Research on Production Decision-Making of Assembly Enterprises Based on Dynamic Bayesian Sampling and Multi-Stage Optimization

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Abstract. This paper focuses on the production decision-making problems of electronic product assembly enterprises, and comprehensively uses probability theory, statistics and optimization theory to construct a series of innovative models and algorithms. For the sampling detection of spare parts, hypothesis testing and Bayesian step-by-step adjustment model based on normal distribution improvement are proposed to effectively balance the detection cost and decision-making reliability. In terms of production decision optimization, dynamic programming, backtracking algorithm and generalized multivariate scheme planning model are used to accurately analyze different production scenarios and determine the optimal strategy at each stage. Through multi-model and algorithm collaboration, it helps enterprises optimize costs and improve benefits in complex production environments, and provides efficient and practical solutions for production decision-making in the assembly industry, which has significant application value.

Keywords: Bayesian derivation; dynamic programming; backtracking algorithm.

1. Introduction

In the electronic product assembly industry, enterprises are faced with challenges such as unstable quality of spare parts and complex production processes, and how to formulate reasonable sampling and testing plans and production decisions to achieve cost control and efficiency improvement has become an urgent problem to be solved [1]. Although the existing research involves related issues to a certain extent, it is difficult to fully take into account the various uncertainties and complex constraints in the production process [2]. Based on this background, this paper innovatively integrates a variety of mathematical models and algorithms. Hypothesis testing and Bayesian method are used to design sampling detection schemes to ensure the accuracy of spare parts quality assessment. With the help of dynamic programming, backtracking algorithm and generalized multivariate scheme planning model, the costs and benefits of each stage of production are deeply analyzed to optimize production decisions. It aims to provide a set of scientific and systematic decision-making methods for assembly enterprises to help them enhance their competitiveness in the fierce market competition.

2. Sampling Detection Scheme Based on Hypothesis Testing and Bayesian Method

Determine whether the defective rate of spare parts provided by the supplier meets the nominal value through sampling inspection. It is necessary to design a sampling test scheme with as few tests as possible, and make a decision on the acceptance or rejection of the spare part given two different levels of reliability. To this end, this paper uses two methods to build the model, including the hypothesis testing method of frequency science and the Bayesian improvement model [3,4]. The hypothesis testing method can calculate the sample size based on a set level of significance to ensure judgment accuracy. The improved Bayesian model can gradually update the information according to the detection results, which can optimize the detection process and improve the confidence of the results.

2.1. Establishment of sampling detection model based on hypothesis testing

Step1: Set p to the real defective rate of spare parts; p_0 of the nominal defective rate 10%; n is the number of spare parts samples, and x is the number of products in the sample.
Step2: Constructs two hypotheses:

Null hypothesis H_0: the defective rate of spare parts is $p < p_0$, that is, the defective rate of spare parts does not exceed the nominal value, and the spare parts can be received;

Alternative hypothesis H_1: the defective rate of spare parts is $p > p_0$, that is, the defective rate of spare parts exceeds the nominal value, and the spare parts are refused;

Step3: If the defective rate of spare parts is p, the pass rate is $1-p$, n is the number of samples, and the number of defective products X in the sampling process obeys the binomial distribution,

$$X \sim B(n,p) \tag{1}$$

When the sample volume n is large, the binomial distribution is approximately normal:

$$X \sim N(np, np(1-p)) \tag{2}$$

The $\hat{p}$ defective rate of the sample, p_0 is the known nominal defective rate, is normalized to the standard normal distribution by the normalized formula of the normal distribution:

$$\hat{p} = \frac{X}{n} \tag{3}$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{4}$$

With this formula, sampling tests can be standardized to test hypotheses at different levels of significance.

2.2. Establishment of improved Bayesian stepwise adjustment model based on normal distribution

Step 1: Build a model framework based on Bayesian method and prior distribution

In this paper, we first construct a framework of Bayesian method, in which by defining the prior distribution, pr_a is the quantity of defects, pr_b is the quantity of genuine products, which indicates the preliminary understanding of the defective rate of spare parts of the enterprise. Here, the prior distribution uses weaker prior information in order to make the actual test data more influential on the results. By progressively updating the posterior distribution, the system adjusts the estimate of the defect rate based on the number of defects observed as the inspection process progresses.
Step2: Optimize the Bayesian model by incorporating the hypothesis testing of normal distribution

Combined with the hypothesis test of frequency, the approximate normal distribution of spare parts is used, and the required sample size under different conditions can be obtained by calculating the critical value sum of the normal distribution under different confidence levels, i.e., the defective product rate exceeds the nominal value under 95% reliability and the rejection rate exceeds the nominal value under 90% reliability. By calculating the critical sum of normal distribution Z_{95} and Z_{90}, separately, the sample size required under different conditions can be obtained.

Let Z_α be the standard normal distribution cut-off for the significance level, p_0 which is the known nominal defect rate, and AEM is the acceptable margin of error, which can be obtained as follows:

$$n = \frac{Z_{\alpha}^{2} \cdot p_0 \cdot (1 - p_0)}{Aem^{2}} \tag{5}$$

Step3: Then determine whether the sample size n of the current test exceeds the minimum sample size calculated by normal distribution, and terminate the detection when the conditions are met, and the minimum sampling detection amount and the upper limit of Bayesian posterior distribution can be obtained.

2.3. Solving and analysis of sampling detection model based on hypothesis testing

The sampling scheme is designed to reject the batch of spare parts when the defective rate exceeds the nominal value under 95% reliability, and the significance level $\alpha = 0.05$ corresponds to the normal distribution critical value $Z_{\frac{\alpha}{2}} = 1.96$.

Null hypothesis H_0: $p = p_0 = 0.10$ that is, the defective rate of spare parts is 10%;

The rejection condition is that the batch of spare parts is rejected when the defective rate detected in the sample $\hat{p}$ is large enough.

When:

$$\hat{p} \geqslant 0.10 + 1.96 \times \sqrt{\frac{0.10(1 - 0.10)}{n}} \tag{6}$$

According to this formula, as the number of samples n increases, the critical value of the defective rate in the sample can be determined, and from this it can be judged whether to reject or not.

The sampling scheme is designed so that the batch of spare parts is received when the defective rate is determined not to exceed the nominal value under 90% reliability, and the significance level $\alpha = 0.10$ corresponds to the critical value of normal distribution $Z_{\frac{\alpha}{2}} = 1.645$.

Null hypothesis H_0: $p = p_0 = 0.10$ that is, the defective rate of spare parts is 10%;

The condition of acceptance is that the defective rate detected in the sample $\hat{p}$ is small enough to receive the spare parts.

When:

$$\hat{p} \leqslant 0.10 + 1.645 \times \sqrt{\frac{0.10(1 - 0.10)}{n}} \tag{7}$$

According to this formula, as the number of samples n increases, the critical value of the defective rate in the sample can be determined, and from this it can be judged whether to reject or not.

2.4. Bayesian stepwise adjustment model based on normal distribution

A sampling detection scheme was designed to reject the batch of parts if the nominal value exceeded 10% under 95% reliability, and the model was required to exclude poor quality parts under high reliability.

The nominal defective rate is known $p_0 = 0.10$, where the defective rate is unknown, the number of samples is n, and the number of defective products X obeys a binomial distribution.

Let's start by assuming a weaker prior distribution,

$$Beat（1，9） \tag{8}$$

Represents a preliminary understanding of the rejection rate, and then updates the Bayesian posterior distribution based on the number of defects observed after each sampling,

$$Beat(1+k, 9+n-k) \tag{9}$$

Using the hypothesis test of normal distribution, the cut-off value at 95% confidence was calculated $Z_{0.95}$ and the sample size was sufficient,

$$n \geqslant \frac{Z_{\frac{\alpha}{2}}^2 \cdot p_0 \cdot (1-p_0)}{Aem^2} \tag{10}$$

Replace $Z_{\frac{\alpha}{2}}$ =1.645 and $Aem = 5\%$ substitution, and test each sample in cycles, and gradually update the Bayesian posterior distribution until the sample size satisfies the condition of normal distribution. When the sample volume is sufficient, calculate the upper limit of Bayesian posterior distribution to determine whether the rejection condition is met.

Under 90% reliability, if the nominal value is less than or equal to 10%, the sampling detection scheme of receiving this batch of parts is designed, and the model is required to receive qualified parts under high reliability.

The nominal defective rate is known $p_0 = 0.10$, where the defective rate is unknown, the number of samples is n, and the number of defective products X obeys a binomial distribution. Using the same Bayesian framework as at 95% reliability, a weaker prior distribution is assumed to be the same (8).

Represents a preliminary understanding of the rejection rate, and then updates the Bayesian posterior distribution based on the number of defects observed after each sampling

Using the hypothesis test of normal distribution, the cut-off value at 90% confidence was calculated $Z_{0.90}$ and the sample size was judged to be sufficient,

$$n \geqslant \frac{Z_{\frac{\alpha}{2}}^2 \cdot p_0 \cdot (1-p_0)}{Aem^2} \tag{11}$$

Substitute $Z_{\frac{\alpha}{2}}$ =1.96 and $Aem = 10\%$ substitute, and test each sample in a cycle, and gradually update the Bayesian posterior distribution until the sample size satisfies the condition of normal distribution. When the sample volume is sufficient, the upper limit of Bayesian posterior distribution is calculated to determine whether the receiving conditions are met.

2.5. Analysis of results

Based on the above model we get the detection results. Figure 1 shows how the rate of defect estimated by Bayesian posterior changes as the sample size increases (solid blue line), along with its 95% confidence interval (shaded in blue). The red dotted line represents the true nominal defect rate (10%). From this plot, it can be observed that as the sample size increases, the Bayesian estimation tends to the true defect rate, the confidence interval gradually converges, and the estimation becomes more and more accurate. Figure 2 shows the cumulative increase in the number of defective products detected as the sample size increases. This helps to observe the variation and cumulative trend of the number of defective products during the inspection process.

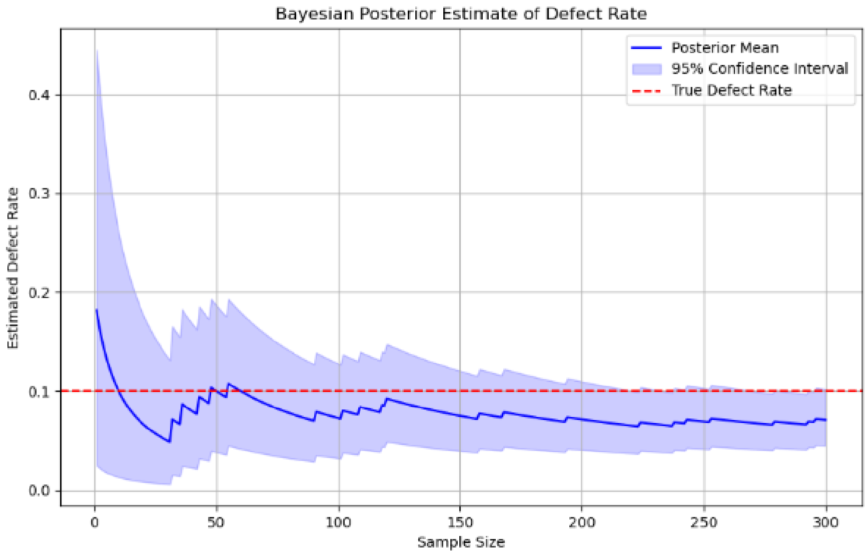


Fig. 1 Bayesian posterior estimation of the defective rate variation

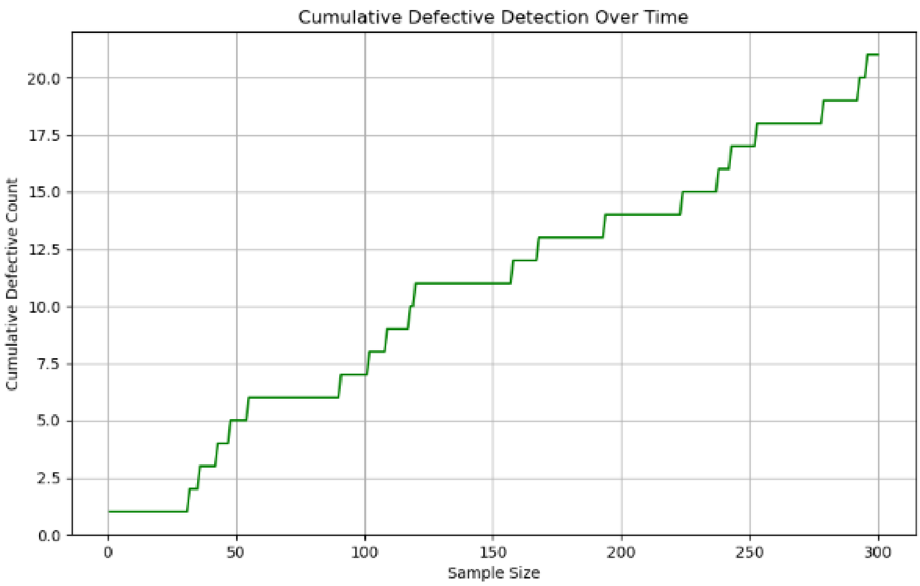


Fig. 2 Cumulative growth trend of the number of defective products

Figure 3 shows how the error (absolute value) between the rate of defect estimated by the Bayesian method and the true rate of defect changes as the sample size increases. In general, as the sample size increases, the estimation error gradually decreases, meaning that the model becomes more accurate. Figure 4 represents the change in Bayesian posterior distribution at different sample sizes. With the increase of sample size, the posterior distribution gradually becomes more concentrated from relatively scattered, indicating that with the increase of data, the model estimates the defective rate more accurately.

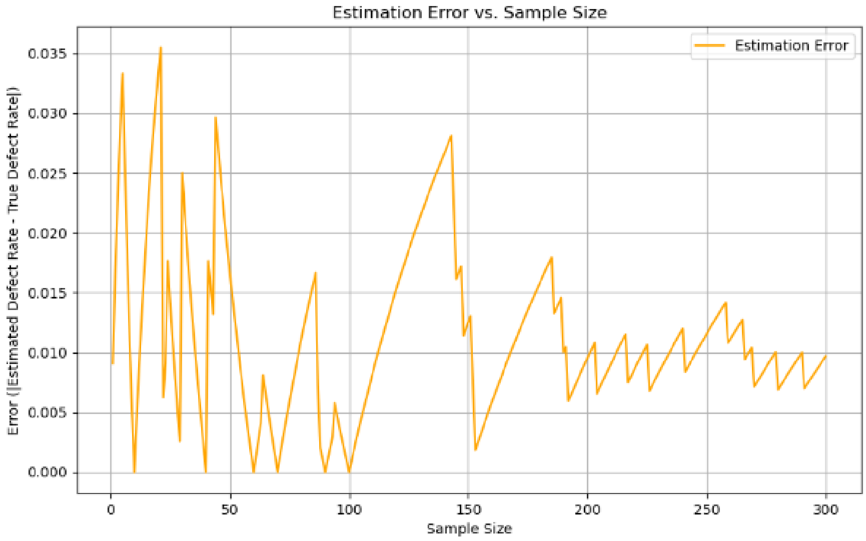


Fig. 3 Relationship between sample size and estimation error

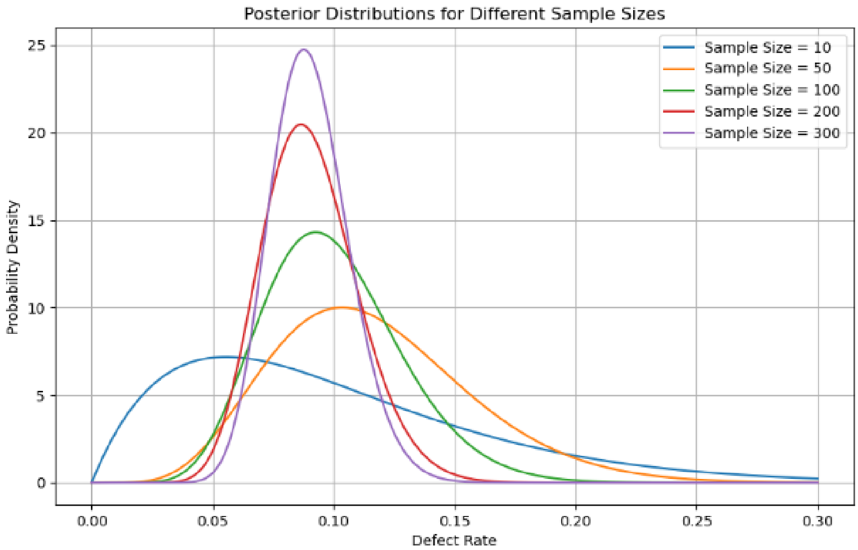


Fig. 4 Variation of posterior distribution under different sample sizes

3. Optimization of Ddecision-making in the Production Phase Based on Dynamic Planning

Comprehensive decision-making optimization for all stages of the production process. It is necessary to design a decision-making plan that will maximize the benefits for the business. Specifically, it includes the detection of spare parts and finished products, the treatment of unqualified finished products, and the treatment of unqualified finished products returned by users, and finally integrates the cost of each decision-making link, so that the decision-making scheme can maximize the benefits for the enterprise, minimize the production cost and the loss of defective products, and ensure product quality to a certain extent. To this end, this paper uses mathematical derivation to quantify the costs and benefits, and constructs a system of equations through dynamic programming retrospective optimization to solve, and finally obtains the optimal decision combination and corresponding costs, which helps enterprises implement specific decisions at each stage [5].

3.1. Model solving and analysis

Memory search optimization model based on dynamic programming and backtracking algorithms, introduce the following variables,

Product defect rate: 1 defective rate of spare parts p_1, 2 defective rate of spare parts p_2, and defective rate of finished products p_f.

Cost variables: spare parts purchase cost a, spare parts testing cost c, finished product testing cost c_d, assembly cost c_a, replacement cost c_s, dismantling cost c_t.

First, decide whether to inspect parts 1 and 2 to reduce the rate of defective parts after assembly, calculate the cost of inspection, and the impact of the reduced defects on the revenue of the finished product.

First of all, assuming that the total number x of parts produced OP_1 is the decision-making factor of whether to execute the first stage, and the rate of 1 defective parts is known p_1, the quantity of spare parts 1 that needs to be purchased x_1 is as follows,

$$x_1 = x \cdot (1 - OP_1) + \frac{x \cdot OP_1}{1 - p_1} \tag{12}$$

The formula shows that if the detection is performed, then OP_1 = 1, the defective parts can be detected and discarded, and the number of spare parts to be purchased is,

$$x_1 = \frac{x}{1 - p_1} \tag{13}$$

If no inspection is performed, it = OP_1 0, that is, all spare parts are regarded as qualified products,

$$x_1 = x \tag{14}$$

And the genuine rate of spare parts 1 is,

$$B_1 = 1 - p_1(1 - OP_1) \tag{15}$$

In the same way, the quantity of spare parts 2 that needs to be purchased can be obtained x_2 as follows,

$$x_2 = x \cdot (1 - OP_2) + \frac{x \cdot OP_2}{1 - p_2} \tag{16}$$

Spare parts 2 genuine rate,

$$B_2 = 1 - p_2(1 - OP_2) \tag{17}$$

Based on the above formula, the detection cost of spare parts 1 is known c_1, and the purchase cost is a_1, and the total cost of spare parts 1 can be obtained C_{p1} as,

$$C_{p1} = a_1 \cdot x_1 + c_1 \cdot x_1 \cdot OP_1 \tag{18}$$

Similarly, the total cost of spare part 2 C_{p2} is,

$$C_{p2} = a_2 \cdot x_2 + c_2 \cdot x_2 \cdot OP_2 \tag{19}$$

Secondly, the decision is whether to test the finished product after assembly, and the OP_3 decision factor of whether to carry out the detection of the finished product is calculated to avoid the loss of the defective product entering the market.

The number of products produced is known x, the assembly cost is, d and the testing cost is c_3, and the total cost of the finished product assembly link can be obtained $C_{assembly}$,

$$C_{assembly} = d \cdot x + c_3 \cdot x \cdot OP_3 \tag{20}$$

The formula shows that if the test is performed, then OP_3 = 1, it can be measured that the defective products of the finished products do not flow into the market, and the cost of finished product testing needs to be added $c_3 \cdot x \cdot OP_3$; If not inspected, the assembly cost is simply the product of the price of assembling a single product and the number of products produced.

Then, the decision is whether to dismantle the unqualified finished product, which OP_4 is the decision-making factor of whether to carry out the dismantling of the finished product, and the dismantled spare parts can be reused, and the dismantling cost and recoverable income after dismantling are calculated.

Knowing the cost of dismantling c_t, equation (15) and (17) obtains the genuine rate of spare parts and B_1, B_2 the defective rate of finished products p_f,

$$C_{disassemble} = c_t \cdot x \cdot (1 - B_1 \cdot B_2 + B_1 \cdot B_2 \cdot p_f) \cdot OP_4 \tag{21}$$

The formula shows that if the disassembly is performed, then OP_4 = 1, if the parts are qualified and successfully assembled, the probability of the event is $B_1 \cdot B_2 \cdot (1 - p_f)$, then the probability of failure is,

$$1 - B_1 \cdot B_2 + B_1 \cdot B_2 \cdot p_f \tag{22}$$

The cost to be dismantled is the product of the failure rate and the number of finished products and the cost of dismantling.

Finally, the finished product returned by the user needs to be exchanged, and the exchange loss will be generated at this stage. The returned finished product can be inspected and dismantled again, and the previous steps can be repeated.

Knowing the replacement loss c_s, the number of finished products x, and the probability of unqualified finished products (27), the decision factor of finished product testing is OP_3, and the available exchange cost is,

$$C_{swap} = c_s \cdot x \cdot (1 - B_1 \cdot B_2 + B_1 \cdot B_2 \cdot p_f) \cdot (1 - OP_3) \tag{23}$$

The formula shows that the product exchange loss will only occur without the inspection of the finished product, and the replacement cost is the product of the number of unqualified finished products and the exchange loss of a single product.

It is known that the selling price of the product is e, the quantity of the product is x, and the probability of the product being qualified is $B_1 \cdot B_2 \cdot (1 - p_f)$, and the profit from the sale of the product is,

$$I = e \cdot x \cdot B_1 \cdot B_2 \cdot (1 - p_f) \cdot OP_3 + (1 - OP_3) \cdot e \cdot x \tag{24}$$

The formula shows that in the case of performing finished product inspection, the revenue is the product of the selling price, the probability of product qualification, and the number of products produced; In the absence of finished product testing, revenue is the product of the number of finished products produced and the selling price.

And because the dismantling of unqualified products will also get part of the income, it is known that the cost of spare parts 1 and 2 products is c_1、c_2, the genuine product rate B_1、B_2, and the finished product assembly failure rate $b_3 = 1 - p_f$ is the dismantling income is,

$$I_{disassemble} = (B_1 \cdot B_2 \cdot b_3 \cdot (a_1 + a_2) + B_1 \cdot (1 - B_2) \cdot a_1 + B_2 \cdot (1 - B_1) \cdot a_2) \cdot x \cdot OP_4 \quad (25)$$

That is, it can be divided into three situations: there is a problem in the assembly link, and the spare parts 1 or 2 are defective products.

To sum up, the net profit is,

$$I_{profit} = I + I_{disassemble} - (C_{assembly} + C_{\det ection} + C_{disassemble} + C_{swap}) \quad (26)$$

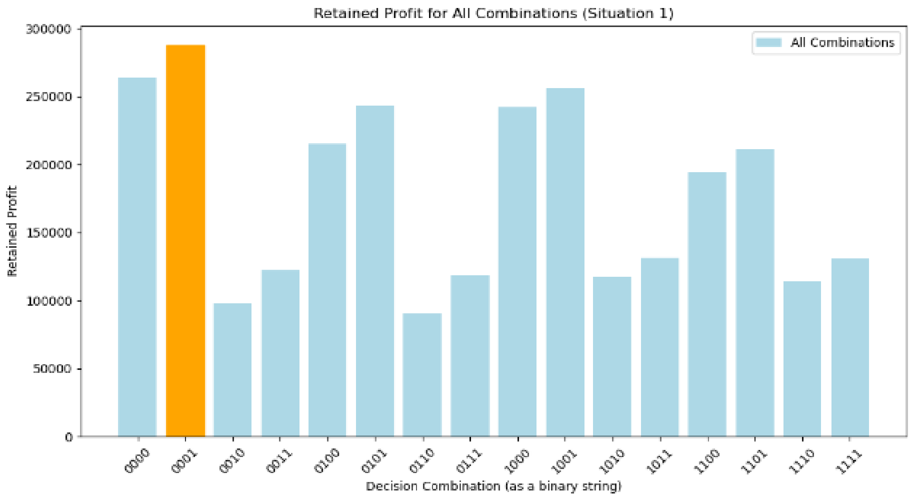
By traversing the decision-making factors at each stage OP_n, the decision-making scheme that maximizes the benefits of the enterprise is found.

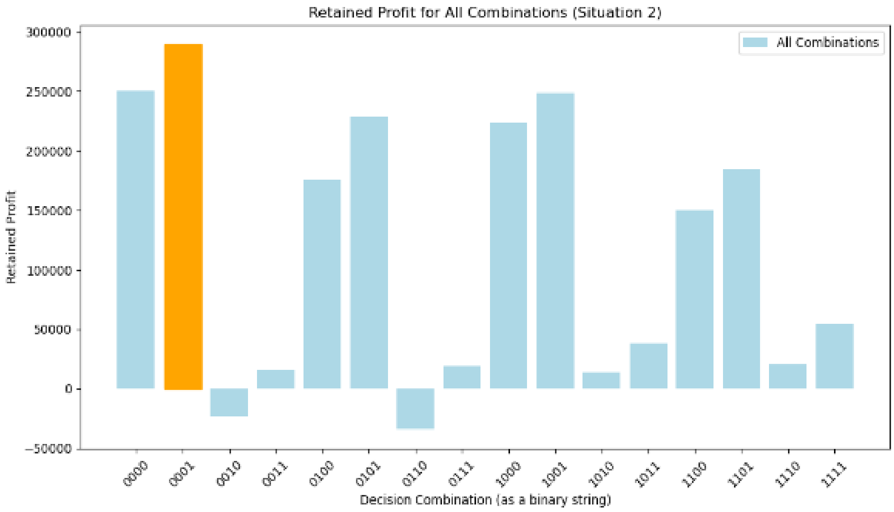
Through this process, we help companies find the best solution to maximize profits under different production and inspection strategies.

When assuming that the number of products produced by the enterprise is 10,000 pieces, the calculation result is shown in Table 1.

Table 1. Decision-making

Circumstance	Net profit (10,000 yuan)	The best combination
1	28.78	[0, 0, 0, 1]
2	28.97	[0, 0, 0, 1]
3	22.28	[0, 0, 0, 1]
4	17.47	[1, 0, 0, 1]
5	26.46	[0, 0, 0, 1]
6	26.57	[0, 0, 0, 0]


Retained Profit for All Combinations (Situation 1)
All Combinations
Retained Profit
Decision Combination (as a binary string)
0000 0001 0010 0011 0100 0101 0110 0111 1000 1001 1010 1011 1100 1101 1110 1111
300000 250000 200000 150000 100000 50000 0


Retained Profit for All Combinations (Situation 2)
All Combinations
Retained Profit
Decision Combination (as a binary string)
0000 0001 0010 0011 0100 0101 0110 0111 1000 1001 1010 1011 1100 1101 1110 1111
300000 250000 200000 150000 100000 50000 0 −50000

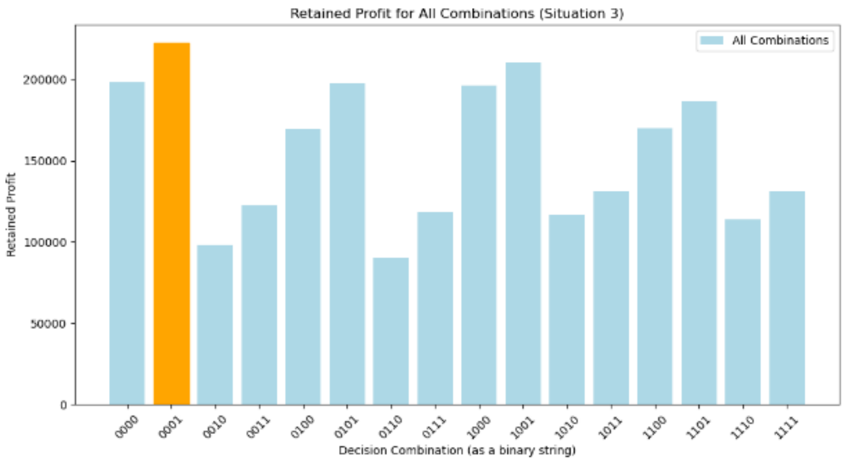

Retained Profit for All Combinations (Situation 3)
All Combinations
Retained Profit
Decision Combination (as a binary string)
0000 0001 0010 0011 0100 0101 0110 0111 1000 1001 1010 1011 1100 1101 1110 1111
200000 150000 100000 50000 0

Fig. 5 Histogram of 16 combinations

Figure.5 shows a histogram containing 16 combinations, the height of the bars represents the benefits of each combination. Set the color of the bar corresponding to the best return to orange, and the color of the rest of the combinations is light blue. For each case, the histogram shows the distribution of returns for all 16 decision combinations, with the best combinations highlighted in orange. With this chart, you can clearly see the effect of each combination.

4. Decision-making Optimization of Complex Production Process Based on Generalized Multi-Scheme Planning

Comprehensive decision-making optimization of the production process of 2 processes and 8 spare parts. It is necessary to design a decision-making plan that will maximize the benefits of the business, and develop a specific strategy for each stage. Specifically, it includes the testing of various spare parts, semifinished products and finished products, the dismantling of unqualified semifinished products and finished products, and the processing of unqualified products returned by users. By integrating the costs and benefits of each decision-making link, this paper aims to maximize the benefits of the decision-making scheme, minimize the production cost and defective product loss, and ensure a certain product quality. To this end, this paper quantifies the costs and benefits of each stage through mathematical derivation, and uses dynamic programming combined with backtracking optimization to construct an equation system [6,7], and finally solves the optimal decision combination and corresponding cost, so as to help enterprises make reasonable decisions in each production stage.

4.1. Model establishment

Step1：Set model parameters and variables

The execution process is quantified into numerical values, and the corresponding decision factors can be introduced into the decision-making at each stage, and the variable has only two results, 1 is execution, and 0 is not executed

Inspection decision for parts 1-8: $\begin{cases} OP_1 \in \{0,1\} \\ \ldots \\ OP_8 \in \{0,1\} \end{cases}$

Inspection decision for semifinished products 1-3: $\begin{cases} OP_9 \in \{0,1\} \\ OP_{10} \in \{0,1\} \\ OP_{11} \in \{0,1\} \end{cases}$

Inspection decisions for finished products: $OP_{12} \in \{0,1\}$;

Disassembly decision for unqualified semifinished products 1-3: $\begin{cases} OP_{13} \in \{0,1\} \\ OP_{14} \in \{0,1\} \\ OP_{15} \in \{0,1\} \end{cases}$

Dismantling decisions for non-conforming finished products: $OP_{16} \in \{0,1\}$;

By exhaustively enumerating the combination of decision variables, the total cost under each combination is calculated, and the combination with the least cost is selected.

Step 2: Calculate the costs and benefits

The costs and benefits associated with each stage of the decision are calculated separately: when inspecting various spare parts, the cost of the inspection and the losses caused by defective products are calculated; When testing three kinds of semifinished products, calculate the cost of testing and the loss caused by defective products; When dismantling unqualified semifinished products, calculate the dismantling cost and the value of dismantled spare parts; When inspecting the finished product, calculate the cost of inspection and possible replacement losses; When dismantling the unqualified finished product, calculate the dismantling cost and the value of the semifinished product after dismantling. Based on the exchange loss and the market selling price, the final gain is calculated.

Step 3: Optimize the model using backtracking and dynamic programming

Through recursive and memory search, each combination is backtracked, the optimal solution of each stage of the decision is calculated layer by layer, and the optimal cost and corresponding decision scheme are saved for each combination. Finally, the optimal decision-making combination and corresponding cost are output to help enterprises judge whether to test spare parts and finished products, and whether to disassemble unqualified finished products.

4.2. Results analysis

The final solution of is [1,1,0,1,1,0,1,0,1,1,1,1,1,0,0,0,1], and the maximum net profit of the enterprise is 508,018.45 yuan. Figure 6 shows the profit distribution of all decision combinations in groups, the number of combinations in different profit intervals can be visualized. If the number of combinations in a certain profit range is large, indicating that the profits of most decision-making combinations are concentrated in this range, enterprises can refer to the corresponding decision-making combination model of this interval when making production decisions, understand the scope of profit concentration distribution, evaluate the impact of different decisions on profits, and judge whether the profit concentration trend of the current production decision is in line with expectations, and if not, the decision-making can be adjusted accordingly. Figure 7 shows the cumulative distribution of profits, you can intuitively understand the distribution range and trend of profits. From the rate of change of cumulative frequency, it is possible to determine where the profit is concentrated. If the cumulative frequency rises rapidly within a certain profit range, it indicates that the profit of a large number of decision combinations is concentrated in the region, and the enterprise can focus on the corresponding decision combinations in the region. Through the cumulative frequency chart, enterprises can clearly know the cumulative proportion of decision combinations at different profit levels, which provides a basis for setting profit targets and assessing decision risks. For example, if an enterprise expects to achieve a certain profit target, the cumulative frequency chart can be used to understand the proportion of decision combinations to achieve the goal and judge the difficulty of the goal.

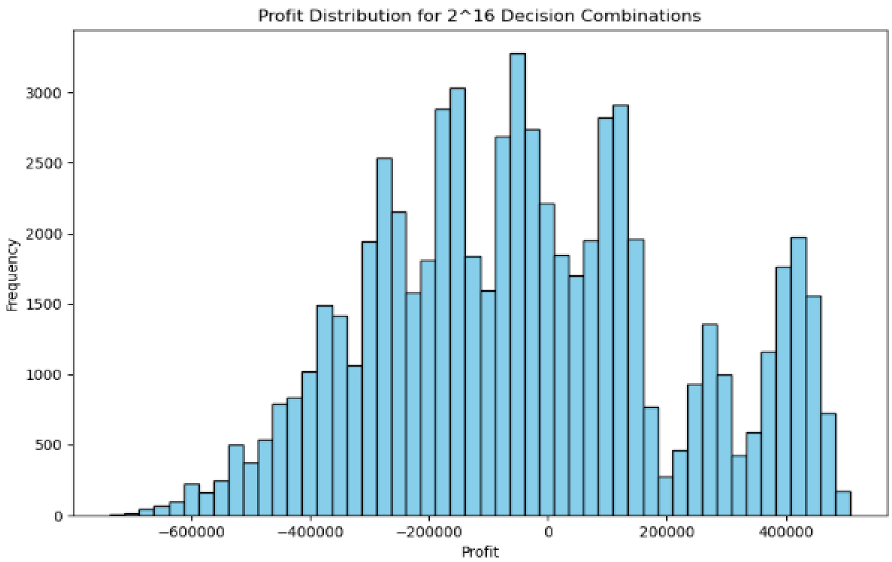


Fig. 6 Histogram of frequency distribution

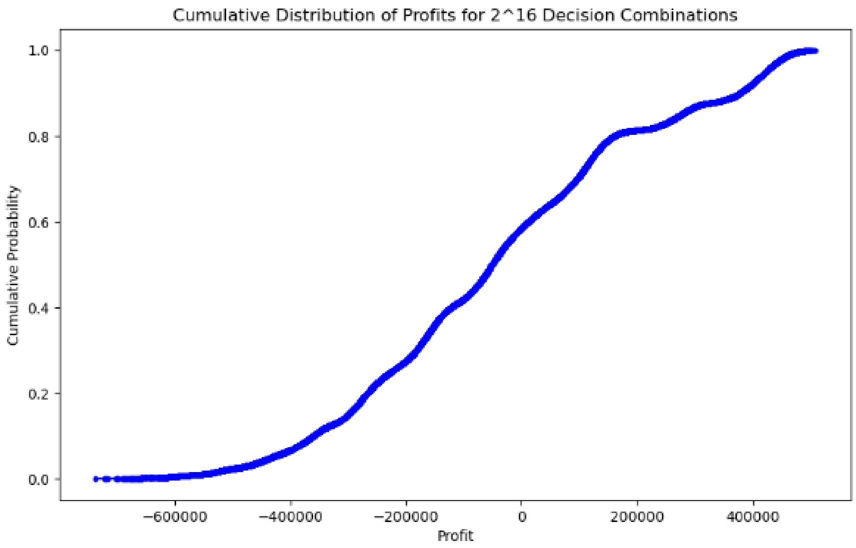


Fig. 7 Cumulative frequency diagram

5. Summary

In this study, a set of production decision-making optimization system suitable for assembly enterprises was successfully constructed, and a variety of models and algorithms were used to effectively solve the key problems in actual production. In the sampling detection process, the model based on hypothesis testing and Bayesian method can accurately determine the number of samples and the number of critical defective products under different reliabilitys, reduce the detection cost and ensure the accuracy of decision-making. In terms of production decision optimization, the combination of dynamic planning, backtracking algorithm and generalized multi-scheme planning model fully considers the complexity of the multi-process and multi-spare parts production process, provides enterprises with the optimal strategy at each stage, and significantly improves the efficiency of enterprises. Verified by actual cases, the system performs well in different production scenarios. Future research can further expand the model to include more practical factors, such as market fluctuations, supply chain risks, etc., to continuously improve the practicability and adaptability of the model, and provide stronger support for the development of assembly enterprises

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