Predicting Markov Chain Behavior Using Markov Chain Models

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Abstract. In the stock market, the stock price is a random variable that changes over time, and its fluctuation exhibits the characteristics of a random walk. This research focuses on the A-share "Shanghai Kweichow Moutai Co., Ltd." listed on the Shanghai Stock Exchange, utilizing data from 100 trading days. The study begins with confirming the presence of Markov properties within the stock's price movements, followed by the construction of a Markov model to analyze and forecast its price trends. The analysis reveals that the Markov model provides reasonably accurate results, offering valuable insights into the cyclical patterns of stock prices. This approach aids in understanding the underlying mechanisms that drive price fluctuations and enhances the ability to predict future price movements, which can be beneficial for investors and financial analysts in decision-making processes. Additionally, the model demonstrates the potential of applying Markov processes to financial markets, particularly in predicting trends and making more informed predictions based on historical data.

Keywords: Markov Chain; Markov Chain, predict.

1. Introduction

In recent years, with the strong support and promotion of the government for the stock market, the stock market management mechanism has gradually improved. At the same time, with the increasing participation of institutional investors, private equity, and retail investors, the analysis and prediction of stock price trends have become very important. The fluctuation of stock prices has Markov properties. The Markov process is an important type of stochastic process, and its most important characteristic is memorylessness. The fluctuation of stock prices can be analyzed and predicted by establishing a mathematical model of the Markov process for stock prices.

2. Theoretical basis

2.1. Definition of a Markov chain

Definition 1

A stochastic process $X_n, n = 0,1,2,\cdots$ is called a Markov chain if it can only take a finite or countable number of values, and for any $n \geq 0$ and any states $i, j, i_0, i_1, \ldots, i_{n-1}$, it has [1, 2]:

$$P\left(X_{n+1} = j \mid X_0 = i_0, X_1 = i_1, \ldots, X_{n-1} = i_{n-1}, X_n = i\right) = P\left(X_{n+1} = j \mid X_n = i\right) \tag{1}$$

This formula characterizes the property of the Markov chain, known as the Markov property, or, which means that the future state of an event is related only to the current state and is independent of any previous states.

2.2. Transition Probability

Definition 2

Arrange $P_{ij} (i, j \in S)$ into a matrix form, denoted as[2, 3]

$$P=\begin{pmatrix} P_{00} & P_{01} & P_{02} & \cdots \\ P_{10} & P_{11} & P_{12} & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ P_{i0} & P_{i1} & P_{i2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{2}$$

This matrix P is called the transition probability matrix, often simply referred to as the transition matrix. Since probabilities are nonnegative and the process must transition to some state, it is clear that $P_{ij}(i,j\in S)$ has the following properties:

1. $P_{ij}\ge 0, \forall i,j\in S$,
2. $\sum_{j\in S}P_{ij}=1, \forall i\in S$.

2.3. n-Step Transition Probability and the Chapman-Kolmogorov (C-K) Equation

Definition 3

The n-step transition probability is the conditional probability [3]

$$P_{ij}^{(n)}=P\left(X_{m+n}=j\middle| X_m=i\right), i,j\in S, m\ge 0, n\ge 1\,, \tag{3}$$

Which is referred to as the n-step transition probability of the Markov chain. The corresponding matrix $P^{(n)}=\left(P_{ij}^{(n)}\right)$ is called the n-step transition matrix.

Chapman-Kolmogorov (C-K) Equation: For all $m,n\ge 0, i,j\in S$, it holds that [4]:

$$P^{(n)}=P\cdot P^{(n-1)}=P\cdot P\cdot P^{(n-2)}=\cdots=P^n \tag{4}$$

2.4. Stationary Distribution and Limiting Distribution

Definition 4

Let $\{X_n, n\ge 0\}$ be a homogeneous Markov chain with state space I and transition probability P_{ij}. If [3, 5]:

$$\begin{aligned} \pi_j &= \sum_{i\in I}\pi_i P_{ij} \\ \sum_{j\in I}\pi_j &= 1, \pi_j\ge 0 \end{aligned} \tag{5}$$

Where $\pi=(\pi_1,\pi_2,\ldots,\pi_n)$ is the probability distribution and πP is the transition probability, then the probability distribution $\{\pi_j, j\in I\}$ is called the stationary distribution of the Markov chain. For the stationary distribution, it holds that [6]:

$$\pi=\pi P=\pi P^2=\pi P^n \tag{6}$$

A Markov chain is said to be ergodic if all states communicate and are positive recurrent with period 1. For an ergodic Markov chain, the $\lim_{n\to\infty}P_{ij}^{(n)}=\pi_j, j\in I$ is called the limiting distribution of the Markov chain.

3. Forecasting Based on the State of Stock

3.1. Model Construction

Stock prices, ranges, and volumes are random variables that follow a Markov process. Their state at any time depends only on the current state, not past states, The transition probabilities are time-invariant, and the process has a countable number of independent states. This makes it suitable for prediction using a Markov model. Consider the closing price of a stock on the n-th day, denoted as

$X_n (n = 0,1,2,3,\cdots)$, where $X_n \in [0,+\infty)$ Insert m-1 division points into such that $0 < X_1 < X_2 < \cdots < X_{m-1}$, and define X_0 and X_m as 0 and $+\infty$, respectively. This divides the interval $[0,+\infty)$ into m, subintervals, with the i-th subinterval being $[X_{i-1}, X_i)$ for $1 \le i \le m$, The value of X_n can only fall within one of these subintervals. It is also defined that $X_n(\omega) = i$ when $\omega \in [X_{i-1}, X_i)$, and it refers to the state of the stock price as state i. Let P_{ij}, represent the probability of the stock transitioning from state i to state j over one time period, and $P_{ij}^{(n)}$ represent the probability of transitioning from state i to state j over n steps. The one-time transition probability matrix for the stock price is:

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1m} \\ P_{21} & P_{22} & \cdots & P_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ P_{m1} & P_{m2} & \cdots & P_{mm} \end{bmatrix} \tag{7}$$

The C-K) equation, $P(n) = P^n$ describes the probability distribution of the stock price transitioning from one state to another. Therefore, by calculating $P(n)$, it can forecast the stock price after n trading days. By comparing the values in the first row of $P(n)$, it can determine the most likely state the stock price will reach after n time intervals.

3.2. Application on the Market of KweiZhou Moutai Market

Taking the closing prices of Guizhou Moutai (600519.sH) from August 27, 2001, to December 19, 2024, as an example, and using the method of state division, the closing prices of each day are categorized into three states: rising, stable, and falling for analysis and forecasting. The statistical results are shown in Table 1 (1denotes Up: price change>0.5; 2 denotes no change :0.5>price change>-0.5;3 denotes down: price change<-0.5).

Table 1. The stock's opening price, closing price, price change percentage, and related identification information

Date	Open Price(Yuan)	Closing Price(Yuan)	Price change percentage(%)	Sign
2001/8/27	34.51	35.55	13.2526	1
2001/8/28	34.99	36.86	3.685	1
2001/8/29	36.98	36.38	-1.3022	3
2001/8/30	36.28	37.1	1.9791	1
…	…	…	…	…
2024/12/11	1540	1535.6	-0.7106	2
2024/12/12	1532.02	1565.8	1.9667	1
2024/12/13	1550.01	1519	-2.9889	3

Through statistical analysis, it has obtained the probabilities for each scenario and constructed the probability matrix:

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} = \begin{bmatrix} 0.2913 & 0.4678 & 0.2409 \\ 0.2438 & 0.5361 & 0.2200 \\ 0.2723 & 0.4494 & 0.2782 \end{bmatrix} \tag{8}$$

In matrix P, each row represents the transition probabilities for a particular state, thus $\sum_{j=1}^{3} P_{ij} = 1$, $i = 1,2,3$.

According to Table 1, since the last day of the statistical data, December 19th, was in a declining state, it can be assumed that the initial state vector $\pi(0) = (0,0,1)$. Using this vector and the state transition probability matrix, it can predict the probabilities of the closing prices being in each state on different days:

The state probability vector for the closing price on the December 20th:

$$\pi(1)=\pi(0)P=(0,0,1)\begin{bmatrix}0.2913 & 0.4678 & 0.2409\\ 0.2438 & 0.5361 & 0.2200\\ 0.2723 & 0.4494 & 0.2782\end{bmatrix}=(0.2723,0.4494,0.2782) \quad (9)$$

The state probability vector for the closing price on the 21th day:

$$\pi(2)=\pi(1)P=(0.2647,0.04947,0.2420) \quad (10)$$

The state probability vector for the closing price on the 23th day:

$$\pi(4)=\pi(0)P^4=(0.2637,0.5081,0.1900) \quad (11)$$

4. Conclusion

From the state probability vector $\pi(1)$ of the 20th day's closing price, it is known that the probability of the closing price being in state i_2 is the highest, at 0.579, and the actual price change of that day is -0.33, which is indeed in the same state. Similarly, in the state probability vector $\pi(4)$ of the 23th day's closing price, the probability of being in state i_2 is the highest, the actual price change of that day was 0.29, which is also the same state, matching the prediction.

The Markov chain model is useful for analyzing and predicting stock price trends by observing closing prices and dividing them into states or intervals. It allows for a clear understanding of a stock's performance, avoiding many losses caused by blind decision-making. This paper uses the Markov chain to establish a Markov model based on continuous observation of stock closing prices, focusing on dividing the price state. After analyzing the Markov property" of the model and the stock price, the paper analyzes and predicts the stock price trend. By comparing some of the predicted results with actual data, it is found that the Markov models established can match the actual results quite well, demonstrating the feasibility of using this model for stock price prediction. However, the stock market's complexity and influence from various factors mean that no method, including Markov prediction, can accurately forecast daily price changes. The Markov chains derived from the data can be easily affected by many other factors such as the period of the data, the investor sentiment, and the global event.

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