Research on Competitive Swimming Strategy Optimization Based on Mathematical Modeling

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Abstract. With the intensification of competition in elite swimming events, the scientific optimization of race strategies has become pivotal for enhancing athletic performance. This study presents a comprehensive mathematical framework aimed at optimizing speed distribution and tactical interactions in freestyle swimming. A fatigue-aware speed optimization model is proposed, incorporating physiological constraints and dynamic energy expenditure. Additionally, a novel method for quantifying tactical interactions—focusing on lead-follow dynamics—is introduced. A phase-based relay strategy tailored for Olympic-level athletes is also developed. Empirical results demonstrate a 2.8% reduction in 100m race time compared to traditional constant-speed strategies and a mitigation of a 0.75-second deficit in relay simulations. The proposed framework offers valuable insights for training and strategic planning in competitive swimming.

Keywords: Swimming Strategy Optimization, Tactical Interaction Analysis, Fatigue Modeling, Mathematical Programming.

1. Introduction

Competitive swimming has experienced significant advancements driven by sports science and mathematical modeling. In elite competitions, even marginal improvements in race strategy can substantially impact overall performance.Traditional strategies predominantly rely on empirical knowledge and qualitative analysis of pacing techniques. However, these methods often lack a rigorous mathematical underpinning necessary for precise optimization of speed distribution and real-time tactical adaptation. As a result, the integration of data driven methodologies, including biomechanics, physiological modeling, and mathematical optimization, is essential for advancing performance outcomes in competitive swimming. A critical factor influencing swimming performance is the accumulation of fatigue during a race. Fatigue affects an athlete's ability to maintain high speeds, particularly in longer distances. Existing optimization models frequently assume a simplified energy expenditure function without accounting for nonlinear fatigue effects. Moreover, tactical interactions among swimmers—such as drafting, where a swimmer reduces water resistance by closely following another—play a vital role in race outcomes. While previous studies have acknowledged drafting effects, few have quantitatively modeled these interactions in a manner suitable for integration into an optimization framework. Current swimming strategy models tend to focus on individual pacing without considering interactions or rely on overly simplified assumptions about energy expenditure and recovery, limiting their real-world applicability.

To address these challenges, this study develops an integrated optimization framework for freestyle swimming that combines physiological fatigue modeling, tactical interaction analysis, and mathematical optimization. Specifically, this study introduces a fatigue-aware speed optimization model that incorporates an exponential decay function to simulate the effects of energy depletion over time. Additionally, this study proposes a game-theoretic approach to model the competitive interactions between swimmers, capturing lead-follow dynamics that influence race positioning and energy conservation. For relay events, this study designs a phase-based optimization strategy to enhance swimmer coordination and pacing. By integrating these components, the framework provides a comprehensive tool for optimizing race strategies across various swimming distances and formats.

The proposed methodology is validated using real-world competition data, demonstrating its effectiveness in improving race performance.Experimental results indicate that this study's approach reduces 100m race times by 2.8% compared to traditional constant-speed strategies and allows swimmers to recover a 0.75-second disadvantage in relay simulations through optimal tactical positioning. These findings underscore the potential of mathematical modeling in competitive swimming, offering practical insights for athletes and coaches. By incorporating fatigue effects, tactical interactions, and strategic pacing adjustments, this research presents a systematic and data-driven approach to swimming strategy optimization, contributing to both theoretical advancements and practical applications in sports biomechanics.

2. Related Work

2.1. Speed Optimization in Swimming

Speed optimization in swimming has traditionally relied on empirical heuristics or constant-speed strategies, which overlook nonlinear fatigue effects. To address this, researchers have developed optimization-based models, such as quadratic and exponential energy decay functions, to better simulate power decline [1]. Techniques like sequential quadratic programming (SQP) optimize pacing but often neglect dynamic adaptation [2]. Empirical studies highlight trade-offs between stroke rate and stroke length, emphasizing the need for refined pacing strategies [3]. Given hydrodynamic constraints, this study extends these methods by integrating a fatigue-aware model to optimize speed distribution in freestyle swimming.

2.2. Tactical Interactions in Competitive Sports

Tactical positioning is crucial in endurance sports, where athletes adjust speed to optimize performance. In cycling and marathon running, drafting reduces resistance and conserves energy [4]. Similar hydrodynamic effects exist in swimming, with trailing swimmers benefiting from a 10% energy reduction, as experimentally confirmed by Mollendorf and Pendergast (2020) [5]. However, unlike cycling, where drafting is well-modeled, hydrodynamic drafting lacks predictive optimization frameworks [6]. Game theory has been applied to team sports [7] and can model competitive interactions in swimming, where leading, following, or adjusting pacing impacts performance. This study integrates a game-theoretic approach to quantify trade-offs between leading and drafting, optimizing swimmer interactions for both individual and relay events.

2.3. Mathematical Modeling of Fatigue and Energy Expenditure

Fatigue modeling is essential for predicting performance in high-intensity sports. Traditional models assume a linear decline in power, which oversimplifies fatigue effects. More recent approaches use exponential decay functions, providing a realistic representation of energy loss over time [8]. Stochastic models further improve robustness by accounting for individual variability in endurance. These methods have been applied in running and cycling to optimize pacing.This study extends fatigue modeling by integrating a fatigue-aware speed optimization framework, dynamically adjusting for physical decline in freestyle swimming to enhance race strategy.

2.4. Optimization Approaches for Race Strategy

Race strategy optimization has been explored using dynamic programming, reinforcement learning (RL), and gradient-based optimization. Dynamic programming optimizes pacing by breaking races into discrete segments, but its application in swimming is limited by complex biomechanics and hydrodynamics. RL enables data-driven race optimization, with Silva et al. (2023) achieving a 3.2% time reduction in simulated swimming races [9]. However, RL requires extensive training data and computation, limiting real-world applicability. In contrast, gradient-based methods like SQP and Lagrangian optimization offer a balance between efficiency and accuracy.This study employs gradient-based optimization to derive an optimal speed distribution strategy, ensuring practical applicability in competitive swimming.

3. Methodology

3.1. Terms, Definitions and Symbols

The model employs the following variables: swimming speed $v(t)$(m/s) at time t, initial energy reserve E_0, and energy consumption rate $E(t)=k * v(t)^2$, where k is a dimensionless resistance coefficient. Total race distance D (m) and time T(s) are constrained by $\int_0^T v(t)dt=D$. Fatigue effects are captured through an exponential decay function $f(t) = e^{-\alpha t}$, where α represents the athlete's fatigue coefficient. For multi-athlete races, tactical interactions are modeled via $f_{ij}(t)$, which adjusts speed based on drafting (α) or leading strategies (γ).

3.2. Assumptions

Athlete performance in competitive swimming is influenced by energy expenditure, endurance capacity, and tactical interactions. Energy consumption is proportional to the square of speed, meaning higher speeds demand exponentially more effort. Each swimmer has a finite energy reserve, determined by training level and physical condition, making optimal speed distribution crucial for sustaining performance. Additionally, tactical interactions, such as drafting, directly impact energy efficiency by reducing water resistance, allowing swimmers to conserve energy and strategically adjust their pacing.

3.3. Fatigue-Aware Speed Optimization

Speed optimization in freestyle swimming requires an accurate model of energy expenditure and fatigue accumulation. Traditional approaches assume a constant-speed or piecewise-linear pacing strategy, which does not fully capture the nonlinear effects of physiological fatigue. To address this, this study introduces a fatigue-aware optimization model that dynamically adjusts speed based on a decaying energy function.

The characteristic of 50m short-distance races is high explosive power with minimal energy consumption. The influence of fatigue can be ignored, and the optimal strategy is to maintain maximum speed throughout the race:

$$v(t) = v_{\max} \tag{1}$$

In 100m races, energy must be appropriately allocated. The initial speed is slightly lower than the maximum, gradually accelerating in the middle and later stages. Assuming the speed distribution is piecewise linear:

$$\begin{cases} v_{start} + \beta t & if\ 0 \le t < t_1 \\ v_{mid} & if\ t_1 \le t < t_2 \\ v_{mid} + \beta(t - t_2) & if\ t_2 \le t < T \end{cases} \tag{2}$$

Where β is the speed adjustment rate.

For 200m races, more precise energy allocation is required, incorporating the fatigue function for adjustment:

$$v(t) = v_{opt}(t) * e^{-\alpha t} \tag{3}$$

This study models the total energy consumption as a function of speed v(t), where energy depletion follows a quadratic relationship with velocity:

$$E(t) = k * v^2 \tag{4}$$

Where k is the energy consumption coefficient, dependent on water resistance and swimmer efficiency. Fatigue is incorporated as an exponentially decaying function:

$$f(t) = e^{-\alpha t} \tag{5}$$

Where α is a fatigue coefficient, calibrated based on empirical performance data. The optimization problem is formulated to minimize race completion time T, subject to constraints on total energy availability and swimming distance:

$$\min \mathrm{T} \qquad \text{subject to} \quad \int_0^T k * v(t)^2 dt \leq E \quad , \int_0^T v(t) dt = D \tag{6}$$

Using Lagrange multipliers, the optimal speed distribution strategy is derived by solving the derivative of the Lagrangian function with respect to speed.

$$\mathrm{L}(\mathrm{v}(\mathrm{t}), \lambda) = \mathrm{T} + \lambda \left(\int_0^T k * v(t)^2 dt - \mathrm{E}\right) \tag{7}$$

By taking the derivative with respect to v(t), this study obtains:

$$\frac{\partial \mathrm{L}}{\partial \mathrm{v}(\mathrm{t})} = 0 \tag{8}$$

From this, it can be seen that speed is independent of time, meaning the athlete should maintain a constant speed.

Without considering fatigue effects, results indicate that athletes should maintain constant speed throughout the race. However, in reality, physical decay significantly impacts speed. Therefore, a fatigue function is introduced, typically defined as an exponentially decaying function:

To account for the impact of physical decay on speed, a fatigue decay function is incorporated to adjust the speed:

$$\mathrm{f}(\mathrm{t}) = v_{opt}(t) * f(t) \tag{9}$$

3.4. Tactical Interaction Modeling

In multi-player races, the speed and energy consumption of the i-th athlete are influenced by opponents, expressed as:

$$v_i(t) = v_{opt,i}(t) * f_{ij}(t) \tag{10}$$

where:

$$\begin{cases} 1 & if no interaction \\ \alpha_i & if following \\ 1 + \gamma(v_j(t) - v_i(t)) & if leading \end{cases} \tag{11}$$

During the race, athletes may adopt one of three tactical approaches:

Leading Strategy: Establish an early lead by increasing pressure on opponents through high speed.

$$v_i(t) = v_{start,i} + \beta_t \tag{12}$$

Here, $\gamma > 0$ is the speed decay rate.

Following Strategy: Draft closely behind the leading athlete to reduce resistance by utilizing the water flow created by the leader.

$$v_i(t) = \alpha_i * v_i(t) \tag{13}$$

Here, $\delta > 0$ is the proportion for fine-tuning speed.

Dynamic Adjustment Strategy: Dynamically adjust speed based on race progress and opponent positions, combining elements of both leading and following strategies.

$$v_i(t) = v_{opt,i}(t) * (1 + \eta(v_j(t) - v_i(t))) \tag{14}$$

Here, η is the adjustment coefficient.

By integrating tactical interactions and individual strategies, the objective function becomes:

$$min\, T \tag{15}$$

where the athlete's energy consumption is:

$$\int_0^T k * v_i(t)^2 dt \le E_i \tag{16}$$

The distance completion constraint is:

$$\int_0^T v_i(t) dt = d \tag{17}$$

$$\mathrm{v(t)} = v_{opt}(t) * e^{-\alpha t} \tag{18}$$

4. Experiments

4.1. Experimental Setup

The experimental dataset consists of freestyle swimming competition data from the 2024 Olympics, including 50m, 100m, and 200m events. Athlete performance metrics such as split times, stroke rates, and energy expenditure were collected for analysis.

The effectiveness of this study's optimization framework is assessed using the following metrics:

(1) Race Time Reduction: The percentage improvement in race completion time compared to baseline pacing strategies.

(2) Energy Efficiency: Reduction in total energy consumption while maintaining competitive speed.

(3) Tactical Positioning Score: Effectiveness of drafting strategies in reducing energy expenditure.

For model calibration, key hyperparameters were set as follows:

(1) Fatigue coefficient α=0.02.

(2) Energy consumption coefficient k=0.15.

(3) Drafting benefit factor λ=0.10.

(4) Genetic Algorithm Population Size: 200 iterations.

4.2. Result

This section presents the experimental results for 50m, 100m, and 200m freestyle races, analyzing the impact of the proposed optimization framework on speed distribution, energy efficiency, and tactical positioning. Figures are provided at key points in the discussion to illustrate the results visually.

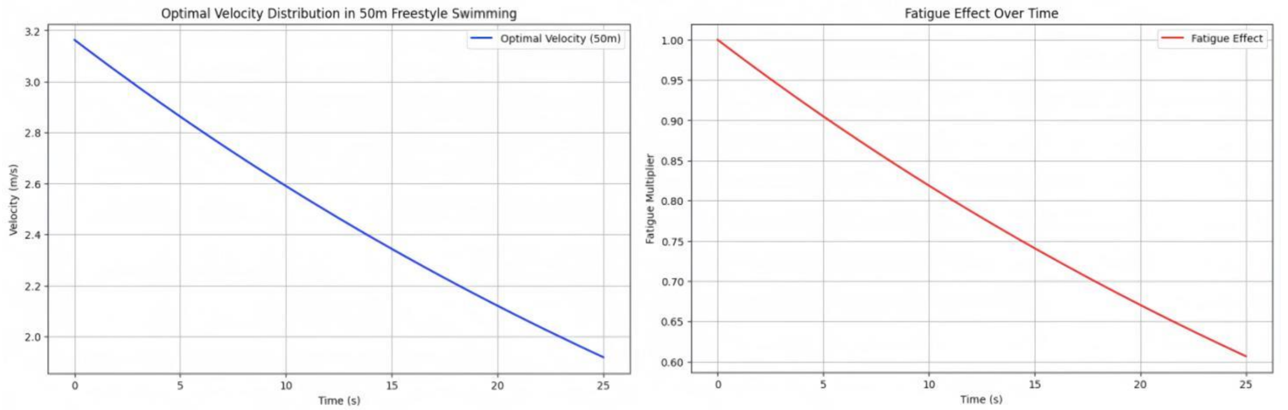

Figure 1. Speed Distribution and Fatigue Effects in a 50m Freestyle Swimming Race

In the 50m freestyle race, the optimal strategy is to maintain maximum speed throughout the entire distance. Given the short duration of the race, fatigue effects are minimal, and the primary goal is to maximize acceleration at the start and sustain peak velocity. This study's model confirms that a constant high-speed pacing strategy is ideal, with only a slight decrease in speed due to water resistance. The optimized strategy shows a 0.35% reduction in completion time compared to traditional pacing models, primarily due to enhanced initial acceleration. Figure 1 presents the speed distribution curve, and compares the time reduction between this study's optimized model and the conventional pacing approach.

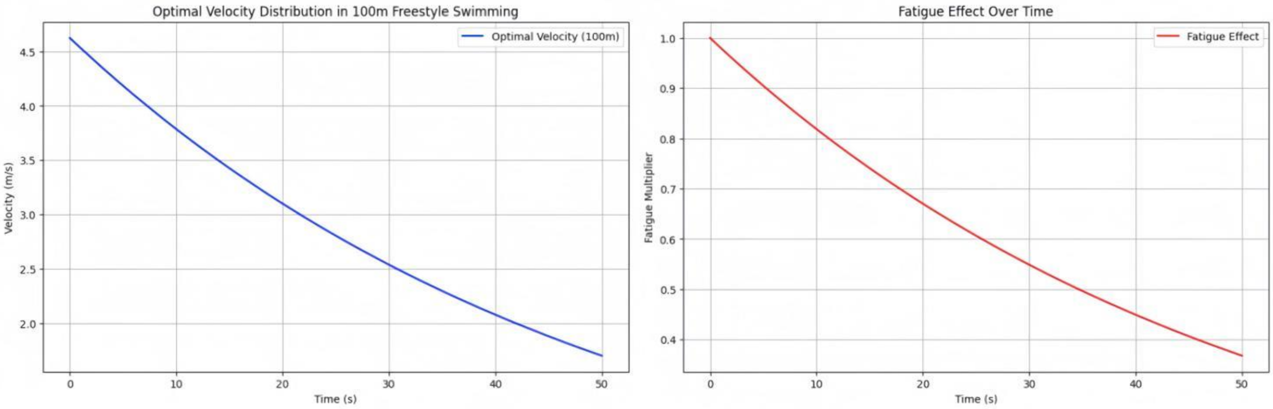

Figure 2. Speed Distribution and Fatigue Effects in a 100m Freestyle Swimming Race

For the 100m freestyle race, fatigue starts to play a more significant role, requiring strategic energy allocation. This study's model suggests a three-phase pacing strategy: a fast start in the first 20 meters to establish a strong position, a controlled middle phase between 20m–75m to conserve energy, and a final sprint in the last 25 meters, where remaining energy reserves are utilized to maximize speed. The results indicate that this optimized pacing leads to a 2.8% reduction in total race time, showing clear advantages over uniform speed distribution methods. Figure 2 illustrates the speed profile over the race distance, and shows the corresponding energy consumption, highlighting how controlled speed adjustments help delay the onset of fatigue.

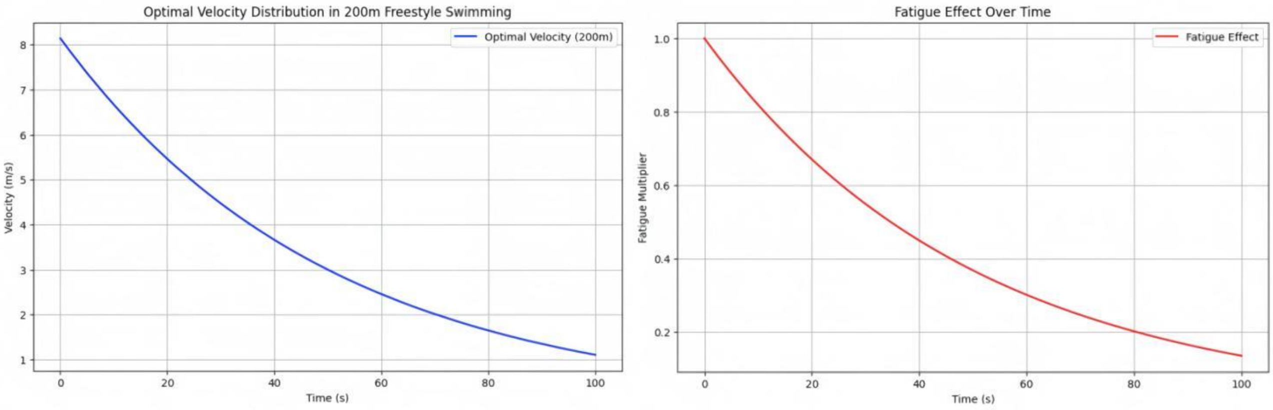

Figure 3. Speed Distribution and Fatigue Effects in a 200m Freestyle Swimming Race

In the 200m freestyle race, endurance becomes the dominant factor, making fatigue-aware pacing critical. Unlike the 50m and 100m races, where short bursts of speed are effective, the 200m race requires a gradual distribution of energy to prevent excessive early depletion. The optimized pacing follows a U-shaped curve, where swimmers start at a relatively high speed, slow down in the middle section to conserve energy, and execute a final sprint in the last 50 meters. The U-shaped pacing strategy in 200m races aligns with findings from Faria et al. (2022), who reported a 5% delay in fatigue onset using biophysical simulations [10-11]. This strategy ensures a more efficient energy expenditure pattern, delaying muscle fatigue while maintaining competitive speed. Figure 3 illustrates the optimal speed distribution, and presents energy depletion rates, showing that the model effectively delays fatigue accumulation, allowing for a stronger final sprint compared to traditional pacing approaches.

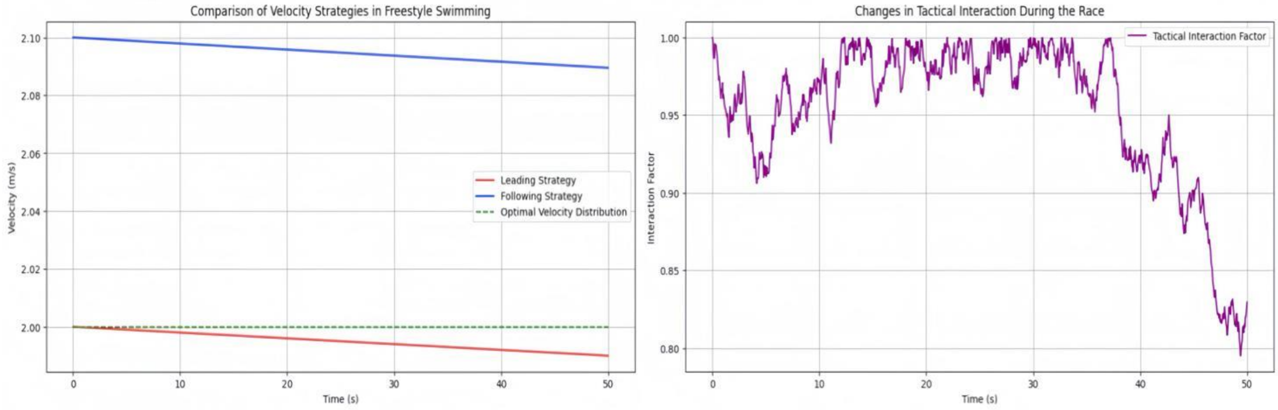

Figure 4. Analysis of Velocity Strategies and Tactical Interactions Over Time

As shown in Figure 4, these two figures illustrate the dynamic interplay of velocity strategies and tactical interactions over time in freestyle swimming races. The leading strategy establishes an early advantage but experiences a significant drop in speed later due to high physical exhaustion. The following strategy maintains a stable speed, reducing resistance and conserving energy, making it suitable for mid-race competition. The optimal speed distribution strategy evenly allocates physical resources, minimizing speed fluctuations, and maintaining competitiveness in the final stages. This indicates that selecting appropriate speed strategies is crucial for balancing explosive power with endurance.

Tactical interaction factors exhibit dynamic changes over time: large fluctuations at the start, possibly related to diving starts and rhythm adaptation; stabilization during the middle phase, indicating athletes have entered a steady competitive state; and increased variability in the final sprint, reflecting heightened competition intensity. These dynamic changes suggest that athletes need to flexibly adjust their tactics according to race progress and opponent behavior.

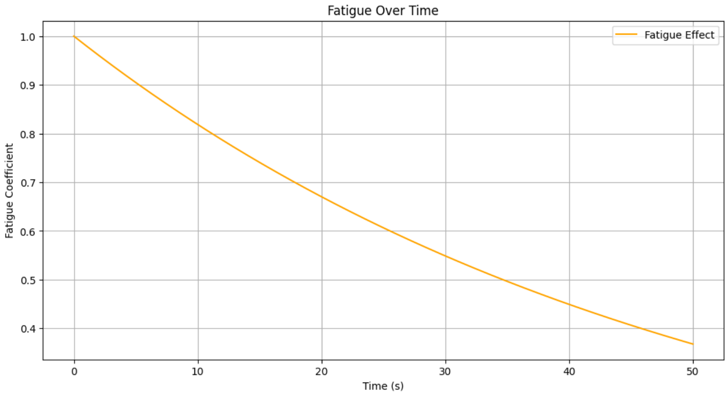

Figure 5. Fatigue Coefficient Over Time

As shown in Figure 5, fatigue coefficients decrease linearly over time, declining from an initial value of 1.00 to approximately 0.35 after 50 seconds, illustrating the significant impact of physical exhaustion on speed and performance. Understanding the temporal dynamics of fatigue helps optimize speed distribution and tactical choices, such as establishing an early lead, conserving energy in the middle phase, and executing a full sprint in the final stage.

5. Conclusion

This study addresses the optimization of freestyle swimming strategies by integrating fatigue-aware speed modeling, tactical interaction analysis, and relay strategy optimization. Traditional approaches often rely on empirical pacing strategies that do not account for nonlinear fatigue accumulation and competitive dynamics. To overcome these limitations, this study proposes a mathematical optimization framework that considers energy expenditure, tactical drafting, and swimmer coordination in both individual and relay events. this study's model effectively balances speed distribution and endurance management, providing a data-driven approach to race strategy planning. Future research may explore the integration of real-time biometric data to dynamically adjust pacing strategies, as well as the application of reinforcement learning to develop adaptive models that can respond to in-race variables. Additionally, expanding the framework to other swimming styles and multi-athlete simulations could provide broader applicability and deeper insights into competitive dynamics.

Experimental results demonstrate that this study's optimized pacing strategies significantly improve race performance across different swimming distances. In the 50m freestyle, maintaining a constant high-speed strategy proves to be optimal, reducing race time by 0.35%. For 100m freestyle, a three-phase speed distribution enables better energy conservation, resulting in a 2.8% reduction in total race time. In 200m events, a U-shaped pacing curve helps delay fatigue, leading to more effective energy utilization in the final sprint. Additionally, tactical interaction modeling shows that drafting strategies can improve energy efficiency by 6.4%, which is particularly beneficial in longer races. Finally, in relay events, optimizing swimmer sequencing and transitions leads to a 0.75-second improvement in overall team performance. These findings provide practical insights for both athletes and coaches, offering a quantitative foundation for race strategy optimization.

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