Research on Production Process Decision-Making Based on Simulated Annealing and Monte Carlo Algorithms

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Abstract. In the production process, the decision - making regarding component inspection exerts a crucial influence on the overall profit. This paper applies a variety of statistical methods to construct models for different production and assembly scenarios, aiming to derive decision - making schemes that achieve optimal profit. Specifically, by utilizing the Central Limit Theorem and the OC curve, an efficient sampling inspection method for determining the component defect rate in the actual production process is proposed. Meanwhile, with the aid of the dynamic programming algorithm and the Monte Carlo simulation algorithm, the simplified production and assembly process is modeled and solved to obtain the optimal decisions under different component defect rates and inspection costs. Subsequently, the simulated annealing algorithm is employed to model and solve the complex multi - stage assembly, and the optimal decisions in different situations are obtained. Moreover, the relationships among the defect rate, inspection cost, and inspection decision - making are visually presented.

Keywords: Production Process Decision, Monte Carlo Simulation, Dynamic Programming, Simulated Annealing Algorithm.

1. Introduction

In the actual production process, the occurrence of substandard products is not uncommon. Enterprises usually deal with such products in two ways: one is to scrap them directly, which can avoid additional costs incurred in subsequent processing; the other is to disassemble the products to recover reusable parts, although this approach involves certain disassembly costs. According to the literature "Incorporating component reuse, remanufacture, and recycle into product portfolio design"[1], reusing or remanufacturing certain components may be more cost-effective than direct disposal and can recover their economic value. This provides enterprises with a new idea, that is, to reasonably evaluate the parts of substandard products to determine which can be reused or remanufactured, thereby controlling costs while maximizing resource utilization efficiency. However, when conducting quality inspections on product parts, even if the parts pass the inspection smoothly, the assembled finished products may still fail due to problems in the assembly process or other unknown factors. Faced with such challenges, enterprises not only need to strictly control the quality of parts procurement but also need to properly deal with substandard finished products caused by improper assembly or compatibility issues between parts [2].

Morato PG's study [3] combines dynamic Bayesian networks with Markov decision processes (POMDPs). The dynamic Bayesian network is used to model the degradation process of structural components and update uncertainty information. Then, in the POMDP framework, the optimal inspection and maintenance strategy is determined based on the current state probability distribution, and the POMDP problem is solved by dynamic programming to minimize the total cost over the structure's life cycle. However, its high complexity leads to significant computational costs, especially for large - scale systems or complex degradation processes. Moreover, it may not adapt well to special cases such as sudden unexpected failures or rare degradation patterns. The particle [4] swarm optimization (PSO) algorithm is adopted to determine the optimal inspection strategy in serial multistage production processes. Decision variables such as inspection types, acceptance limits, and sample sizes at each stage are considered, and the total inspection cost is used as the performance metric. The optimal solution is searched through the iterative operation of the PSO algorithm. Nevertheless, PSO is prone to getting trapped in local optimal solutions. Additionally, the model's consideration of uncertain inspection costs may not accurately reflect the complex cost changes in actual production, affecting its practicality and accuracy. The time - based maintenance (TBM) and condition - based maintenance (CBM) models [5] are formulated as Markov decision processes (MDPs), and dynamic programming is used to find the optimal strategy. For continuous degradation processes, discretization is required, and the discretization level needed to obtain a near - optimal strategy is investigated. Based on the concept of the value of information (VOI), a non - periodic sequential inspection strategy [6] determines the optimal time for the next inspection or replacement according to the current condition of the component and the expected benefits of possible inspections, arranging inspections in a piece - wise optimal manner. The paper [7] proposes a two - stage inspection policy model, aiming to integrate inspection methods with different accuracy and cost characteristics. In the mathematical model, the situation where the second - stage inspection is not necessarily perfect is taken into account. A set of rules are formulated to support decision - making. When searching for the cost - effective parameters of the two - stage policy, the application of these rules is verified through numerical examples. The existing research is still lacking in some aspects: the model [3] is highly complex and incurs significant computational costs. Especially when dealing with large - scale systems or complex degradation processes, it may face the issue of low computational efficiency. Moreover, for some special situations in practical applications, such as sudden unexpected failures or rare degradation patterns, the model may not be able to adapt to and handle them well. The PSO algorithm may get trapped in local optimal solutions, resulting in the inability to find the globally optimal inspection strategy. Additionally, although the uncertain inspection costs are considered in the model, it may not accurately reflect the complex cost variations in actual production, thus affecting the practicality and accuracy of the algorithm. When solving the CBM model, the discretization of the continuous degradation process may lead to certain precision loss, and the determination of the discretization level requires further research and optimization. For systems with stochastic dependencies, the computational requirements may increase rapidly as the number of components grows. There is a curse - of - dimensionality problem, which limits the application of this method in large - scale and complex systems. The strategy [6] mainly focuses on single - component analysis and preventive maintenance, insufficiently considering the complex interactions and collaborative optimization in multi - component systems. The model [7] assumes that the inspection results in the first stage are accurate. However, in actual production, errors may also occur in the first - stage inspection. For complex production systems, there may be various types of components and multiple inspection methods, and this model may not be able to handle such complexity well. To sum up, the research goal of this paper is to solve the optimal decision of parts inspection in the production process through statistical modeling, so as to maximize the profit.

The innovation points of this paper include: the use of diversified sampling scheme can make the most suitable sampling scheme according to different actual conditions. Monte Carlo algorithm is used to simulate the profit under different decisions. The simulated annealing algorithm is suitable for solving the difficulties of high dimensional traversal in complex decision making, and it is easy to jump out of the local optimal solution and make the results visible.

2. Methodology

2.1. Central limit theorem theory and OC curve

If the number of samples is large enough and each sample has little effect on the population, and the result of the random variable depends on a large number of samples, and the effect of each sample is relatively uniform, then the random variable follows a normal distribution.[8]

The OC curve shows the probability that a whole batch of products will be accepted under different conditions of defective (or nonconforming) rates. The steeper OC curve indicates that the selection sampling scheme is very sensitive to the change of the defective rate, that is, a small change in the defective rate will lead to a large change in the acceptance probability. This usually means that the scheme has a high level of differentiation and can identify batches of different quality levels better.

2.2. Dynamic Programming and Monte Carlo simulation

Dynamic programming is a method for solving multi-stage optimal decision problems. For each process (decision stage), the basic flow of dynamic programming is as follows:

State definition: The state is defined as the state of the current part (or finished product) (such as whether it is defective) and the process that has arrived.

Decision-making: In each process (decision-making stage), choose "test" or "no test", for non-conforming products, choose whether to disassemble.

State transfer: If the inspection, in the next step to exclude defective products, enter the next process; If not, proceed directly to the next process.

Goal: Maximize profit.

The core idea of dynamic programming is that each decision will have an impact on the subsequent state and final cost, recursively traverse each decision, and dynamically find the optimal solution of each decision.

For a finished product, the cost changes brought about by different decisions in the three stages are as follows. Let the decision variables X_a, X_b, X_p, X_d be 01 variable. Let e be a random variable, with a value of 0 or 1 indicating that it may be defective. C_a is the purchase cost of part a. C_b is the purchase cost of part b. $C_{assembly}$ is the assembly cost. $C_{return}$is the return cost. $C_{disassemble}$is the dismantling cost.

Cost of the first decision stage C1:

$$C_1 = C_a + C_b + I_a \cdot X_a + I_b \cdot X_b \tag{1}$$

If the two parts are not defective, the cost of the second decision stage C2:

$$C_2 = C_1 + C_{assembly} + I_p \cdot X_p + e \cdot C_{disambly} \tag{2}$$

If the finished product is non-defective, the cost of the third decision stage C3:

$$C_3 = C_2 + (1 - X_d) \cdot e \cdot C_{return} \tag{3}$$

If it is still non-defective, the profit is as follows:

$$w = P - C_3 \tag{4}$$

Monte Carlo simulation uses a large number of random samples to simulate the cost under different strategies, and then uses dynamic programming to find the strategy with the least expected cost [9]. This method can be used to simulate multiple production processes and observe cost performance under different inspection strategies. The state and decision space of the decision problem in this problem are very complex, and it is difficult to solve it by using only dynamic programming. Monte Carlo simulation is a good method of approximate solution.

Suppose there are n assembled products, the inspection decision is x_i, and $f_n(x)$ represents the profit of the product. Then the profit under this decision can be expressed as

$$F(x_i) = \lim_{n \to \infty} \frac{\sum_{j=i}^{n} f j(x_i)}{n} \tag{5}$$

2.3. Simulated annealing algorithm

The simulated annealing algorithm is a global optimization algorithm. Its principle is similar to the solid annealing process in physics [10]. Simulated annealing is achieved by appropriately controlling the "temperature" reduction process (so that the objective function reaches its extreme value) through a specific algorithm (the Metropolis algorithm). This algorithm has strong generality and robustness. It is not prone to getting trapped in local optimal solutions and is suitable for high - dimensional decision - making, thus being applicable to this problem.

The core ideas of simulated annealing are as follows.

Set an initial solution, control the "temperature", introduce a "temperature" parameter, and allow the solution to accept a solution worse than the current one with a certain probability. The "temperature" gradually decreases as the number of iterations increases. At the current "temperature", a new solution (neighboring solution) is generated through a certain strategy, and then this new solution is accepted according to a certain probability.

The derivation of this probability formula is as follows:

The total energy E of the system:

$$E = \sum_{i,j} V(rij) \tag{6}$$

Where $V(r_{ij})$ represents the potential energy between i and j, and r_{ij} is the distance between the two particles.

The probability $P(accept)$ is given by the total energy of the system:

$$P(accept) = \begin{cases} 1, E(n+1) < E(n) \\ e^{-\frac{E(n+1)-E(n)}{T}}, E(n+1) \geq E(n) \end{cases} \tag{7}$$

This probability is determined by the difference between the current solution and the new solution as well as the current "temperature". The above process generally follows the Metropolis algorithm criterion.

3. Results and discussion

The sampling plan is based on simple random sampling to select defective items. The minimum number of samples is calculated using a formula that takes into account factors such as error range, confidence level, and Z-value. The specific details are shown in Table.1.

Table 1. The specific details of samples

Error Range	Sample Proportion Estimate	Z-value	Confidence Level	Sampling Times	Critical Value
0.02	0.1	1.96	95%	864	96
0.05	0.1	1.96	95%	138	17
0.02	0.1	1.96	90%	608	70
0.05	0.1	1.96	90%	97	14

According to the results generated by the model code, under an error range of 0.02 and a confidence level of 95%, the minimum number of sampling times is 864, with an acceptance or rejection critical value of 96. Under an error range of 0.02 and a confidence level of 90%, the minimum number of sampling times is 608, with an acceptance/rejection critical value of 70.

When deciding whether to disassemble and recycle the components, there are multiple scenarios. Taking two components as an example, it is possible to choose whether to inspect them. If inspection is conducted, additional inspection costs will be incurred, but the probability of defective items appearing in the assembly stage can be reduced. If inspection is performed in the assembly stage, the probability of customers purchasing defective items and incurring replacement costs can be reduced. For the decision on whether to disassemble, disassembly can allow materials to enter a new cycle, thereby reducing the cost of purchasing raw materials. Each decision has a significant and complex impact on subsequent costs, as well as the quantity and number of defective items. Finally, the decision on whether to disassemble will also lead to the components entering a new processing flow. Due to the complexity and variability of the situation, this paper will use a dynamic programming and Monte Carlo simulation model to solve this problem.

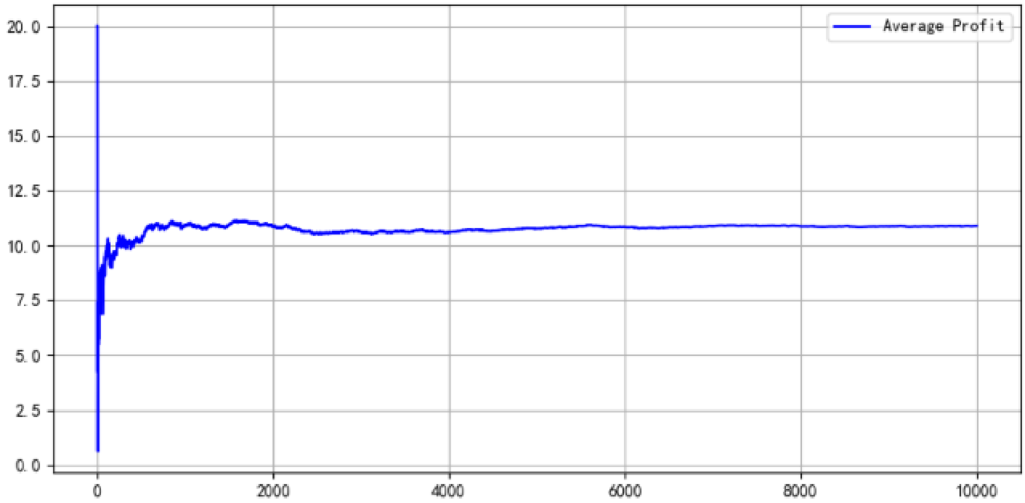

Figure 1. Monte Carlo simulation schematic drawing

As the number of iterations increases, the average profit curve becomes smoother and quickly converges to a value. This demonstrates the implementation of the Monte Carlo simulation method.

The decision results for the six specific scenarios are shown in the Table 2.

Table 2. Results of Monte Carlo simulation

Whether to Inspect a	Whether to Inspect b	Whether to Inspect Final Product	Whether to Disassemble	Average Profit
0	0	1	1	14.074
0	1	1	1	5.215
0	0	1	1	13.767
0	1	1	1	7.619
0	1	1	1	13.263
0	0	1	0	18.780

Further, considering the actual production process, we will discuss more complex scenarios (assuming 8 components whether to inspect + 3 semi-finished products whether to inspect + 3 semi-finished products whether to disassemble + 1 final product whether to inspect + 1 final product whether to disassemble = 16 decisions). Here, we will use the simulated annealing method for research.

First, the initial parameters of the simulated annealing algorithm are as follows.

Table 3. Parameter of simulated annealing algorithm

Parameter Name	Value
Initial Temperature	10000000
Maximum Iterations	100000
Temperature Decrease Rate	0.9

The profit changes with the number of iterations during each iteration are shown in the following figure.

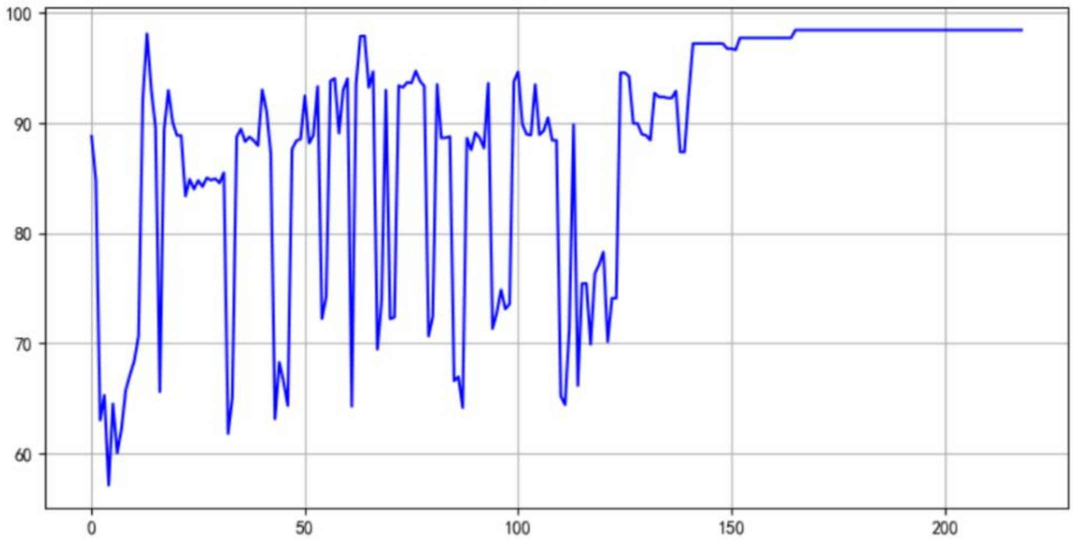

Figure 2. Simulated annealing algorithm schematic drawing

It can be seen that at the beginning, when the temperature is high, there are some cases with lower profits. This is because a larger range of profit differences can be accepted when the temperature is high. As iterations proceed, the profit gradually stabilizes and converges to a value. At this point, the temperature is low, and new solutions with much lower profits than the original solution will no longer appear. In the final stage, the neighborhood solutions of the original solution no longer have lower profits than the original solution, and this solution is the optimal solution of the simulated annealing algorithm. This is the annealing process. Model Initial Parameters

Since the simulated annealing process involves randomness, the decision results after running the program multiple times are as follows.

Table 4. Result of simulated annealing algorithm

Item	Code	Inspection/Disassembly Required
Component Inspection	Part 1	1
	Part 2	1
	Part 3	0
	Part 4	1
	Part 5	1
	Part 6	0
	Part 7	1
	Part 8	1
Semi-finished Product Inspection	Semi-finished Product 1	1
	Semi-finished Product 2	1
	Semi-finished Product 3	0
Semi-finished Product Disassembly	Semi-finished Product 1	1
	Semi-finished Product 2	1
	Semi-finished Product 3	1
Final Product Inspection	Final Product	0
Final Product Disassembly	Final Product	0

4. Conclusion

This study investigates the optimal decision-making strategies for part inspection in production processes and proposes a series of innovative methods, including those based on statistical modeling, dynamic programming, Monte Carlo simulation, and simulated annealing algorithms, to assist enterprises in enhancing production profitability through rational inspection strategies. By applying the Central Limit Theorem and OC (Operating Characteristic) curves, effective sampling inspection methods are developed, which enable the selection of appropriate sampling plans according to varying defect rates and inspection costs. Dynamic programming and Monte Carlo simulation are employed to model simplified production and assembly processes, revealing optimal decisions under different conditions. For complex multi-stage assembly problems, the simulated annealing algorithm successfully overcomes the issue of local optima, providing more practical decision-making solutions. Simulations and experiments validate the relationship among defect rates, inspection costs, and inspection decisions, demonstrating that a rational inspection scheme can significantly improve overall production profitability. The simulated annealing algorithm exhibits strong global optimization capabilities in multi-stage decision-making problems, capable of identifying optimal solutions in high-dimensional decision spaces.

In summary, this study provides theoretical foundations and methodological support for enterprises to make optimal part inspection decisions in complex production systems. The proposed models and algorithms are characterized by good practicability and robustness, and can be widely applied to various production and assembly scenarios, assisting enterprises in optimizing resource allocation, reducing costs, and enhancing benefits under conditions of uncertainty.

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