Research on Production Decision Optimization Based on Sampling Inspection and Cost Analysis

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Abstract. With the increasing competition in the manufacturing industry, how to optimize the production process to reduce production costs has become a key issue for enterprises. This paper addresses the optimization of production processes in the manufacturing industry, focusing on sampling inspection schemes and production decision-making models. Based on binomial distribution hypothesis testing with unknown incoming defective rates, this paper develops an optimal sampling program by controlling confidence conditions and simulating rejection domains. This approach minimizes detection frequency while maintaining effective quality control. Furthermore, this paper proposes a comprehensive decision equation model that considers various cost factors including parts procurement, assembly, inspection, dismantling, exchange losses, and market sales of finished products. The model dynamically adjusts the inspection proportions of spare parts and finished products, the dismantling ratio of substandard products, and the procurement ratio of two types of spare parts. Through optimization within specified constraints, the model achieves optimal resource allocation. Analysis of empirical data reveals optimal sampling inspection ratios of 0.0797, 0.0438, and 0.0201 for spare parts 1, spare parts 2, and finished products respectively. Additionally, the model yields an optimal dismantling ratio of 0.9511 for defective products, an optimal proportion of 0.5009 for spare parts 1, and a normalized expected profit value of 11.5548.

Keywords: Production Decisions, Resource Allocation, Hypothesis Testing, Optimization Models.

1. Introduction

With the increasingly fierce competition in the global manufacturing industry, enterprises face increasing cost pressure and quality requirements in the production process. In this context, how to ensure product quality and reduce production costs has become a key issue in enterprise management and production decisions. Controlling the rate of defective products and optimizing inspection strategies are particularly important in the production process. Traditional quality inspection methods often rely on total inspection or simple sampling inspection [1], but these methods are difficult to meet the real needs of large-scale production in enterprises. Therefore, how to develop an efficient sampling and testing program and scientific cost analysis has become the focus of current research. Many scholars have explored different decision-making methods for such problems [2-5]. However, the current research results for the production decision-making problems of small and medium-sized enterprises are still lacking, so this paper analyses the production decision-making problems of small and medium-sized enterprises and establishes a corresponding model, hoping to inspire the production decision-making problems of small and medium-sized enterprises.

The specific structure of this paper is as follows. This paper first through the binomial distribution hypothesis test, in the nominal value of 10%, the confidence conditions were in 95% and 90% of the case of spare parts defective rate restrictions, and based on the critical conditions of confidence, in the specific results of the total number of different samples of defective products for the number of rejection domains, to help enterprises to achieve the development of the number of tests as few as possible sampling and testing program, to achieve the production process as much as possible to reduce testing costs. The purpose of reducing the testing cost in the production process is to achieve rational decision-making in the production process. Further, to decide on the specific production process of the enterprise, it is reasonable to analyze the defective rate of spare parts and finished products, the unit price of purchase, the cost of testing, and other factors. By combining the decision variables to construct a target profit function, the optimization model is further constructed to help the enterprise solve the key problem of how to maximize profits.

2. Hypothesis Testing for Defective Rate and Inspection Optimization

The data for this study was obtained from www.mcm.edu.cn. This paper addresses the problem of purchasing spare parts in the production process of an enterprise, in order to reduce the production cost of the enterprise by designing a reasonable sampling and testing program of spare parts for the enterprise.

The specific problem is as follows: a supplier claims that the defective rate of a batch of spare parts will not exceed a certain nominal value. The enterprise is going to use sampling and testing to decide whether or not to accept this batch of spare parts from the supplier, and the cost of testing will be borne by the enterprise. Design a sampling program for the enterprise that will result in as few tests as possible.

To more closely approximate the real situation, two scenarios are set up in this paper based on a nominal value of 10 percent. Scenario 1 is a 95 percent confidence level that the defective rate of spare parts exceeds the nominal value, then reject the spare parts. Scenario 2 is a 90 percent confidence level that the defective parts rate does not exceed the nominal value, and the parts are accepted.

The study involves the sampling rate of defective products obeying the binomial distribution $X \sim b(n, p)$, and the key to converting it to the hypothesis testing problem of the binomial distribution lies in the construction of its rejection domain. When the significance level ∂ and the nominal value are given, the boundary value of the rejection domain can be obtained, and the original hypothesis is rejected when the number of defective samples reaches the boundary. The study gives a decision plan for the lowest possible number of tests at a nominal value of 10 percent for the two different scenarios described above:

Original hypothesis H_0 : $p \le p_0$ the rate of defects in the batch of spare parts will not exceed the nominal value.

Alternative scenario H_1 : $p > p_0$ the defect rate in the batch of spare parts exceeds the nominal value.

For hypothesis testing, given a confidence level, a rejection domain is set out, and when a critical value in the rejection domain is reached, the corresponding variable can be found to determine the final decision-making solution.

For the binomial distribution problem, the formula for calculating the number of defective products L is $L = np$, assuming its rejection domain is $(k, +\infty)$

Given a significance level ∂ , makes $P(L \ge k) \le \partial$ or $P(L \le k) \le \partial$ so that a specific value of k can be determined. the specific formula for the value of k is:

$$k_1 = \min\left\{ l : \sum_{k=l+1}^{n} C_n^k p_0^k (1 - p_0)^{n-k} \le \partial \right\} \tag{1}$$

$$k_2 = \min\left\{ l : \sum_{k=l+1}^{n} C_n^k p_0^{n-k} (1 - p_0)^{k} \le \partial \right\} \tag{2}$$

In case (1) $\partial = 0.05$, the spare parts are found to be defective beyond the nominal value and the consignment is rejected. Bringing $p_0 = 0.1$ into the equation gives:

$$k_1 = \min\left\{ l : \sum_{k=l+1}^{n} C_n^k 0.1^k 0.9^{n-k} \le 0.05 \right\} \tag{3}$$

The formula for k_2 in case (2) can be found in the same way:

$$k_2 = \min\left\{ l : \sum_{k=l+1}^{n} C_n^k 0.1^{n-k} 0.9^k \le 0.1 \right\} \tag{4}$$

At this point, bringing in a different total number of samples n can result in different k values, that is, in the total number of samples n, the number of detected defective more than k_1, that is, the rejection of this batch of parts.

The correspondence between n and k in the scenario (1) is shown in Table 1:

Table 1. Scenario (1) results table

n	10	50	100	200	1000	2000
k	2	8	15	27	115	222
accurate	0.02125	0.00828	0.01064	0.00692	0.00363	0.00163

The correspondence between n and k in the scenario (2) is shown in Table 2:

Table 2. Scenario (2) results table

n	10	50	100	200	1000	2000
k	2	7	13	25	112	217
accurate	0.03312	0.02461	0.02653	0.00051	0.00510	0.00314

As can be seen from the change in the precision of the results, the higher the number of samples tested, the more precise the results are in the case of significance levels $\partial = 0.05$ and $\partial = 0.10$. The results of this paper are based on the results of a test where the number of samples detected is greater than the number of samples. In summary, when the defective rate is unknown, this paper gives the least costly test for a given nominal value, i.e., when the total number of samples is n, and the number of defective parts detected exceeds k, the batch of parts is rejected. For example, in the case of significance level $\partial = 0.05$, when the total number of samples inspected is 100, the given precision is 0.01064, and the rejection domain is （15， $+\infty$）, the batch is considered rejected if the number of defective parts exceeds 15 under this condition; In the case of significance level $\partial = 0.10$, when the total number of samples entered for testing is 50, the given accuracy is 0.02461 and the rejection domain is （7， $+\infty$）. It is considered that the batch of parts is rejected when the number of defective parts exceeds 7 under this condition. It was also concluded that the larger the number of samples tested, the more accurate the results were at significance levels $\partial = 0.05$ and $\partial = 0.10$. The results were also found to be more accurate at significance levels $\partial = 0.05$ and $\partial = 0.10$.

3. Decision Making for Cost Minimization and Profit Maximization in Production Processes

The fundamental purpose of the enterprise production process to make decisions is to achieve the maximization of enterprise profits [6]. For such problems, many scholars have established relevant models using linear programming methods [7-10]. The linear programming method has the characteristics of a simple structure and efficient solution. Therefore, this paper constructs the corresponding planning model based on linear programming. First of all, it is necessary to determine the determinants of the enterprise's obtainable profit, namely, the production cost and the degree of sales. In the establishment of the model, as long as the minimization of production costs while maximizing sales is the optimal solution strategy for the enterprise. The specific idea of this paper is shown in Figure 1.

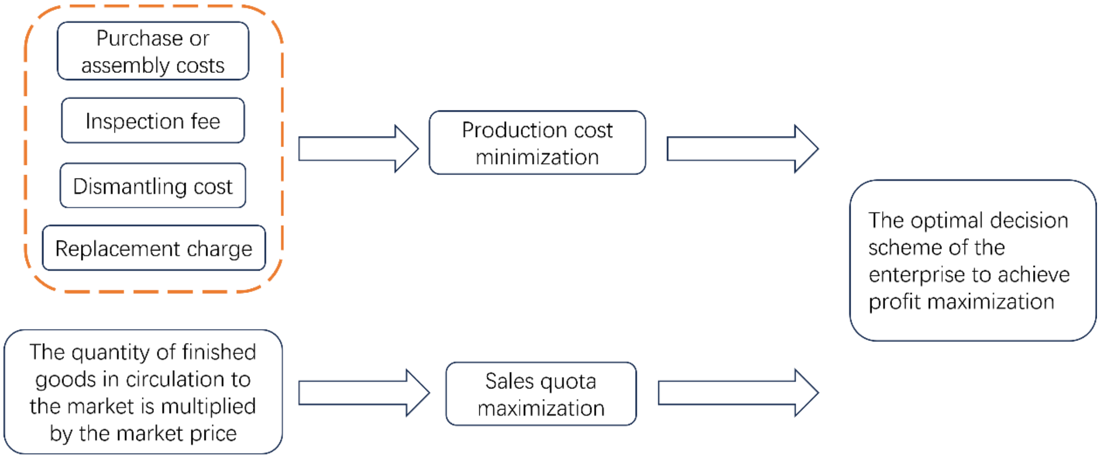

Figure 1. Schematic diagram of the idea

The production process in a business requires decisions to be made at various stages, including whether to make inspections of spare part 1, and spare part 2, whether to inspect the combined finished product, and whether to dismantle the non-conforming finished product. These are all important components of cost. For the production cost of a firm, this paper divides into four dimensions, i.e., the firm's purchase or assembly cost, inspection cost, dismantling cost, and exchange loss. To achieve the minimum production cost of the firm, i.e., to achieve the minimum case of the cumulative sum in the above four dimensions. The specific cost variables are defined as shown in Table 3.

Table 3. Definition of Cost Variables

Level 1 variable symbols	Nature of the variable	Level 2 variable symbols	Variable Definition
Z_1	Purchase or assembly costs	CA_1 / CA_2	Purchase cost of spare parts 1 and 2
		CA_3	Assembly cost of the finished product
Z_2	Testing costs	CB_1 / CB_2	Testing costs for spare parts 1 and 2
		CB_3	Inspection costs for finished products
Z_3	Disassembly costs	CC_3	Cost of dismantling non-conforming finished goods
Z_4	exchange cost	CD_3	Replacement cost of non-conforming finished goods

The degree of sales of a firm is defined as the product of the quantity of finished goods assembled by the firm and its market unit price. In considering the degree of sales, the market unit price is fixed, i.e., when the number of finished products assembled by the enterprise reaches the maximum value, the degree of sales of the enterprise also reaches the extreme value.

Through the analysis, the key to solving the difficulty lies in how to achieve the balance between the minimization of production costs and the maximization of sales, and to determine the maximum value of the target profit function when the two are balanced, that is, the optimal decision-making scheme for the enterprise. To clearly express the model process, this paper simplifies the production flow chart shown in Figure 2, to correspond to the subsequent variables.

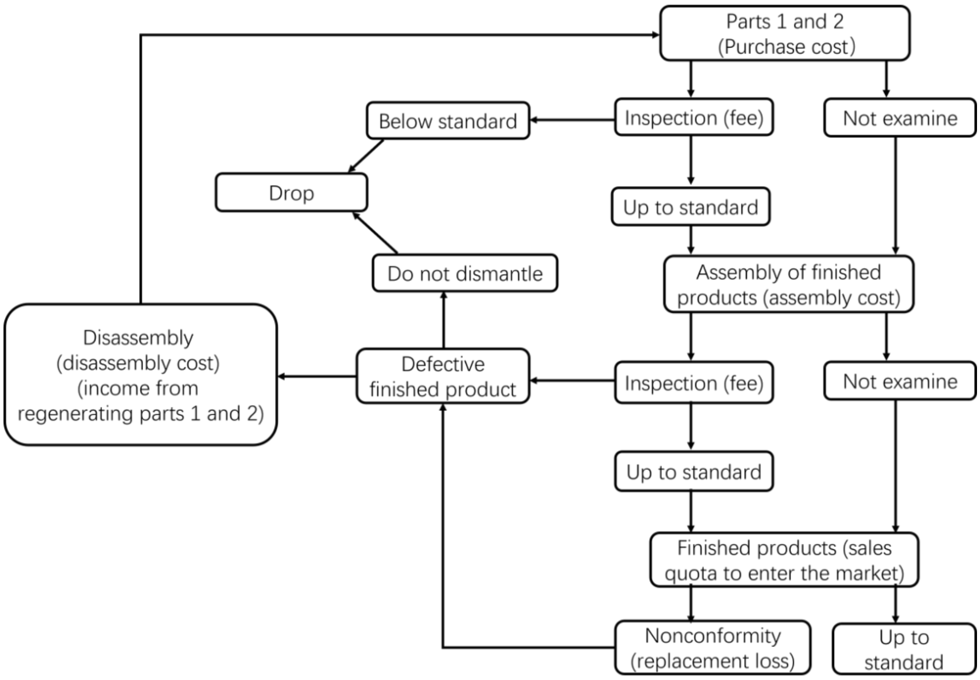

Figure 2. Simplified production flow chart

After a more precise definition of each cost in the process, to further quantitatively portray the nature of the sample, this paper defines some numerical variables as shown in Table 4:

Table 4. Table of definitions of some numerical variables

variable symbol	Variable Definition	Variable symbol	Variable Definition
n_1	Quantity of spare parts 1	Q_1	Raw reject rate for spare parts 1
n_2	Quantity of spare parts 2	Q_2	Raw reject rate for spare parts 2
n_3	Overall number of finished and non-conforming products	Q_3	Raw defective rate of finished products
n_4	Number of non-conforming finished products to be dismantled	R	Enterprise sales volume
S	The market price of the finished product	f	Profitability of the enterprise

Record x_1, x_2, x_3, x_4, as the proportion of parts 1 entering the inspection, the proportion of parts 2 entering the inspection, the proportion of finished products entering the inspection, and the proportion of dismantling of unqualified finished products, respectively. Then the quantity of each item of Part 1 in Step 1 is shown in Table 5:

Table 5. Schematic representation of the quantities of each part of part 1 in step 1

Spare part 1	Inclusion in testing	Exclusion from testing
Eligible	$n_1x_1(1-Q_1)$	$n_1(1-x_1)(1-Q_1)$
Substandard	$n_1x_1Q_1$	$n_1(1-x_1)Q_1$
Subtotal	n_1x_1	$n_1(1-x_1)$

The formula for the corrected reject rate P_1 for Part 1 after removing the quantity that should be discarded (i.e., failed inspection) in Step 1 is:

$$P_1 = \frac{n_1(1-x_1)Q_1}{n_1x_1(1-Q_1)+n_1(1-x_1)(1-Q_1)+n_1(1-x_1)Q_1} = \frac{Q_1 - Q_1x_1}{1-x_1Q_1} \tag{5}$$

Similarly, Part 2 can be given the following schematic table 6 for each variable in step one:

Table 6. Schematic representation of the quantities of each part of part 2 in step 1

Spare part 2	Inclusion in testing	Exclusion from testing
Eligible	$n_2x_2(1-Q_2)$	$n_2(1-x_2)(1-Q_2)$
Substandard	$n_2x_2Q_2$	$n_2(1-x_2)Q_2$
Subtotal	n_2x_2	$n_2(1-x_2)$

The corrected defective rate P_2 for Part 2 after removing the discarded (i.e., failed inspection) quantity in Step 1 is given by the formula:

$$P_2 = \frac{n_2(1-x_2)Q_2}{n_2x_2(1-Q_2)+n_2(1-x_2)(1-Q_2)+n_2(1-x_2)Q_2} = \frac{Q_2 - Q_2x_2}{1-x_2Q_2} \tag{6}$$

During the process of composing the finished product in step 1, the overall corrected defective rate P_3 of the assembled finished product is calculated by the formula:

$$P_3 = 1-(1-P_1)(1-P_2)(1-Q_3) \tag{7}$$

For the assembled finished product, the variables are specified as shown in Table 7:

Table 7. Schematic representation of the variables of the assembled finished product

Finished assembly	Inclusion in testing	Exclusion from testing
Eligible	$n_3x_3(1-P_3)$	$n_3(1-x_3)(1-P_3)$
Substandard	$n_3x_3P_3$	$n_3(1-x_3)P_3$
Subtotal	n_3x_3	$n_3(1-x_3)$

The formula for calculating the corrected defective rate P_4 after removing the number of inspection rejects from the finished assembly is:

$$P_4 = \frac{n_3(1-x_3)P_3}{n_3x_3(1-P_3)+n_3(1-x_3)(1-P_3)+n_3(1-x_3)P_3} \tag{8}$$

After each corrected defective rate is found, this paper gives the formula for calculating the overall quantity n_3 of finished products and non-conforming finished products:

$$n_3 = \min\{n_1 - x_1Q_1n_1 \text{ , } n_2 - x_2Q_2n_2\} \tag{9}$$

Firstly, each cost is calculated separately. Purchase and assembly cost Z_1 includes the purchase cost of spare parts I and II and the assembly cost of the finished assembly, which is calculated by the formula:

$$Z_1 = CA_1n_1 + CA_2n_2 + CA_3n_3 \tag{10}$$

Inspection cost Z_2 consists of the cost of inspection of spare parts I and II and the finished assembly, which is calculated by the formula:

$$Z_2 = n_1CB_1x_1 + n_2CB_2x_2 + n_3CB_3x_3 \tag{11}$$

The dismantling cost Z_3 includes the dismantling cost of non-conforming finished products and the proceeds of re-generating spare parts I and II after dismantling, and the formula for calculating the dismantling cost is as follows:

$$Z_{31} = \left[x_3 n_3 P_3 + n_3 P_3 (1 - x_3) \right] x_4 CC_3 \tag{12}$$

Re-generate the proceeds Z_{32} of Parts I and II as:

$$Z_{32} = \left[x_3 n_3 P_3 + n_3 P_3 (1 - x_3) \right] x_4 [(1 - p_1) CA_1 + (1 - p_2) CA_2] \tag{13}$$

The dismantling cost Z_3 is calculated using the formula:

$$Z_3 = Z_{31} - Z_{32} = n_3 P_3 x_4 \left[CC_3 - (1 - P_1) CA_1 - (1 - P_2) CA_2 \right] \tag{14}$$

The loss on exchange Z_4 is calculated by the formula:

$$Z_4 = n_3 P_3 (1 - x_3) CD_3 \tag{15}$$

Remember that the unit price of the finished product is S, then the formula for calculating the firm's sales degree R is:

$$R = (n_3 - x_3 n_3 P_3) S \tag{16}$$

The final objective function for the profit of a given firm is:

$$\max f = (n_3 - x_3 n_3 P_3) S - \{ CA_1 n_1 + CA_2 n_2 + CA_3 n_3 + n_1 CB_1 x_1 + n_2 CB_2 x_2 + n_3 CB_3 x_3 + n_3 P_3 x_4 \left[CC_3 - (1 - P_1) CA_1 - (1 - P_2) CA_2 \right] + n_3 P_3 (1 - x_3) CD_3 \}$$

This is equivalent to:

$$\max f = R - (Z_1 + Z_2 + Z_3 + Z_4) \tag{17}$$

There are five decision variables in the model that jointly determine the normalized profit expectation during the decision-making scenario development process. In the solution process of this paper, the manufacturer's decision recommendation is given as the relative proportion of the number of spare parts 1 and spare parts 2 purchased.

At the same time, this paper gives the optimal sampling and testing ratio of spare parts 1, spare parts 2, finished products, and the optimal ratio of dismantling of unqualified samples to achieve the most effective allocation of resources. The combination of the two conditions to maximize profits.

Each of the above variables is brought into the objective function in turn to obtain the final decision scheme as shown in Table 8:

Table 8. Table of decision-making results

situations	1	2	3	4	5	6
Optimal ratio for sampling and testing of spare parts1	0.0437	0.0312	0.0797	0.0114	0.0199	0.0796
The optimal ratio for sampling and testing of Spare Part 2	0.0301	0.0219	0.0438	0.0569	0.0286	0.0475
Optimal ratio of finished products for sampling and testing	0.0122	0.0074	0.0201	0.0141	0.0111	0.0211
Optimal ratio for dismantling of non-conforming finished products	0.9517	0.9682	0.9511	0.9674	0.9577	0.0709
Optimal ratio of spare parts1 to total spare parts	0.5003	0.5005	0.5009	0.5028	0.4991	0.5004
Normalized profit expectation extremes	14.8027	15.2176	11.5548	9.3703	14.1360	12.9017

In summary, under the condition that the defective rate is known, this paper gives the production decision corresponding to the enterprise to obtain the maximum profit based on the defective rate in different cases. For example, under the condition of case 1, the optimal ratio of sampling and testing of parts 1, parts 2, and finished products are 0.0437, 0.0301, and 0.0122 respectively; the optimal ratio of the dismantling of unqualified finished products, the optimal ratio of parts 1 to the total number of parts, and the extreme value of the normalized profit expectation are 0.9517, 0.5003, and 14.8027 respectively. Under the condition of case 1, it is firstly recommended that when the manufacturer purchases parts 1 and 2, it should purchase them by the ratio of spare parts 1 to the overall quantity of spare parts 1 and 2 of 0.5003, which can achieve the maximum utilization of resources, and under this utilization condition, 4.37% and 3.01% of the testing of spare parts 1 and 2, respectively, 1.22% of the testing of the assembled finished products, and 95.17% of the dismantling of the unqualified finished products, can achieve the most efficient deployment of resources and get the highest profit value expectation of $14.8027.

4. Conclusion

This paper provide a sampling testing scheme and production decision-making model for small and medium-sized enterprises, which can significantly reduce the production cost improve the profit of enterprises, and prove the expandability and feasibility of linear programming methods in the field of production decision-making. In the design of the sampling testing scheme, this paper gives different testing schemes for different confidence conditions and different sample sizes, which has a wide range of practical value. In the construction of the decision equation model, this paper considers the proportion of different spare parts purchased as a decision variable of the model, and creatively considers the range of the decision variable in the value of the decision variable, instead of considering it as a simple 0-1 planning model, which results in a higher universality of the decision equation and better decision-making scheme. In the construction process of the decision equation, a large number of parameters are introduced to make the model more stable and reliable, and in the actual production process, when the parameters are given the actual value of the change over time, the model is still applicable, more scientific and more feasible. However, the model constructed in this paper does not take into account the changes in testing costs, market selling prices, and other influencing factors, and when disturbed by external random factors, the model may produce large errors. In the future, related research can try to further maintain the stability of the model under the interference of changing factors.

The decision equation constructed in this paper is not limited to the problem of assembling electronic components. For other industrial production directions of the most complex production process decision-making, this model can provide a certain reference value. The model can be extended from the one-to-one relationship between spare parts and semi-finished products to one-to-many production relationships to solve more complex production process problems. When the variables in the model are given other meanings, the applicable scenarios of the model will be migrated, and this migration has more far-reaching and broader impacts on the generalization of the model.

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