Optimal Crop Planting Strategies Based on Multi-Constraint Optimization Models

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Abstract. This paper takes crop cultivation in a village as the object of study, and aims to formulate the optimal cultivation scheme for 2024-2030 under a specific situation by establishing a mathematical model, i.e., the optimal scheme for stagnant sales and half-price treatment for the over-expected sales volume, respectively, while the relevant factors of the crops remain stable. In order to achieve profit maximization, the optimization model is established. Firstly, establish decision variables, according to the principle of economics, and at the same time consider the planting time, crop type, plot type to determine the three-dimensional decision variables planting area and 0-1 variable planting. Secondly, establish the sales volume under the super-expected to take the stagnant sales and half-price treatment as well as not exceeding the expected two types of objective function.Finally, the constraints are quantified, including arable land area limitations, crop growth conditions, and crop rotation requirements to prevent continuous monocropping. Additionally, constraints are imposed on the total marketing revenue of a given year, which affects the planting costs for the following year, along with six other key constraints. The optimal solution is obtained through programming, and conclusions are drawn based on the results.

Keywords: Optimal Planting Strategies, Optimization Models, Constraint Quantification.

1. Introduction

With the continuous advancement and transformation of global agricultural production models, achieving efficient crop cultivation and sustainable development under constrained land and resource conditions has become a central issue in agricultural research [1]. This challenge is particularly prominent in the mountainous areas of North China and other regions with limited resources, where optimizing the allocation of scarce arable land while enhancing crop yield and quality to maximize economic benefits remains a key concern for agricultural scientists [2].

In recent years, numerous studies have explored crop cultivation optimization in mountainous regions. For instance, research on the Wuling Mountain Area has highlighted the impact of rural labor migration on crop cultivation structures [3], while studies in Ningxia have examined the role of water scarcity in driving adjustments to planting strategies [4]. Although these studies provide valuable data and theoretical foundations, most focus on general crop structure optimization and resource allocation. However, limited research has addressed optimization strategies for specific challenges, such as managing surplus production and ensuring sustainable crop rotation [5].

One underexplored issue in agricultural production is handling unexpected surplus sales, which can lead to market fluctuations and financial losses. Effective market strategies to mitigate these risks remain insufficiently studied [6]. To address this gap, this paper introduces a multi-constraint optimization model for crop planting, developed through the analysis of agricultural data from a specific region. This model considers critical factors such as planting schedules, crop-plot compatibility, and land resource allocation. Unlike previous models, it explicitly integrates over-expected sales volume management, employing stagnation control and half-price disposal mechanisms to minimize financial losses and enhance farmers' profitability [7].

Moreover, conventional agricultural research often overlooks the impact of continuous monocropping, which can deplete soil nutrients and reduce yields [8]. This study introduces an adaptive optimization model with crop rotation constraints, ensuring sustainable soil management and enhanced long-term yield stability [9]. Compared with traditional models that primarily focus on crop structure and resource allocation, this research incorporates dynamic sales volume handling mechanisms, addressing both stagnant inventory risks and market price adjustments. Additionally, by implementing adaptive no-stubble cropping constraints, this model optimizes soil fertility while improving long-term agricultural productivity.

Unlike purely theoretical frameworks, the proposed model is validated using real-world agricultural data, ensuring its practical applicability in mountainous rural regions. The results demonstrate that this optimization approach not only enhances economic returns for farmers but also provides a sustainable strategy for long-term agricultural development. Through the construction and solution of this model, this paper aims to offer an actionable framework for rural agricultural planning, particularly in resource-constrained areas, and to contribute valuable insights for future farmland cultivation planning, resource management, and agricultural policy formulation [11].

2. Data processing and analysis of results

2.1. Data sources

The data used in this paper comes from the website: http://cumcm.cnki.net.

This data is related to agricultural cultivation in a rural village in the mountainous region of North China, and contains the types of crops cultivated, cultivation conditions, crop yields, and sales.

2.2. Data processing

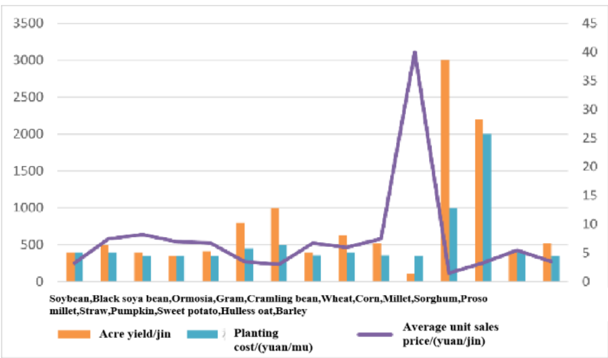
Organized linkage: The two sets of existing data were naturally linked (NJ) through Python programming to integrate and link cropland and crop information.

New column: A new column “Is bean” was added to Sheet1 to identify whether the crop is bean or not by determining the crop type.

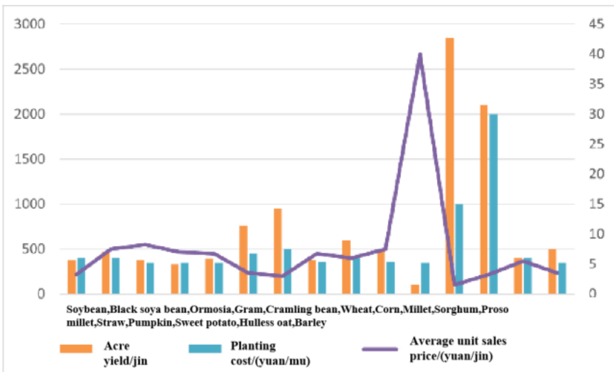
Transformation of data: The range of unit sales prices is averaged out and the average value is used as the expected selling price in 2023 to facilitate later calculations.

2.3. Visualization and analysis of results

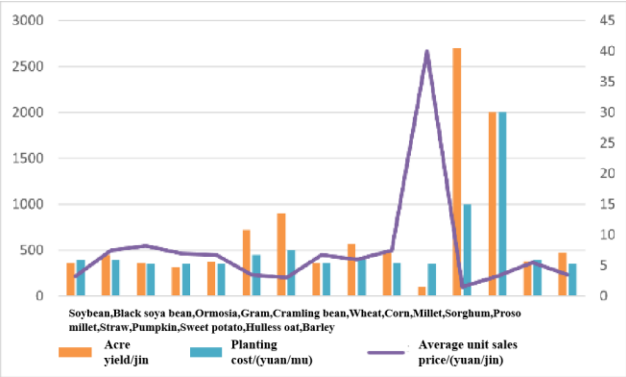
Using 2023 planting and sales data as the base input data for the model, the acreage yield, planting costs, expected sales volume, and prices of the different crops were preprocessed (as shown in Figure 1).



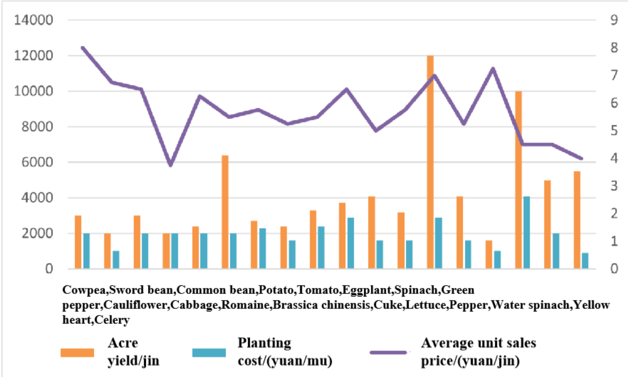
(a)single season in arid lands



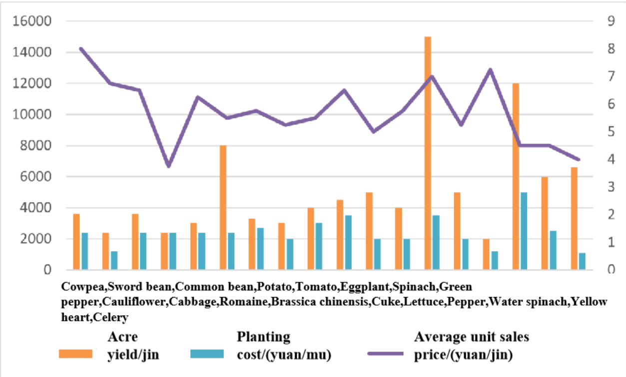
(b)terraces single season



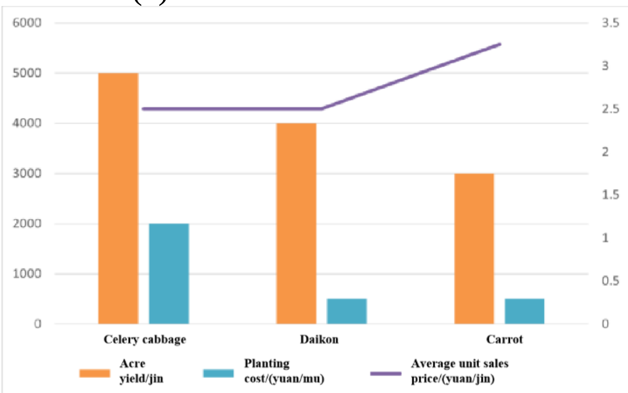
(c)hillside single season



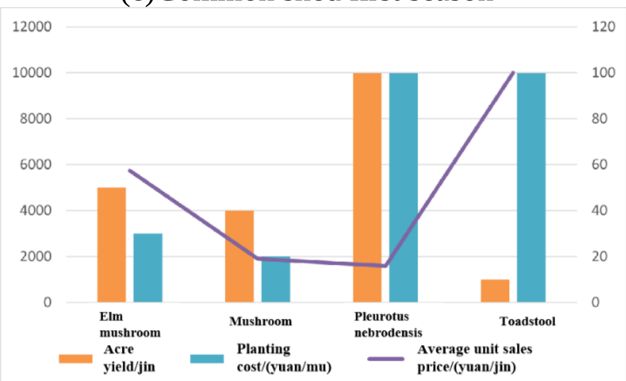
(d)watered land first season



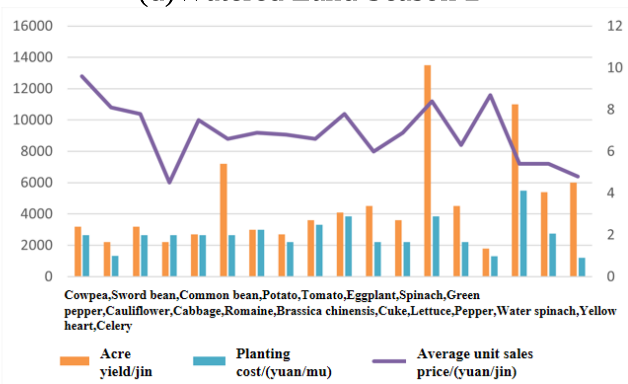
(e)Common shed first season



(d)Watered Land Season 2



(e)Common Shed Season 2



(f)Smart Shed Season 2

Figure 1. Yield per acre, cost of cultivation and average selling unit price of different monocultures in different types of plots

The data in Figure 1 on crop yields per acre, planting costs, and average selling unit prices on different plots are instructive in terms of profitability and planting strategies in the following ways.

(1)Data analysis

In terms of yield: of these, the range of yields for the first season on watered land (d) was between 0 and 14,000 pounds; the range of yields for the first season on ordinary greenhouses (e) was between 0 and 3,300 pounds; the range of yields for the single season on hillslopes (c) was between 0 and 2,500 pounds; and the yield data for the single season on arid lands (a) and single season on terraced fields (b) were more scattered, but relatively low overall. Overall, yields were relatively high in the first season on watered land, followed by the first season in ordinary greenhouses and the single season on hillsides, and relatively low in the single season on dry land and the single season on terraces.

As for planting costs: in the dryland single season (a), terraced single season (b), and hillside single season (c), the planting costs were different for yields of about 1,500 pounds, but were relatively similar overall; the planting costs in the first season of the irrigated field (d) and the first season of the ordinary greenhouse (e) were relatively high.

As for the average selling unit price: the average selling unit price for the first season in watered land (d) ranged from 0 to 7 yuan/catty; the average selling unit price for the first season in ordinary greenhouses (e) ranged from 23 yuan/catty to 53 yuan/catty; while the average selling unit price for the single season in hillside land (c), the single season in arid land (a), and the single season in terraced land (b) ranged from 10 yuan/catty to 45 yuan/catty. In summary, the average selling unit price was relatively low for the first season (d) of watered land with higher yields and the first season (e) of ordinary greenhouses; while the average selling unit price was relatively high for the single season (a) of arid land, the single season of terraced land (b), and the single season of hillside land (c), which had lower yields.

(2)Practical significance

Choosing suitable planting sites: If you pursue high yield, you can give priority to the first season on watered land or the first season in ordinary greenhouses; if you focus on the quality of the product and a higher selling price per unit, you can choose a single season on arid land, a single season on terraced fields or a single season on hillside land. If consider planting cost, land resources, water resources and other factors. Watered land and common greenhouses require higher input and management costs, while arid land, terraced land and hillside land may face problems such as water shortage.

Balance between yield and selling price per unit: The relationship between yield and selling price per unit needs to be considered comprehensively when formulating planting strategies. Generally speaking, planting sites with high yields may have relatively low selling unit prices, while planting sites with low yields have relatively high selling unit prices. Planting varieties and planting densities can be adjusted according to market demand and price trends in order to achieve a balance between yields and selling unit prices.

2.4. Modeling and solving

2.4.1 Setting of decision variables

First, the basic elements of planting planning need to be considered, including plot, planting season, crop type, and planting area A_{ijk}, which directly influence the development of the planting program. In order to differentiate and manage the 41 different plots, unique numbers were assigned to each plot (see Table.1). Finally, "denotes that plot i planted crop j in year k (0-1 variable)" is introduced, when this variable $X_{i,j,k}$ is 1, it means that planting is carried out, and when it is 0, it means that no planting is carried out, so that it can clearly indicate the relationship between each plot and crop planting in different years, so as to accurately construct specific planting programs, and also facilitate the model to constraints and optimization in the model.

Table.1. Key decision variables

variable names	Mathematical implications
$i \in \{1,2, \ldots, 54\}$	Plot number
$j \in \{1,2, \ldots, 41\}$	Crop type number
$k \in \{1,2, \ldots, 14\}$	Planting season (biannual)
$X_{i,j,k}$	Indicates that plot i is planted with crop j in year k (0-1 variable).
A_{ijk}	Area under crop cultivation

2.4.2 Determination of the objective function

For the first case, the part that exceeds the expected sales volume is stagnant and causes waste, when the actual sales volume is less than or equal to the expected sales volume, the revenue = the selling price of a single crop × the actual sales volume - the acre cost of a single crop × the acreage of the crop; when the actual sales volume is greater than the expected sales volume, the revenue = the selling price of a single crop × the expected sales volume - the acre cost of a single crop × the acreage of the crop.

When $Q_{ij} \geq E_{ij}$:

$$\pi = S - C = \sum_{j=1}^{n} E_{ij} P_j - A_{ijk} \cdot CM_{ij} \tag{1}$$

When $Q_{ij} < E_{ij}$:

$$\pi = S - C = Q_{ijk} \cdot P_j - A_{ijk} \cdot CM_{ij} = A_{ijk} \cdot QM_{ij} \cdot Pj - A_{ijk} \cdot CM_{ij} \tag{2}$$

For the second scenario, the portion that exceeds the expected sales volume is treated as a 50% reduction in price based on the 2023 selling price.

When $Q_{ij} \geq E_{ij}$:

$$\pi = S - C = \sum_{j=1}^{n} E_{ij} P_j + \sum_{j=1}^{n} Q_{ijk} P_j \cdot 50\% - A_{ijk} \cdot CM_{ij} \tag{3}$$

When $Q_{ij} < E_{ij}$:

$$\pi = S - C = Q_{ijk} \cdot P_j - A_{ijk} \cdot CM_{ij} = A_{ijk} \cdot QM_{ij} \cdot Pj - A_{ijk} \cdot CM_{ij} \tag{4}$$

2.4.3 Quantification of constraints

The purpose of this paper is to plan the cultivation of crops in a rural area under the same conditions as the expected sales volume, planting cost, mu yield and sales price in 2023. The following are the limits of the relevant constraints, according to which they are quantified:

(1) Cultivated land area constraints:

The total cultivated area of each plot cannot exceed its actual area, viz:

$$\sum_{j=1}^{n} A_{i,j} \leq A_{\max,i} X_{i,j,\mathrm{k}} \quad i \in \forall \tag{5}$$

Where $A_{i,j}$ denotes the acreage of crop j on plot i and $A_{\max,i}$ denotes the maximum acreage of plot i.

(2) Crop growing conditions:

(a) Only one season of food crops can be grown on flat dry land, terraces and hillsides.

$$C_{i,\mathrm{k}} = 1 \quad i \in \forall, k \in \{1, 2, ..., 14\} \tag{6}$$

(b) Water-consumed land can grow one season of rice or two seasons of vegetables:

If two seasons of vegetables are grown on a particular piece of watered land, a variety of vegetables can be grown in the first season (except Chinese cabbage, white radish and red radish); in the second season, only one of Chinese cabbage, white radish and red radish can be grown (for ease of management). According to the seasonal requirements, cabbage, white radish and carrot can only be grown in the second season in a watered field.

$$\begin{cases} C_{i,j1,k1} \,\&\, C_{i,j2,k2} \\ i \in \forall \\ j1 \in \{17,...,34\} \\ \mathrm{j}2 \in \{35,36,37\} \\ k1 \in \{1,2,...,14 \,\&\, k1\} \\ k2 \in \{1,2,...,14 \,\&\, k2 \text{ is an even number}\} \end{cases} \tag{7}$$

(c) Ordinary greenhouses grow two seasons of crops per year. In the first season, a variety of vegetables can be grown (with the exception of Chinese cabbage, white radish and carrot), and in the second season, only edible mushrooms can be grown. Since edible fungi are adapted to grow in a lower and suitable temperature and humidity environment, they can only be grown in ordinary greenhouses in the fall and winter.

$$\begin{cases} C_{i,j3,k3} \,\&\, C_{i,j4,k4} = 1 \\ i \in \forall \\ j3 \in \{17,...,34\} \\ \mathrm{j}4 \in \{38,36...,41\} \\ k3 \in \{1,2,...,14 \,\&\, k1 \text{ is an odd number}\} \\ k4 \in \{1,2,...,14 \,\&\, k2 \text{ is an even number}\} \end{cases} \tag{8}$$

(d) Smart greenhouses can grow two seasons of vegetables every year (except cabbage, white radish and carrot).

$$\begin{cases} C_{i,j5,k5} \,\&\, C_{i,j6,k6} \\ i \in \forall \\ j3 \in \{17,...,34\} \\ \mathrm{j}4 \in \{17,...,34\} \\ k5 \in \{1,2,...,14 \,\&\, k1 \text{ is an odd number}\} \\ k6 \in \{1,2,...,14 \,\&\, k2 \text{ is an even number}\} \end{cases} \tag{9}$$

(e) Non-re-cropping constraints:

The same plot cannot be planted with the same crop in two consecutive years without yield reduction:

$$X_{i,j,\mathrm{k}} \neq X_{i,j,k-1} \quad i,j,k \in \forall \tag{10}$$

where $X_{i,j,k}$, denotes that plot i is planted with crop j in the k-th year.

(f) Constraints on planting legume crops once in three years:

Each plot shall be planted with legume crops at least once in every three years.

$$\sum_{k=k_0}^{k_0+2} X_{i,\mathrm{legume,k}} \geq 1 \quad i \in \forall \tag{11}$$

(g) Constraints on non-dispersed planting areas:

There is a need to avoid the planting area of each crop being too spread out, and constraints can be added to limit the planting area of the same crop to adjacent plots:

$$|A_{i,j} - A_{k,j}| \le \epsilon \quad i,k \in \forall \quad \text{(Among them, i and k are adjacent plots.)} \tag{12}$$

where $A_{i,j}$ denotes the area of crop j planted on plot i and ϵ denotes the acceptable range of area error.

Avoiding the period 2024-2030 if there exists a year in which the total marketing is less than the cost of planting in the following year

Assume that R_k denotes the total marketing (total sales revenue) in the k th season and C_{k+1} denotes the planting cost in the $k+1$ st season. The formula constraints are:

$$\forall t \in \{1,2,...,14\}, R_k \ge C_{k+1} \text{(Among them, k means the season, two seasons a year, a total of seven years.)} \tag{13}$$

3. Solving the model

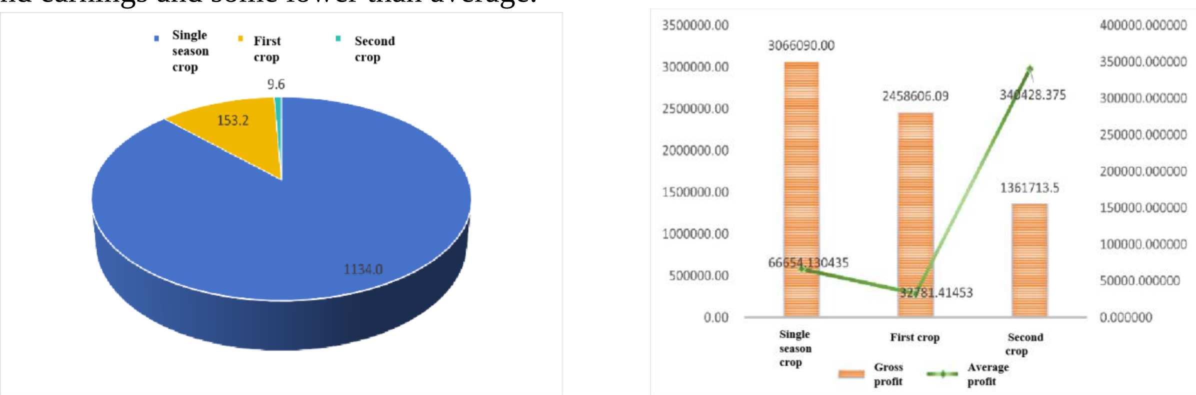
3.1. algorithmic solution model

In the process of solving the model,the linear programming algorithm is the main idea, calling the pulp.LpProblem function to point to the maximization goal, the decision variables are defined using three-dimensional arrays of decision variables, and loops, judgments, etc. are mainly used when adding constraints. By using interior point method, which is applied to the matrix problem with dense constraints between crops and plots in this problem, iterations are performed continuously in the feasible domain, and the decision variables are adjusted to get closer and closer to the optimal solution.

3.2. Qualitative analysis of results

The aim of this part is to solve the model to maximize profit based on the given cropping constraints and market demand. The sales returns of different crops are maximized without violating the constraints such as non-recropping and frequency of legume planting.

First of all, the paper starts with the total sales and total profitability of the crops. From the data, it can be seen that the total sales and total profit of different crops vary significantly (as shown in Figure 2). For example, the total sales of cabbage is as high as 150,000, chili pepper is only 1200, the total profit of elm yellow mushrooms reaches 517,396.5, and chili pepper is 8,695.65. Farmers can choose the crops according to the market demand and profitability, and the analysis of the data shows that the average total sales is about 33,695.48, and the median is 38,400, and the average total profit is about 156,975.88, and the median is 265016.07, with some crops having higher than average sales and earnings and some lower than average.



(a) Area under crops in different seasons

(b) Total and average profits for different crop types

Figure 2. Analysis of crop cultivation indicators

Flat dry land, terraces and slopes are mainly planted with single-season crops such as cereals, buckwheat and so on due to the limitation of climatic conditions, the planting area is large but the yield is low, and its planting area allocation needs to take into account the adaptability of the soil and climatic conditions in order to improve the yield per unit area; water-consumed land and greenhouses are able to plant two seasons of crops, with a wide range of vegetables in the first season, and a wide range of cold-hardy crops or edible fungi in the second season, and the greenhouses planting area is concentrated on high-value vegetables and edible fungi, because they can provide the best growing conditions. The greenhouse planting area is concentrated on high-value vegetables and mushrooms because they provide the best growing conditions; according to the profit distribution map, the planting plan should prioritize high-profit crops and reduce resource inputs for low-profit crops or optimize management to balance market demand and costs.

Crop acreage for the two seasons of 2024 is shown in Table.2 and Table.3.

Table.2. Crop Planting of A,B,C,D,E and F Plots in the First Quarter of 2024

Serial Number	Agrotype	Cultivated Area	Serial Number	Agrotype	Cultivated Area
	Corn	24		Corn	13.2
A1	Pumpkin	24	B8	Pumpkin	13.2
	Sweet Potato	24		Sweet Potato	13.2
	Corn	16.5		Corn	15
A2	Pumpkin	16.5	B9	Pumpkin	15
	Sweet Potato	16.5		Sweet Potato	15
	Corn	10.5		Corn	7.5
A3	Pumpkin	10.5	B10	Pumpkin	7.5
	Sweet Potato	10.5		Sweet Potato	7.5
	Corn	21.6		Corn	18
A4	Pumpkin	21.6	B11	Pumpkin	18
	Sweet Potato	21.6		Sweet Potato	18
	Corn	20.4		Corn	13.5
A5	Pumpkin	20.4	B12	Pumpkin	13.5
	Sweet Potato	20.4		Sweet Potato	13.5
	Corn	16.5		Corn	10.5
A6	Pumpkin	16.5	B13	Pumpkin	10.5
	Sweet Potato	16.5		Sweet Potato	10.5
	Corn	18		Corn	6
B1	Pumpkin	18	B14	Pumpkin	6
	Sweet Potato	18		Sweet Potato	6
	Corn	13.8		Corn	4.5
B2	Pumpkin	13.8	C1	Pumpkin	4.5
	Sweet Potato	13.8		Sweet Potato	4.5
	Corn	12		Corn	3.9
B3	Pumpkin	12	C2	Pumpkin	3.9
	Sweet Potato	12		Sweet Potato	3.9
	Corn	8.4		Corn	4.5
B4	Pumpkin	8.4	C3	Pumpkin	4.5
	Sweet Potato	8.4		Sweet Potato	4.5
	Corn	7.5		Corn	5.4
B5	Pumpkin	7.5	C4	Pumpkin	5.4
	Sweet Potato	7.5		Sweet Potato	5.4
	Corn	25.8		Corn	8.1
B6	Pumpkin	25.8	C5	Pumpkin	8.1
	Sweet Potato	25.8		Sweet Potato	8.1
	Corn	16.5		Corn	6
B7	Pumpkin	16.5	C6	Pumpkin	6
	Sweet Potato	16.5		Sweet Potato	6
D1	Ripe	15	E8	Buckwheat	0.1

D2	Ripe	10		Black Soya Bean	0.1
D3	Ripe	14			
D4	Ripe	10	E9	Common Bean	0.4
D5	Ripe	10	E10	Cowpea	0.4
D6	Ripe	12	E11	Cowpea	0.2
D7	Ripe	22	E12	Common Bean	0.1
D8	Ripe	20		Black Soya Bean	0.1
E2	Cowpea	0.2	E13	Common Bean	0.2
	Pumpkin	0.2		Black Soya Bean	0.2
E3	Cowpea	0.2	E14	Common Bean	0.1
	Pumpkin	0.2		Black Soya Bean	0.1
E4	Common Bean	0.1	E15	Common Bean	0.2
	Mung Bean	0.1		Cowpea	0.2
E5	Common Bean	0.2	E16	Common Bean	0.4
E6	Common Bean	0.1		Cowpea	0.2
	Cowpea	0.1			
E7	Cowpea	0.2			
	Pumpkin	0.2			

Table.3. Crop Planting in D,E and F Plots in the Second Quarter of 2024

Serial Number	Agrotype	Cultivated Area	Serial Number	Agrotype	Cultivated Area
D1	Ripe	15	E12	Soya Bean	0.1
D2	Ripe	10		Mung Bean	0.1
D3	Ripe	14	E13	Red Bean Shrub	0.1
D4	Ripe	6		Naked Oat	0.1
D5	Ripe	10	E14	Mung Bean	0.1
D6	Ripe	12		Crawling Beans	0.1
D7	Ripe	22		Green Pepper	0.1
D8	Ripe	20	E15	Cucumber	0.2
E1	Wheat	0.2	E16	Chinese Cabbage	0.2
	Black Soya Bean	0.1	E9	Red Bean Shrub	0.1
	Crawling Beans	0.1		Naked Oat	0.1
	Chinese Sorghum	0.1	E7	Crawling Beans	0.1
	Buckwheat	0.1		Barley	0.1
	Barley	0.1		Buckwheat	0.1
E6	Soya Bean	0.1			
	Mung Bean	0.1			

4. Conclusion

The results from the model indicate significant differences in the sales and profitability of different crops. For instance, Chinese cabbage has higher sales volume, while elm mushrooms offer greater profitability. This highlights the varying market demands and profit margins of crops, directly influencing planting decisions and revenue allocation. Further analysis reveals that the arrangement of irrigated land and greenhouse planting is more efficient, improving crop yield and economic benefits. This demonstrates that the multi-constraint optimization model, which considers factors like arable land limits, crop planting conditions, and non-repetitive planting, effectively optimizes planting plans and improves overall agricultural production efficiency.

However, real-world agricultural planning is influenced by various factors such as geographical differences, climate change, and market demand. Thus, the planting data from a mountainous region in North China may not be directly applicable to other regions. Future work could improve the model's generalizability by obtaining agricultural data from various regions and conducting comparative analysis, enhancing the operability and effectiveness of the optimization scheme.

Additionally, while the optimization model offers effective support for planting decisions, there are limitations, particularly in local optimal solutions. Although the objective function can reach optimal values for specific plots or periods, it may not represent the global optimum. Future research can explore global optimization algorithms or a combination of multiple iterations and local search strategies to find a better global solution. This would enhance the model's accuracy and reliability, making it suitable for more complex agricultural environments.

With the continuous advancement of agricultural technology, including big data, the Internet of Things, and artificial intelligence, future models could integrate more complex variables such as soil quality and climate change. Real-time data monitoring and dynamic adjustments will enable more accurate and intelligent agricultural planning. Thus, the optimization model in this study has considerable potential for improvement and wide application prospects, contributing to more efficient agricultural resource allocation and sustainable agricultural development.

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