Research on the collision mechanism and U-turn curve of bench dragon

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Abstract. The Bench Dragon is one of the traditional folk activities originating in the River Luo region and is an annual event in southern Chinese provinces and cities. Villagers connect the bench drills one after another to form a bench dragon, which is more than 200 metres long and very spectacular. However, the bench dragon will be too close or the routes overlap during the movement leading to accidents. In order to reduce the injuries, this paper constructs a two-dimensional collision model based on the spiral motion mechanism and the curve planning problem to study its motion law, the collision occurrence conditions and route optimisation problem in the two-dimensional plane.

Keywords: Spiral model, 2D collision model, Curve planning issues.

1. Introduction

As an important part of traditional Chinese folk activities, the bench dragon often causes collision accidents due to speed and route problems during the process of coiling in and out, which seriously threatens the safety of villagers. In order to explore this problem in depth and find a solution, a two-dimensional bench dragon spiral and collision model is constructed using physical modelling and dynamic simulation techniques. Through this model, we systematically analysed the optimal routes of the bench dragon's spiral in and out, the collision danger points, and the limiting speed of the villagers' movement.

At present, scholars at home and abroad have carried out research on spiral and collision models. Bonetti et al[1]. established a mathematical modeling and simulation method for traditional cultural activities, which has an enlightening effect on the study of traditional folk activities such as bench dragons. Zhao et al[2]. studied the trajectory optimization problem of robots and introduced the concept of trajectory optimization. Holmes et al[3]. conducted a literature review of the helix, including information on the mechanism of helix motion and mathematical models. Doshi et al[4]. reviewed the spiral model and its applications, and introduced a method of substituting the spiral model into the display problem. Fowler et al[5]. studied the collision-based helix model, which has implications for the collision mechanism in the spiral in this paper. Orefice et al[6]. established a 2D collision avoidance method based on spiral trajectories, and introduced the collision model based on spiral trajectories. Jansson et al[7]. have studied the theoretical basis of vehicle collision avoidance, including collision prediction, collision risk assessment, and collision avoidance strategies and methods. Banerjee et al[8]. elucidated the model establishment, analysis, and application of mathematical models, and provided modeling methods and applications of mathematical modeling in the bench dragon model for mathematical modeling in this paper. Toomre[9] discusses the theory of helical structure, which provides a theoretical basis for the establishment of the spiral model in this paper. Zafar et al[10]. discussed the route planning and route optimization of vehicles, which provided guidance for the planning of the path and the selection of the shortest path in the adjustment curve of the bench dragon in this paper.

The contribution of this thesis is that it is the first time to construct a two-dimensional collision model based on the isometric spiral motion mechanism for the motion mechanism and collision problem of Bench Dragon, a traditional folklore activity. The model not only deeply analyses the speed and route characteristics of the bench dragon in the process of coiling in and out, but also provides a powerful theoretical support and computational tool for accurately analysing the possible dangers in the process of the bench dragon's movement, which enriches the theoretical and methodological system of the research on folklore sports activities.

2. Description of the motion mechanism of the bench dragon and calculation of the velocity at each position

2.1. Background introduction of the bench dragon

"Bench Dragon", also known as "Coiling Dragon", is a traditional local folk cultural activity in Zhejiang and Fujian. People connect dozens of benches to hundreds of benches end to end to form a winding bench dragon. When coiling the dragon, the dragon head is in front of the leader, and the dragon body and dragon tail hover, and the whole is in the shape of a disc. Generally speaking, under the premise that the dragon dance team can freely enter and exit, the smaller the area required and the faster the speed of the dragon, the better the ornamentation. A bench dragon is composed of 223 benches, of which the first section is the dragon's head, the back 221 sections are the dragon's body, and the last one is the dragon's tail. The length of the board of the dragon head is 341 cm, the length of the body and tail of the dragon is 220 cm, and the width of all benches is 30 cm. There are two holes on each bench with a bore diameter (diameter of the hole) of 5.5 cm and the center of the hole 27.5 cm from the nearest head (shown in Figure 1 ， The tail and body of the dragon are similar to those in Figure 1).The two adjacent benches are connected by handles (shown in Figure 2.).

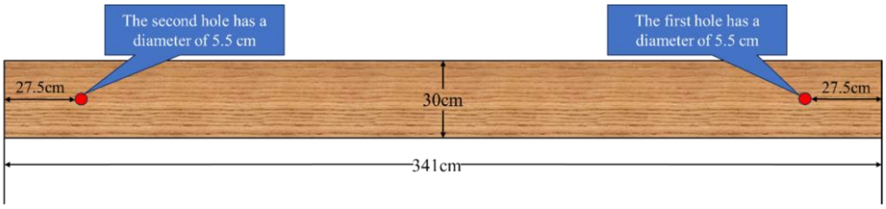

Figure 1. Top view of a faucet

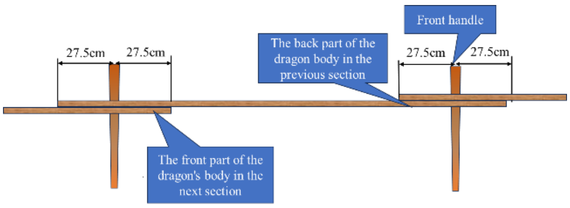

Figure 2. Front view of the bench

The dragon dance team coiled clockwise along an equidistant spiral with a pitch of 55 cm, and the center of each handle was located on the spiral. Dragon The travel speed of the head front handle is always 1 m/s. Initially, the faucet is located at point A on the 16th turn of the spiral.

2.2. The position and velocity of the dragon head part of the bench dragon body are derived from the recursive formula

2.2.1 Establishment of the equidistant spiral model:

Since the dragon dance team is inserted clockwise along the equidistant spiral with a pitch of $d = 0.55$m, the dragon head at the initial moment is located on the right side of the 16th circle of the spiral, we take the center of the circle as the pole and the positive direction of the x-axis as the polar axis to establish a polar coordinate system[3,4,9].

Defined in terms of angular velocity ω:

$$\frac{d\theta}{dt} = \omega \tag{1}$$

According to the relationship between angular velocity and linear velocity, it can be obtained:

$$\omega = \frac{v}{\rho} \tag{2}$$

At time 0, $\rho(0)$ for ρ is 16d, and $\theta(0)$ for θ is 32. Therefore, the polar diameter at time t is:

$$\rho(t) = \rho(0) - b \cdot \theta(t) \tag{3}$$

Among them, $\rho(t)$ is the polar diameter at time t of the spiral, $\theta(t)$ is the polar angle at time t of the spiral, and $b = \dfrac{d}{2\pi}$ is the coefficient of the polar angle.

2.2.2 Solution for the position of the faucet:

To determine the faucet at each moment, by combining equations (1) and (2), we can obtain:

$$\frac{d\theta}{dt} = \frac{v}{\rho} \tag{4}$$

According to the Euler method, equation (4) can be approximated as follows:

$$\theta(t + \Delta t) = \theta(t) + \Delta t \cdot \frac{v_0}{\rho(t)} \tag{5}$$

Among them, Δt is the time step, $v_0 = 1m/s$ is the speed of the dragon head, and $\rho(t)$ is the polar diameter at time t. Starting from $t = 0$, the iteration can solve for the polar angle of the faucet at each moment. In the Cartesian coordinate system, $x(t) = \rho(t)\cos\theta$ and $y(t) = -\rho(t)\sin\theta$ can be used, and then the polar angle can be substituted to determine the position of the faucet at each moment.

2.2.3 Solving the position of the dragon body:

For solving the position of the dragon body, we first draw a top view of the connected bench dragon, as shown in Figure 3[1,8].

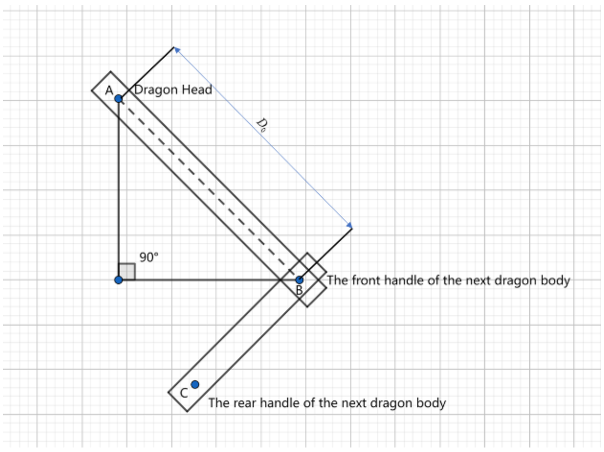

Figure 3. Schematic diagram of bench dragon connection

According to Figure 5, we obtain the relationship diagram between the front handle of the dragon head and the front handle of the first section of the dragon body. Based on the Pythagorean theorem, we can obtain the connection formula between the dragon head and the dragon body:

$$D_0 = \sqrt{(x_0(t) - x_1(t))^2 + (y_0(t) - y_1(t))^2} = 341 - 55cm \tag{6}$$

The formula for connecting dragon bodies is:

$$D_1 = \sqrt{(x_i(t) - x_{i+1}(t))^2 + (y_i(t) - y_{i+1}(t))^2} = 220 - 55cm \tag{7}$$

Among them, D_0 is the distance between the two holes of the dragon head stool, D_1 is the distance between the two holes of the dragon body stool.

2.2.4 Solution for the speed of the dragon head:

According to the definition of limits, the derivative of displacement with respect to time is velocity, and we combine the idea of difference with the derivative of displacement with respect to time to obtain the head velocity as follows:

$$v_0 = \frac{\sqrt{(x_0(t+\Delta t) - x_0(t))^2 + (y_0(t+\Delta t) - y_0(t))^2}}{\Delta t} \tag{8}$$

Similarly, the speed of the dragon's body can be obtained as follows:

$$v_i(t) = \frac{\sqrt{(x_i(t+\Delta t) - x_i(t))^2 + (y_i(t+\Delta t) - y_i(t))^2}}{\Delta t} \tag{9}$$

Among them, v_0 is the speed of the dragon head at time t, and $v_i(t)$ is the speed of the i -th dragon body at time t.

3. Analysis of the Mechanism of Bench Dragon Collision Motion

3.1. Construction of the collision motion equation for the bench dragon

In order to reduce the accidents caused by collisions during the movement of the villagers of the dragon dance team, based on the above background, we analyze the moment when the bench dragon collides when it is coiled in according to the set spiral.

Due to the fact that the length of the dragon head is greater than that of the dragon body, and the bench dragon is wound clockwise along the spiral from the dragon head, the size of the spiral will gradually decrease. Therefore, the collision point is a point between the front end of the dragon head and the previous circle of the dragon body. For easier understanding, we have drawn a local schematic diagram of the collision point as shown in Figure 4[1,5,6,7,8]:

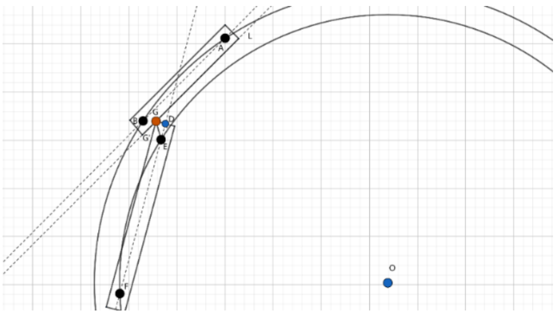

Figure 4 Schematic diagram of collision points

Among them, points A and B are the front and rear handles of the dragon body, points E and F are the front and rear handles of the dragon head, and point G is the point where the dragon head collides with the dragon body. G ' is the vertex of the front end of the bench dragon head.

Translating line AB inward by 0.15m yields line L. If they intersect, line L $\cap$ line GE intersect at point G. GE is the counterclockwise rotation angle α of line EF. Based on geometric knowledge, we can obtain:

$$\alpha = \arctan \frac{GD}{ED} \tag{10}$$

The equation for the straight line EF is:

$$\frac{y - y_E}{x - x_E} = \frac{y - y_F}{x - x_F} \tag{11}$$

+Similarly, the equation for line AB is:

$$\frac{y-y_A}{x-x_A}=\frac{y-y_B}{x-x_B} \tag{12}$$

The general formal equation for the line AB is:

$$Ay+Bx+C=0 \tag{13}$$

The oblique section of the straight line EF is:

$$y=kx+b_0 \tag{14}$$

Since the straight line GE is the counterclockwise rotation angle α of the straight line EF, and then the point E is substituted into the equation GE of the straight line, the expression of the straight line GE is:

$$y=\tan(\arctan k+\alpha)x+b_1 \tag{15}$$

The straight line L is obtained by translating the straight line AB, that is, the expression of the straight line L is:

$$Ay+Bx+C+0.15\sqrt{A^2+B^2}=0 \tag{16}$$

By combining equations (15) and (16), the coordinates of point G can be determined, and based on the Euclidean distance, D_{GE} can be determined as:

$$D_{GE}=\sqrt{(x_G-x_E)^2+(y_G-y_E)^2} \tag{17}$$

When the Euclidean distance D_{GE} is not greater than $D_{G'E}$, it is considered as a collision between the dragon head and the dragon body. The formula is as follows:

$$D_{GE}=\sqrt{(x_G-x_E)^2+(y_G-y_E)^2}\le\sqrt{GD^2+ED^2} \tag{18}$$

3.2. Solution of the collision motion equation of the bench dragon

Figure 6 only shows the case where the collision point is in the second quadrant. For other specific situations, equations (15) and (16) need to be rewritten:

$$y=\tan(\arctan k\pm\alpha)x+b_1 \tag{19}$$

$$Ay+Bx+C\pm0.15\sqrt{A^2+B^2}=0 \tag{20}$$

When the collision point is located in the first or fourth quadrant, equation (19) is $-\alpha$; when the collision point is located in the second or third quadrant, equation (19) is $+\alpha$; when the horizontal axis of the front handle of the dragon body is greater than that of the rear handle of the dragon body, equation (20) is $-0.15\sqrt{A^2+B^2}$; when the horizontal axis of the front handle of the dragon body is smaller than that of the rear handle of the dragon body, equation (20) is $+0.15\sqrt{A^2+B^2}$.

Assuming the extreme angle of the front handle of the faucet at time t is θ_{ft}, search for the $(\theta_{ft}-2\pi,\theta_{ft})$ range and find benches within the range such that D_{GE} equals $D_{G'E}$. The specific algorithm is as follows:

$step1$:: Parameter initialization, GD=0.15, ED=0.275. The initial value of θ_{ft} is 2π, the step size of $\Delta\theta$ is 0.1π, and the termination value is 32π.

$step2$: Substitute the initial parameters into equations (6)and(7) in problem one to solve for the coordinates of the dragon head and each dragon body, and then determine the collision conditions through equation (18).

$step3$: Find the limit moment where D_{GE} equals $D_{G'E}$, which is the termination time of this question.

4. Optimization of bench dragon turning curve

In order to make the action of the bench dragon more beautiful, it is now known that the pitch of the disc in spiral line is 1. 7 m, the disc out spiral line and the disc in spiral line are symmetrical around the centre of the spiral line, and the bench dragon completes the turn in the turning space (9 m in diameter) set up in Fig. 5, and the turning path is an S-curve consisting of two tangent arcs, and the radius of arc of the first arc is twice as much as that of the latter arc, which is tangent to both the disc in spiral line and the disc out spiral line. The latter arc is tangent to both the disc-in and disc-out solenoids. The arcs are adjusted so that the path of the turning curve is the shortest and the movement of the handle in front of the tap is always 1 m/s. In this case, the relevant data are recorded for 100 seconds before and 100 seconds after the turn.

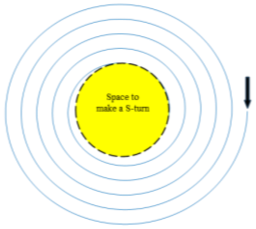

Figure 5 Schematic diagram of turning space

From the background, it can be inferred that the turning path is formed by two sections of arcs that are tangent to each other, and it is also tangent to the spiral that enters and exits the disc. The spiral that exits the disc and the spiral that enters the disc are symmetrical about the center of the spiral. Based on the definition and properties of the spiral, it can be determined that this turning curve is the sum of two semicircles. Based on the background of the problem, the radius of the previous arc is twice that of the subsequent arc, and the sum of the radii of the two circles is positively correlated with the length of the turning curve, while the sum of the radii of the two circles is negatively correlated with the polar angle. Therefore, this problem can be transformed into a linear programming problem by adjusting the angle θ of the arc to make the turning curve shorter.

Due to the fact that the turning curve is the sum of two semicircles, and the radius of the first arc is twice that of the second arc, the equation for the length of the turning curve is:

$$C = \pi r_1 + \pi r_2 \tag{21}$$

$$r_1 = 2r_2 \tag{22}$$

Where C is the length of the curve, r_1 is the radius of the previous arc, and r_2 is the radius of the next arc.

Due to the starting point of the spiral in this question, we take $\rho_0 = -4.5\text{m}$ as the starting point for coiling in the spiral, and adjust the end of the curve as the starting point for coiling out the spiral. The spiral equations for the insertion and extraction of the disk are:

$$\rho_{in} = 4.5 - \frac{1.7}{2\pi}\theta \tag{23}$$

$$\rho_{out} = \rho_j + \frac{1.7}{2\pi}\theta \tag{24}$$

Among them, ρ_{in} is the spiral equation of the disk in, ρ_{out} is the spiral equation of the disk out, and ρ_j is the polar diameter of the end point of the adjustment curve.

For the critical point of a circular arc, we can use the spiral equation. The spiral in and spiral out are symmetrical about the center of the spiral, and the sum of the radii of the two arcs is recorded as:

$$r_1 + r_2 = 4.5 - \frac{1.7}{2\pi}\theta \tag{25}$$

According to the collision conditions, we combine (23) and (25). Since the length of the faucet is the longest, to ensure the minimum length of the arc, the diameter of the smallest semicircle must be equal to the distance between the front and rear handles of the faucet. Finally[2],[10], we calculate $r_1 = 1.43m, r_2 = 2.86m, \rho_j = 4.29m, C = 13.48m$.

For $r_1 = 1.43m, r_2 = 2.86m$, , $C = 13.48m$, in order to facilitate the analysis of the parameters of the turning curve, this question takes the center of the spiral as the coordinate origin, the direction from the center of the large circle to the center of the small circle as the positive x-axis direction, and the vertical upward direction perpendicular to the x-axis as the positive y-axis direction. This article draws an internal schematic diagram of the turning space, as shown in Figure 6[1],[3],[4],[8]:

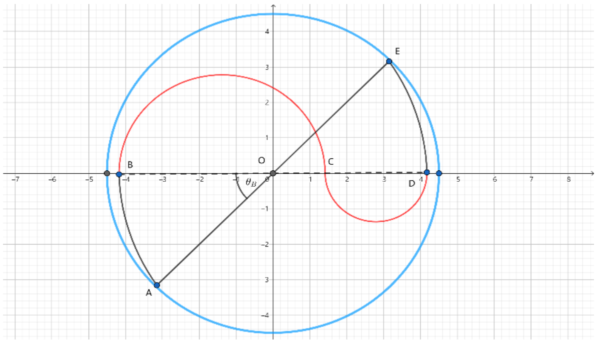

Figure 6 Schematic diagram of the interior of the turning space

As shown in the figure, we can see that $3\rho_C = \rho_B$, $\rho_B = (4.29, \theta_B)$, and C is collinear with B, so $\rho_c = (\frac{4.29}{3}, \theta_B + \pi)$. Given the polar coordinate formula of BC and $d_{bc} = 2r_2 = 5.72m$, we can substitute $\rho_B = (4.29, \theta_B)$ into equation (23) to obtain the polar coordinate equation of circle BC. Similarly, we can substitute $\rho_D = (4.29, \theta_D)$ into equation (24) to obtain the polar coordinate equation of circle CD as follows:

$$\begin{cases} x_{BC}(\varphi) = -1.43\cos\theta_B + 2.86\cos\varphi \\ y_{BC}(\varphi) = 1.43\sin\theta_B + 2.86\sin\varphi \end{cases} \quad (\varphi \in (-\theta_B, \pi - \theta_B)) \tag{26}$$

$$\begin{cases} x_{CD}(\varphi) = -2.86\cos\theta_B + 1.43\cos\varphi \\ y_{CD}(\varphi) = -2.86\sin\theta_B + 1.43\sin\varphi \end{cases} \quad (\varphi \in (\theta_B - \frac{\pi}{2} - \arctan\frac{1}{2}, \theta_B - \pi)) \tag{27}$$

where $\theta_B = 44.47°$.

For the time period of -100s~0s, we adopt the model of the first question and set the starting point as point B. This question is mainly complex in 100s~0s. For this time period, three types of curves need to be considered, and the dragon body of each curve needs to be classified. The specific algorithm is as follows:

$step1$: Divide $t > 0s$ into a large semicircle BC, a small semicircle CD, and a coil-out spiral line DE.

$step2$: When the faucet is located in the large semicircle BC, its position is updated using the Euler method of radians as follows:

$$L(\theta + \Delta\theta) = L(\theta) + \Delta\theta \cdot \rho(t) \tag{28}$$

According to the fixed linear speed of the faucet, the relationship between θ and t can be established, and its formula is as follows:

$$\frac{d\theta}{dt} = \omega = \frac{v}{\rho(t)} \tag{29}$$

The same applies to small semicircles and large semicircles.

$step3$: Based on the known distance between the front handle of the dragon head and the first section of the dragon body, and the known distance between each section of the dragon body, which is consistent with the iterative formula for the dragon head body in Problem 1:

$$D_0 = \sqrt{(x_0(t) - x_1(t))^2 + (y_0(t) - y_1(t))^2} = 341 - 55cm \tag{30}$$

$$D_1 = \sqrt{(x_i(t) - x_{i+1}(t))^2 + (y_i(t) - y_{i+1}(t))^2} = 220 - 55cm \tag{31}$$

$step4$: Set the step size Δt to 0.00001s and solve equations(23)(24)(28)(29)(30)(31) simultaneously. The final solution is to determine the position of each dragon dance team between 0s and 100s.

Based on the model in the second part. According to the definition of limit, the derivative of displacement with respect to time is velocity, and we combine the idea of difference with the derivative of displacement with respect to time to obtain the head velocity as:

$$v_0 = \frac{\sqrt{(x_0(t+\Delta t) - x_0(t))^2 + (y_0(t+\Delta t) - y_0(t))^2}}{\Delta t} \tag{32}$$

Similarly, the speed of the dragon's body can be obtained as follows:

$$v_i(t) = \frac{\sqrt{(x_i(t+\Delta t) - x_i(t))^2 + (y_i(t+\Delta t) - y_i(t))^2}}{\Delta t} \tag{33}$$

Where v_0 is the speed of the dragon head at time t, and $v_i(t)$ is the speed of the i dragon body at time t.

5. Conclusions

In this paper, the motion mechanism and collision test of the bench dragon are abstracted by mathematical methods and computer simulation technology, and the velocity formula of the bench dragon head and each handle of the dragon body is deduced in the second part. For the third part, this paper abstracts the collision conditions of the bench dragon in reality by using geometric knowledge and linear transformation, and obtains that when $D_{GE} = \sqrt{(x_G - x_E)^2 + (y_G - y_E)^2} \le \sqrt{GD^2 + ED^2}$ is true, the collision is assumed. The fourth part of the paper mainly studies the optimization of the U-turn curve of the bench dragon, discusses how to optimize the path of the U-turn curve, and involves the process of modeling and simulating the U-turn curve by using mathematical methods and computer simulation technology.

The tool or method is simple to operate, fully functional, easy to adjust, with high accuracy of output results, and can establish corresponding modelling frameworks for different problems with good reusability. However, it should also be seen that the tool and method also have limitations, such as ignoring the complexity of real-world scenarios, relying on human intervention to ensure smooth operation, relatively narrow application scenarios, being based on a certain degree of simplified assumptions, and lack of flexibility in specific use cases. In order to overcome these limitations, this paper intends to start from multiple dimensions. On the other hand, we will enhance public education on science and technology to increase social awareness and acceptance of the tool or method. In addition, in this paper, we will introduce the collision warning function to identify and warn the problems before they occur in order to improve the processing efficiency and accuracy.

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