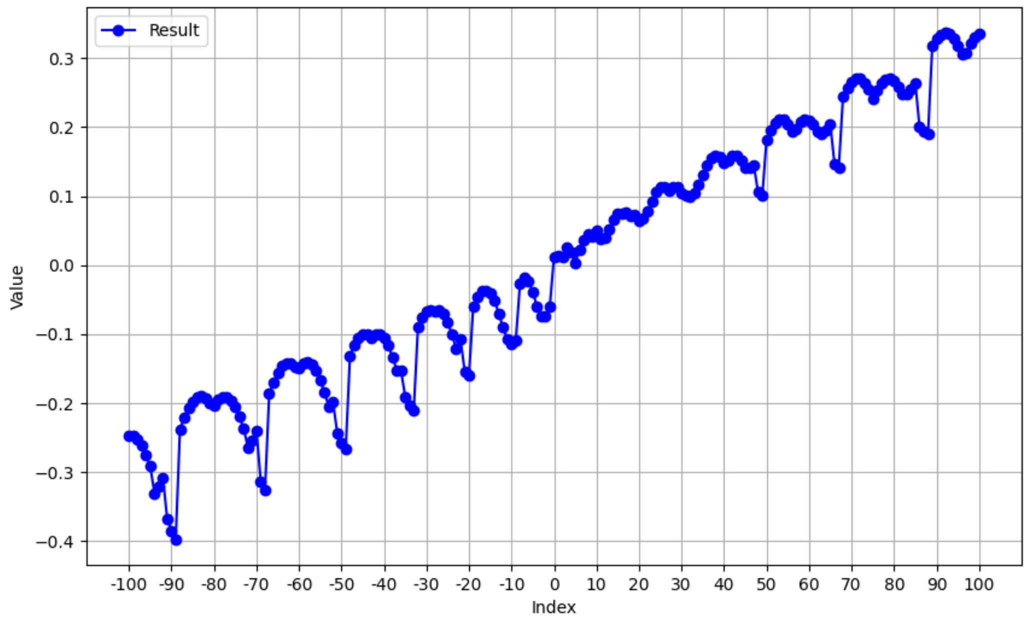


Figure 10. Difference in Distance from the Origin

When we take the intersection of the two spiral lines where the front handle of the faucet passes through the U-turn area as 0s, we use the coordinate information of the front handle of the faucet to find the distance between the rear handle of the faucet and the origin from -100s to 100s, calculate the difference relationship of the distance, and draw a line graph as shown in Figure 10.

It can be seen that the difference value changes from negative to positive, so the distance between the rear handle and the origin first decreases and then increases, and there is no backward movement. Therefore, the minimum turning distance can be obtained to be about 5.969m.

4. Conclusion

During the performance of the bench dragon, it is necessary to ensure that there is no collision during the process of moving, and to leave enough space for the dragon dance team to turn around. We designed the Archimedean spiral model, and based on this model, we conducted research on collision avoidance and path planning for the bench dragon performance. This model is close to application needs and real life, and makes full use of methods such as combination of numbers and shapes, iteration method, differential method, and Python visualization. On the premise of proposing some basic assumptions, it has done its best to accurately analyze the problems of collision avoidance and path planning during the performance of the dragon dance team. The study shows that the model combines numbers and shapes, depicts the path and position information on the coordinate axis, so that it can be converted into a specific geometric problem to solve. Through Python, the mathematical analysis is visualized, and the trajectory and turning path of the dragon dance team can be intuitively displayed, making the model more vivid and intuitive, and easy to understand and apply. At the same time, the model takes into account that as a festival custom, its meaning cannot contain unlucky factors such as retreat, so as to ensure that the bench dragon will not be separated from the actual application of the festival. Path planning and collision avoidance have certain practical value. They can be used in the platform economy, such as the design of delivery routes and service routes for express delivery, postal services, takeout, and online car-hailing. They are the key to enhancing user experience and improving service efficiency. At the same time, in the era of artificial intelligence, driverless cars, mobile robots, etc. should plan a path from the initial position to the target position that can avoid static and dynamic obstacles along the way based on user instructions, and should meet a series of requirements such as short path, high efficiency, and high safety.

Finally, the model still has certain shortcomings. For example, the structure and movement of the dragon dance team are simplified in the model, and the connection of all benches is regarded as a rigid connection, without considering the flexibility factors in the actual performance. Moreover, the model uses spirals at both ends instead of arcs at both ends for solution. Although the error is within the controllable range, the result is still an approximation, not an exact value. In subsequent research, these shortcomings can be optimized to better spread Chinese traditional culture.

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