Manufacturing Decision Optimization Based on Sequential Testing and Cost-Benefit Analysis

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Abstract. In the production process of enterprises, the reasonable decision of each link often affects the profits and costs of enterprises. In order to help enterprises reduce production costs, this paper uses the sequential test method to calculate the reliability of 95% and 90% of the two cases of the significance level and testing force, according to the results to calculate the upper and lower limit threshold and the likelihood ratio of the two cases, the likelihood ratio and the upper and lower threshold of the relationship is compared, so as to obtain the number of tests as little as possible sampling detection scheme; At the same time, this paper establishes a cost-benefit analysis model, calculates and compares the costs and losses (gains) of the four links of parts testing, finished product testing, disassembly of unqualified products and disposal of unqualified products, and obtains specific decision plans for each situation, so as to help enterprises make efficient production decisions.

Keywords: Sequential Probability Ratio Test, Production Decision Optimization, Cost-Benefit Analysis.

1. Introduction

In the competitive modern market environment, enterprises are constantly striving for survival and development. The quality and cost of a product play a crucial role in determining its market position. Ensuring product quality while reducing production costs has become a top priority for enterprises. Substandard products can not only damage a company's reputation, but also lead to major financial losses such as returns and customer complaints. Therefore, optimizing the production decision-making process has extremely important practical significance and is also an important factor in promoting the healthy development of the industry and economy.

Many explorations have been made in the field of production decision making by predecessors. Dynamic programming has been widely used in solving multi-stage decision making problems. It decomposes complex problems into a series of interrelated sub-problems, and obtains the optimal solution of the whole problem by solving the sub-problems. However, its application usually requires a clear definition of state variables and transition equations, and there is no unified standard model, which brings certain difficulties to the practical application [1-3]. Monte Carlo simulation is effective when dealing with problems with an element of uncertainty. It can estimate the expected benefits and risks of different strategies by randomly generating samples, but it requires a lot of simulation and consumes a lot of computational resources [4-7]. Genetic algorithm can quickly search for a good solution in a large solution space. However, the determination of parameters such as fitness function and crossover probability has a certain impact on the optimization results, and it is difficult to ensure that the global optimal solution is obtained [8-10]. As a result, these methods often have limitations in dealing with the integration of defective rate estimates and the balance between detection costs and quality control.

According to the above situation, a series of innovations and improvements are made in this paper. Firstly, the sequential inspection method is improved in the design of sampling inspection scheme. By setting parameters and thresholds reasonably, combined with the actual production situation, a more efficient sampling inspection scheme is designed, which can accurately judge the quality of parts with a smaller sample size, and effectively reduce the inspection cost and time cost. Secondly, in the production decision model, the cost-benefit analysis model and dynamic programming are deeply integrated. A more comprehensive cost-benefit calculation method has been established, taking into account various factors in the production process, such as parts inspection, assembly, disassembly and replacement costs. Through dynamic programming optimization, the optimal decision path can be obtained more accurately, and the rationality and economy of decision making can be improved. Finally, in dealing with the uncertainty of the defective rate, the Bayesian inference method is innovatively applied. By assuming the prior distribution of the defective rate and combining with the sampling results, the defective rate is updated in real time, which makes the production decision more suitable to the actual situation and improves the quality control ability of enterprises. Through the above contributions and innovations, this paper provides a more effective solution for enterprises' production decision-making, helping enterprises to improve production efficiency, reduce costs and enhance market competitiveness.

2. Methodology

2.1. Sequential testing

The sampling inspection method is adopted to check whether the defective rate of spare parts exceeds the nominal value, so as to determine whether the enterprise accepts the batch of spare parts purchased from the supplier, and to design a sampling inspection scheme for the enterprise with as few inspection times as possible. If the nominal value is 10%, according to the sampling testing program, specific results are given for the following two situations:

(1) Under 95% reliability, the defective rate of parts is identified as exceeding the nominal value, then reject this batch of parts;

(2) In 90% of the reliability of the parts defective rate does not exceed the nominal value, then accept this batch of parts.

In order to ensure the optimal sample size, we do not presuppose a fixed sample size, but take samples one by one through sequential inspection, and calculate whether the conditions for stopping inspection are met according to the current sample information. If the condition is not reached, continue sampling; If it is reached, a decision is made.

2.1.1. Set sequential test parameters

For case 1: The scheme with 95% confidence

There is a pair of hypotheses, the original hypothesis and the alternative hypothesis, let the original hypothesis is H_0, the alternative hypothesis is H_1, the defective rate is p, then there are:

$$H_0: p \leq p_0 = 10\% \tag{1}$$

$$H_1: p > p_0 = 10\% \tag{2}$$

From the reliability of 95%, we can know that the significance level is the probability of accepting unqualified parts incorrectly, which is denoted as α; The test force is the probability of rejecting the null hypothesis when it is not true, denoted as β, then:

$$\alpha_1 = 0.05 \tag{3}$$

$$\beta_1 = 0.10 \tag{4}$$

Where, α_1 represents the probability of incorrectly accepting unqualified parts under 95% confidence, and β_1 represents the probability of rejecting the null hypothesis under 95% confidence when it is not true.

According to the set parameters, the threshold value of the sequential test can be calculated as:

$$\text{Upper Critical Value (Rejection Region): } A_1 = \frac{1-\beta_1}{\alpha_1} = 18 \tag{5}$$

$$\text{Lower Critical Value (Acceptance Region): } B_1 = \frac{\beta_1}{1-\alpha_1} \approx 0.105 \tag{6}$$

Where A_1 and B_1 respectively represent the upper and lower threshold values of the sequential test in the case of 95% reliability.

For case 2: the scheme with 90% confidence

Suppose the null hypothesis is H_0, the alternative hypothesis is H_1, the defective rate is p, then:

$$H_0: p \le p_0 = 10\% \tag{7}$$

$$H_1: p > p_0 = 10\% \tag{8}$$

The goal is to receive the batch of parts at a 90% confidence level (i.e., the probability of incorrectly rejecting qualified parts β=0.10) by judging that the defective rate of parts does not exceed the nominal value with as few samples as possible. Then, there is:

$$\alpha_2 = 0.10 \tag{9}$$

$$\beta_2 = 0.05 \tag{10}$$

Where, α_2 represents the probability of incorrectly accepting the unqualified part under the condition of 90% confidence, and β_2 represents the probability of rejecting the null hypothesis under the condition of 90% confidence when it is not true.

According to the set parameters, the threshold value of the sequential test can be calculated as:

$$\text{Upper Critical Value (Rejection Region): } A_2 = \frac{1-\beta_2}{\alpha_2} = 9.5 \tag{11}$$

$$\text{Lower Critical Value (Acceptance Region): } B_2 = \frac{\beta_2}{1-\alpha_2} \approx 0.0556 \tag{12}$$

Where A_2 and B_2 respectively represent the upper and lower threshold values of sequential test in the case of 90% reliability.

2.1.2. Sequential Probability Ratio Test (SPRT)

To further reduce the number of tests, it is possible to use the test process to dynamically adjust whether to continue sampling according to the results of each test until an acceptance or rejection decision is made.

Assuming that there is a rejection rate compliance distribution, the cumulative likelihood ratio is:

$$S_i = S_{i-1} + \log \Lambda_i \tag{13}$$

For 95% confidence, set: p_0=0.10, p_1=0.12 (slightly greater than the nominal value, used to test the case of exceeding the nominal value)

The likelihood ratio function is Λ_1

$$\Lambda_1 = \left(\frac{p_1}{p_0}\right)^X \left(\frac{1-p_1}{1-p_0}\right)^{n-X} \tag{14}$$

Where:

$\frac{p_1}{p_0} = \frac{0.12}{0.10} = 1.2$, $\frac{1-p_1}{1-p_0} = \frac{0.88}{0.90} \approx 0.9778$

Therefore, the likelihood ratio is:

$$\Lambda_1 = (1.2)^X * (0.9778)^{n-X} \tag{15}$$

For 90% confidence, let: p_0=0.10,p_2=0.08 (slightly less than the nominal value, used to test reception)

The likelihood ratio function is Λ_2

$$\Lambda_2 = \left(\frac{p_2}{p_0}\right)^X \left(\frac{1-p_2}{1-p_0}\right)^{n-X} \tag{16}$$

Where:
$\frac{p_2}{p_0} = \frac{0.12}{0.10} = 0.8$, $\frac{1-p_2}{1-p_0} = \frac{0.88}{0.90} \approx 1.0222$

Therefore, the likelihood ratio is:

$$\Lambda_2 = (0.8)^X * (1.0222)^{n-X} \tag{17}$$

2.2. Cost-benefit analysis

2.2.1. Establishment of cost-benefit analysis model

In the production process, every decision (such as whether to inspect spare parts or finished products, whether to disassemble unqualified finished products, etc.) involves the cost, and determines the final profit or loss of the enterprise. Therefore, this paper uses the cost-benefit analysis model to optimize the decision-making of each link. Its core goal is to minimize the total cost, which is denoted as C, the formula is as follows:

$$\mathrm{C} = \mathrm{C}_1 + \mathrm{C}_2 + \mathrm{C}_3 + \mathrm{C}_4\mathrm{R}_0 \tag{18}$$

Where, the cost required C_1 to inspect spare parts or finished products; C_2 is the cost of assembling the finished product; C_3 is the cost of disassembling the nonconforming finished product; C_4 for the replacement loss that the company needs to pay if the customer returns the unqualified product; R_0 is the profit from the sale of the finished product.

2.2.2. Cost decision of parts testing

In the production process, enterprises can choose to test parts 1 and 2 to reduce unqualified parts into the assembly link. For whether to detect spare parts, two factors are mainly considered: detection cost and defective rate. If the defective rate is very low and the inspection cost is high, it may be more cost-effective not to test: on the contrary, if the defective rate is high and the inspection cost is low, it is recommended to test to prevent unqualified parts from entering the assembly process.

If the inspection cost is low relative to the defective rate (for example, the inspection cost is lower than the assembly cost or the market price), the inspection can be selected to eliminate the defective product before the assembly process; If the inspection cost is relatively high and the defective rate is low, you can choose not to inspect and directly enter the assembly link to save inspection costs.

The formula of inspection cost is:

$$\mathrm{C_x} = \mathrm{n} \times \mathrm{C_d} \tag{19}$$

Where, $\mathrm{C_x}$ is the total cost of parts testing, $\mathrm{C_d}$ is the number of parts tested, n is the testing cost of each part.

Potential loss without testing:

If the enterprise chooses not to detect spare parts, there will be a certain number of defective products into the assembly link, thus affecting the quality of the finished product. Set the defective rate to be p_3, the assembly defective rate to be $\mathrm{p_a}$, then the expected loss without detection is:

$$\mathrm{E}_0 = \mathrm{n} \times \mathrm{p}_3 \times \mathrm{C_e} + \mathrm{n} \times \mathrm{p}_3 \times \mathrm{p_a} \times \mathrm{C_f} \tag{20}$$

Among them, $\mathrm{C_e}$is the loss caused by the defective parts entering the assembly, and $\mathrm{C_f}$ is the replacement loss of the customer returning the unqualified finished products.

Parts testing decision criteria are: if $\mathrm{C_x}<\mathrm{E}_0$, then select detection; Otherwise, no detection.

2.2.3. Cost decision of finished product testing

The finished product after assembly may contain a combination of qualified parts and unqualified parts, so the enterprise needs to decide whether to test the finished product after assembly. The purpose of finished product testing is to eliminate the unqualified products and avoid the unqualified products entering the market and causing the loss of return and replacement.

Cost of finished product testing:

$$C_y = n \times C_g \tag{21}$$

Where,C_g is the inspection cost of each finished product.

Potential losses from not testing:

If the enterprise chooses not to test the finished product, there may be unqualified finished products flowing into the market, triggering customer returns and exchanges, and damaging the reputation of the enterprise. The expected losses are:

$$E_1 = n \times p_4 \times C_h \tag{22}$$

Where, p_4 is the defective rate of finished products, C_h is the replacement loss of each unqualified product.

The decision criteria for the detection of finished products are: if C_y<E_1, then select the detection; Otherwise, no testing.

2.2.4. Cost decision for disassembly of unqualified products

For the detected unqualified products, the enterprise can choose to discard them directly, or disassemble the unqualified products. There will be a certain cost for the recycled parts to be reused and disassembled, so it is necessary to determine whether the disassembly is worthwhile.

Disassembly cost:

$$C_z = n_0 \times C_s \tag{23}$$

Where, n_0 is the number of unqualified finished products, C_s is the dismantling cost of each finished product.

Disassembly income:

The disassembled parts can be reused. If qualified parts can be obtained after disassembly, the expected income is:

$$E_2 = n_0 \times p_5 \times \left(C_j C_2\right) \tag{24}$$

Where, p_5 is the probability of qualified parts after disassembly,C_j is the value of each recovered part.

If C_z< E_2, choose disassembly; If not, do not disassemble.

2.2.5. Return nonconforming products for disposal decisions

For the unqualified products returned by the user, the enterprise needs to unconditionally exchange, which will lead to certain exchange losses (such as logistics costs, corporate reputation loss, etc.). At the same time, the enterprise can choose to disassemble the returned unqualified products and recycle them after disassembly.

$$C_t = C_l + C_3 E_3 \tag{25}$$

Among them,C_l is the logistics cost in the return process and E_3 is the expected benefit from the qualified parts recovered from the disassembly.

2.2.6. Comprehensive decision model

By synthesising the entire production process and considering decisions at each horizon, the total cost expectation model can be constructed:

$$E_0 = \sum_{i=1}^{n} \left(C_x(i) + C_y(i) + C_2(i) + C_z(i)\right) \tag{26}$$

Here, the cost of each process is calculated according to the above formula. The cost of each item can be determined by the expected total cost. horizon inspection and disassembly, so as to optimize the cost structure of the entire manufacturing process.

There are two kinds of spare parts, a total of 2^2=4 kinds of strategy combination of spare parts, a total of 2*2=4 kinds of strategy combination of finished products, a total of 4*4=16 kinds of strategy combination, each combination represents the detection decision of spare parts 1, spare parts 2, the detection and disassembly decision of finished products.

Table 1. Product production strategy combination

ID	Inspection Parts 1	Test parts 2	Finished Product Inspection	Disassemble unqualified finished products
1	Testing	Testing	Testing	Disassemble
2	Testing	Testing	Testing	Not disassemble
3	Detection	Testing	Not detect	Disassemble
4	Testing	Detection	Not detect	No dismantling
5	Detection	Not detect	Detect	Disassemble
6	Testing	Not detect	Detect	Not disassemble
7	Detection	Not detect	Do not detect	Disassemble
8	Testing	Do not detect	Do not detect	No dismantling
9	No testing	Detect	Testing	Disassemble
10	Undetected	Detect	Testing	Not disassemble
11	No testing	Detect	Not detect	Disassemble
12	Undetected	Detection	Not detect	No dismantling
13	No testing	Do not detect	Detect	Disassemble
14	Undetected	Do not detect	Detect	Not disassemble
15	No testing	Do not detect	Do not detect	Disassemble
16	Undetected	Do not detect	Do not detect	No dismantling

We put the 16 sets of strategies in the problem into the model and get specific results.

3. Results And Discussion

3.1. Plan of sampling inspection

The inspection process will be dynamically adjusted according to the inspection results. According to the above analysis, the sampling inspection plan is as follows:

(1) Sampling from parts one by one, testing one part each time, record the number of samples n and the number of defective products X.

(2) After each sample, the updated value.Λ

(3) The decision to reject or continue sampling is made according to the inspection rules.

Through the above steps, we can get the sequential inspection decision rules at 95% confidence level and 90% confidence level, that is, the specific sampling inspection scheme:

(1) The conditions of H_0rejection:

When the reliability is 95%, if $\Lambda_1 \geq A_1$= 18, it is rejected H_0, that the defective rate exceeds the nominal value, and the parts are rejected;

When the reliability is 90%, if $\Lambda_2 \geq A_2$=9.5, it is rejected H_0, it is considered that the defective rate exceeds the nominal value, and the parts are rejected.

(2) Conditions of H_0acceptance:

When the reliability is 95%, if $\Lambda_1 \geq B_1$= 0.105, then accept H_0, consider that the defective rate does not exceed the nominal value, receive parts;

When the reliability is 90%, if $\Lambda_2 \geq B_2$=0.0556, then accept H_0, consider that the defective rate does not exceed the nominal value, receive parts.

(3) Continue sampling conditions:

When the reliability is 95%, if 0.105 < Λ_1 < 18, then continue sampling;

When the confidence is 90%, if 0.0556 < Λ_2 < 9.5, then continue sampling.

3.2. Specific decision scenarios

According to the above model processing, for the six situations given in the question, the specific decision scheme is obtained as shown in Table 2 and Figure 1.

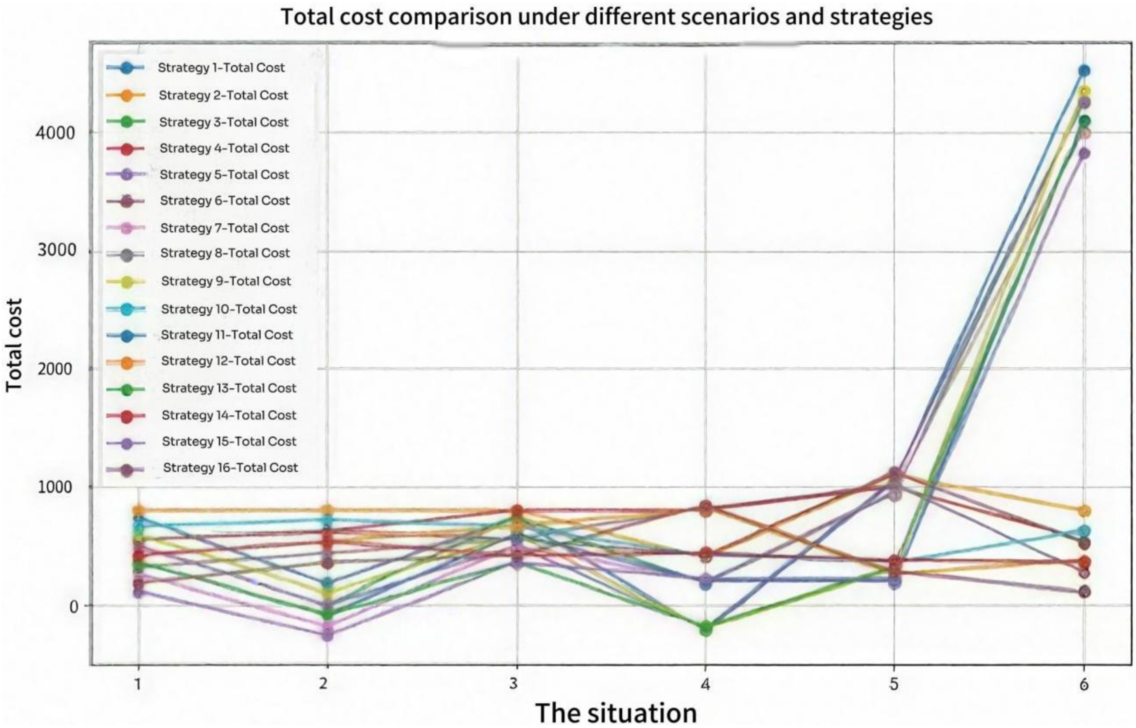

Figure 1. Total cost comparison chart under each strategy

Table 2. Decision-making of situations encountered by enterprises in production

Scenario	Detecting Parts 1	Detect Parts 2	Finished Product Inspection	Disassemble unqualified finished products
1	Testing	Testing	Not detect	Do not detect
2	Detect	Testing	Not detect	Do not detect
3	Detect	Testing	Not detect	Do not detect
4	Detect	Testing	Testing	Testing
5	Not detect	Detect	Not detect	Do not detect
6	Detect	Testing	Not detect	Do not detect

3.3. Decision-making process and summary

Through the above analysis, every decision point in the production process can be optimized by comparing the cost with the benefit. The following is a simplified decision process:

(1) Parts testing decision: When the testing cost is low and the defective rate is high, it is recommended to test.

(2) Finished product testing decision: When the finished product defective rate is high and the testing cost is low, it is recommended to test.

(3) Disassembly of unqualified finished products: When the disassembly cost is lower than the value of recovered parts, it is recommended to disassemble.

(4) Disposal of returned unqualified products: If the value of recovered parts is greater than the dismantling cost, it is recommended to disassemble.

By calculating the costs and benefits of different schemes through the model, specific decision schemes can be provided for enterprises in the production process to optimize production efficiency and reduce costs.

4. Conclusion

This study provides a comprehensive and practical solution for optimizing production decisions in enterprises by integrating sequential testing and cost-benefit analysis models. The improved sequential inspection method ensures the accuracy of part quality assessment while reducing sample size and inspection costs. The cost-benefit analysis model, which considers both production costs and potential benefits, helps enterprises more accurately determine the optimal decision path. Overall, the mathematical modeling approach adopted in this paper offers a scientific and efficient tool for production decision-making, significantly enhancing market competitiveness. However, the current algorithm still has certain limitations, such as the dependence of sequential testing on defect rate distribution assumptions, which may lead to deviations in practical applications. Future research can explore more flexible testing techniques and incorporate advanced data analysis methods to better adapt to real-world data distributions and complex cost estimations.

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