Research on the Layout Strategy of Bench Dragon Based on the Archimedean Spiral Model

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Abstract. This article focuses on the innovative transmission of the intangible cultural heritage folk activity known as "Bench Dragon." Considering its diverse performances, a qualitative study is conducted to investigate layout strategies through the establishment of mathematical models. This approach aims to enhance bench formations, increase aesthetic appeal, and integrate people's movement patterns, thereby adding athleticism and diversity to the performance. Various methods, including the bisection method, backward recursion, and collision threshold verification, are proposed to develop the Archimedean Bench Dragon model and models for collision and turning problems using both Cartesian and polar coordinate systems. MATLAB simulations are employed to track trajectories, determining the velocities of each segment and optimal arrangements of the model under specific parameter settings. This work aims to achieve the best layout and aesthetic quality for the "Bench Dragon," providing insights for the protection and transmission of cultural heritage.

Keywords: The Bisection Method, Recursive Method, Archimedes Spiral, Optimization Model.

1. Introduction

"Bench Dragon," is a traditional folk cultural activity prevalent in the Zhejiang and Fujian regions of China. This activity, characterized by its large scale, fosters ethnic sentiment and serves as an essential means for people to transmit emotional memories. It has been included multiple times in China's national intangible cultural heritage list. A distinctive form of "Bench Dragon" performance involves connecting dozens to over a hundred benches head-to-tail to create a spiraling dragon formation, whose trajectory resembles a special form of the Archimedean spiral. During the performance, the dragon's head leads, with the body and tail following its trajectory. Generally, participants hope that the dragon dance team achieves better visual effects; this entails performing the spiral with a smaller area and a faster speed while maintaining smooth entrances and exits. To enhance the visibility of the Bench Dragon's movement, considerations must include the specific position of each bench, interactions due to the pitch of the spiral that might lead to collisions, the required turning radius during entry and exit, and the velocity differences between each bench caused by the speed of the dragon's head, which should not exceed a certain limit.

Literature on the study of the Bench Dragon cultural activity shows that in various regions and cultural contexts, it comprises not only an essential part of traditional folk rituals but also plays a role in shaping cultural memory and stimulating emotional energy. Taking the Huizhou Right Dragon Bench Dragon as an example, Chen Wenyuan[1] points out that cultural performances can adjust social structural boundaries and promote the active utilization of cultural heritage. Additionally, Liu Chuanwen[2] conducts an in-depth exploration of the cultural memory associated with the Pujiang Bench Dragon. Hu Yang[3] analyzes the relationship between the Anren Bench Dragon, local culture, natural environment, and rural society through a three-dimensional perspective and proposes design ideas for cultural tourism development. Furthermore, Xiang Yunhua[4] discusses the intrinsic connection between rural sports-related intangible cultural heritage and rural revitalization, emphasizing the importance of integrating sports-related intangible cultural heritage into rural tourism. However, existing research primarily focuses on exploring cultural values and dissemination strategies, while literature specifically addressing the movement trajectories and regularity of the Bench Dragon remains relatively scarce. The application of the Archimedean spiral has garnered attention from numerous scholars. For instance, Peng Huanhuan et al[5].proposed a ground-penetrating radar antenna based on the Archimedean spiral with wide bandwidth and circular polarization characteristics; Chen Weison et al[6].designed a single-loop spiral differential microphone array based on the Archimedean spiral structure, showing superior performance; and Wan Junyang[7] studied the wear characteristics of polishing wheels using the Archimedean spiral model to control their movement. Nevertheless, the literature on the qualitative study of the Bench Dragon's movement trajectory utilizing the Archimedean spiral is still limited. This article aims to address the challenges faced by the Bench Dragon in its inheritance and innovation by establishing a mathematical model based on the Archimedean spiral and simulating the trajectory morphology of the Bench Dragon through mathematical software, thereby achieving an optimal layout. This research not only provides theoretical support for further improvements in the Bench Dragon but also opens new avenues for exploring optimization schemes.

2. Bench Dragon Model Based on the Archimedean Spiral

2.1. Model Establishment

First, we simplify the model by assuming that a dragon composed of benches consists of 223 segments, with the first segment serving as the dragon's head, the next 221 segments as the dragon's body, and the final segment as the dragon's tail. The length of the head segment is 341 cm, while both the body segments and the tail segment measure 220 cm in length. All bench segments have a width of 30 cm. Each segment features two holes, with a diameter of 5.5 cm, located such that the center of each hole is 27.5 cm from the nearest end of the segment. Adjacent segments of the dragon are connected by handles, and the trajectory of the bench dragon approximates an Archimedean spiral.

The Archimedean spiral, also known as the "equidistant spiral," describes the path traced by a point P that moves at a constant speed along a ray OP while simultaneously rotating around point O at a constant angular speed. The trajectory of point P is referred to as an "Archimedean spiral."

Assuming the double-hole joints of the bench dragon are situated on the spiral, we need to reverse-engineer the positions of all bench segments corresponding to specific time instances from the starting position of the head segment. Furthermore, we need to consider the overall motion trajectory. To begin, we can establish the equation for the motion trajectory of the spiral using polar coordinates[8].

$$\rho = a\theta \tag{1}$$

That is, it is assumed that the trajectory of the model starts at the center of the coordinate system, and because the polar equation between the pitch l and the spiral coefficient a is

$$l = 2\pi a \tag{2}$$

For the convenience of research, it is hypothesized that the faucet is initially located at point A of the 16th circle of the spiral. θ_F corresponding to the angle at the t_F is $\theta_F = 0$, and the initial polar radius of the faucet is $32\pi a$.

Here using the recursive thinking [9], the polar coordinates of the two adjacent holes of a bench at time t are set to $\mathrm{A_n}(\rho_\mathrm{n}, \theta_n), A_{n+1}(\rho_{\mathrm{n}+1}, \theta_{n+1})$ respectively. Then, we can calculate the arc length $s_{(n,n+1)}$ between these two points through these polar coordinates. The specific expression for the arc length can be obtained by referring to [10].

$$s_{(n,n+1)} = \int_{\theta_n}^{\theta_{n+1}} \sqrt{\rho^2(\theta) + \rho'^2(\theta)}\, d\theta \tag{3}$$

Consequently, we get

$$s_{(n,n+1)} = \int_{\theta_n}^{\theta_{n+1}} \sqrt{a^2\theta^2 + a^2}\, d\theta \tag{4}$$

The integral can be used to obtain a recursive relationship between the coordinates of the two adjacent points and the arc length

$$s_{(n,n+1)} = a[\frac{\theta_{n+1}}{2}\sqrt{\theta_{n+1}^2+1}+\frac{1}{2}\ln(\theta_{n+1}+\sqrt{\theta_{n+1}^2+1})]-a[\frac{\theta_n}{2}\sqrt{\theta_n^2+1}+\frac{1}{2}\ln(\theta_n+\sqrt{\theta_n^2+1})] \quad (5)$$

It is also known that the relationship between the polar coordinates and the Cartesian coordinates of each point is $x=\rho\cos\theta, y=\rho\sin\theta$, The relationship between the distance between the two points (string length) A_nA_{n+1} and polar coordinates is as follows

$$(\rho_n\cos\theta_n-\rho_{n+1}\cos\theta_{n+1})^2+(\rho_n\sin\theta_n-\rho_{n+1}\sin\theta_{n+1})^2 = A_nA_{n+1}^2 \quad (6)$$

The recursive relationship between the coordinates of two adjacent points and the length of the cord is obtained by unfolding and organizing.

$$\rho_n^2+\rho_{n+1}^2-2\rho_n\rho_{n+1}\cos(\theta_{n+1}-\theta_n) = A_nA_{n+1}^2 \quad (7)$$

The relational expression (5) can also be employed to determine the polar coordinates of another point given a known point and arc length, and these polar coordinates can further be converted into rectangular coordinates. Using this formula, all the positions of the bench holes can be deduced from the position of the dragon's head (or any reference point), and by inputting specific coordinate points, the coordinates of the four corners of the bench can be obtained.

Regarding the calculation of velocity, the goal is to find the instantaneous velocity of each handle of the bench at certain moments. The concept of limits is adopted, where another node is sought near the point of interest, and this node is gradually made to approach the point of interest to simulate the process of Δt tending to zero. Finally, the expression for velocity is derived by dividing the arc length traversed by the point of interest during a given time interval Δt by the elapsed time Δt.

$$v_s(t) = \lim_{\Delta t\to 0}\frac{s_{t+\Delta t}-s_t}{\Delta t} \quad (8)$$

(In this context, s_t represents the arc length traversed within time t, while $s_{t+\Delta t}$ represents the arc length traversed within time $t+\Delta t$.)

When solving for velocity, the derivative of position with respect to time can be used. However, during the process of solving the differential equation, due to the inability to obtain an analytical solution, it is necessary to seek a numerical solution to the differential equation, which can result in significant errors. Therefore, the concept of limits is adopted instead of solving the differential equation directly.

2.2. Solving the Model and Analyzing the Results

Using MATLAB software, a program is written to first determine the position coordinates of the dragon's head. Based on the recursive relationship, relevant equations are listed. Since the equations are nonlinear, analytical solutions cannot be obtained, and thus the results are represented using numerical solutions.

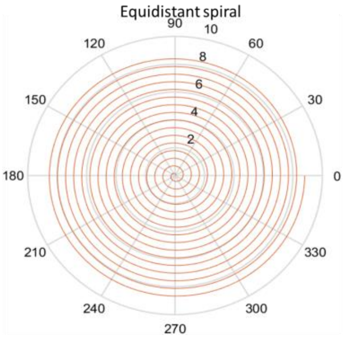

Figure 1. Equidistant Spiral Diagram

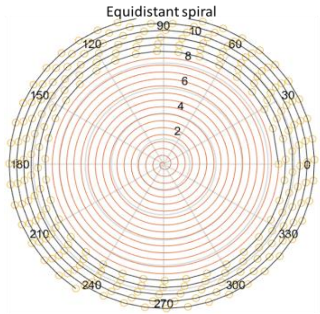

Figure 2. Approximate Distribution Map of Benches

Through the spiral formula, the approximate trajectory of the dragon dance and the starting position of the dragon's head can be determined, as shown in Figure 1. By utilizing the positions of the front and rear connection points of each bench at the 60th second, an approximate illustration of the specific position of the dragon dance at the 60th second can be drawn, as shown in Figure 2. By observing and comparing with the obtained data, it can be roughly determined that the radius of the spiral will not exceed 12 meters. (Except for the tail node, all other nodes point to the preceding node, with identical data in the subsequent table.)

Tables 1 and 2 show the positions and velocities at selected time points and bench segments.

Table 1. Location of some Nodes

	0 s	60 s	120 s	180 s	240 s	300 s
Head x (m)	8.800000	5.799209	-4.084887	-2.963609	2.594494	4.420274
Head y (m)	0.000000	-5.771092	-6.304479	6.094780	-5.356743	2.320429
Body 1 x (m)	8.363824	7.456758	-1.445473	-5.237118	4.821221	2.459489
Body 1 y (m)	2.826544	-3.440399	-7.405883	4.359627	-3.561949	4.402476
Body 51 x (m)	-9.518732	-8.686317	-5.543150	2.890455	5.980011	-6.301346
Body 51 y (m)	1.341137	2.540108	6.377946	7.249289	-3.827758	0.465829
Body 101 x (m)	2.913983	5.687116	5.361939	1.898794	-4.917371	-6.237722
Body 101 y (m)	-9.918311	-8.001384	-7.557638	-8.471614	-6.379874	3.936008
Body 151 x (m)	10.861726	6.682311	2.388757	1.005154	2.965378	7.040740
Body 151 y (m)	1.828753	8.134544	9.727411	9.424751	8.399721	4.393013
Body 201 x (m)	4.555102	-6.619664	-10.627211	-9.287720	-7.457151	-7.458662
Body 201 y (m)	10.725118	9.025570	1.359847	-4.246673	-6.180726	-5.263384
Tail x (m)	-4.194781	8.260258	10.823952	6.476241	2.070483	0.577990
Tail y (m)	-11.169324	-7.976364	2.050077	8.302344	9.803013	9.464529

Table 1. Part of the Node Speed

	0 s	60 s	120 s	180 s	240 s	300 s
Body 1 (m/s)	0.999971	0.999961	0.999945	0.999917	0.999859	0.999709
Body 51 (m/s)	0.999742	0.999662	0.999538	0.999330	0.998941	0.998064
Body 101(m/s)	0.999575	0.999453	0.999269	0.998970	0.998435	0.997300
Body 151 (m/s)	0.999448	0.999299	0.999077	0.998727	0.998114	0.996860
Body 201 (m/s)	0.999348	0.999180	0.998934	0.998551	0.997893	0.996573
Tail (m/s)	0.999304	0.999128	0.998873	0.998477	0.997802	0.996458

During the program implementation process, it is necessary to consider the spiral equation, as well as the updates to velocity, angle, and specific position.

3. Analysis of Collision and Turnaround Space Model Concepts and Model Establishment

3.1. Establishment of the Collision Model

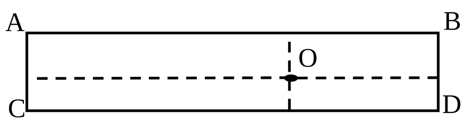

Figure 3. The Point is Inside

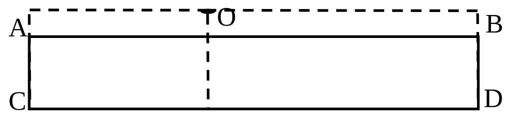

Figure 4. The Point is Outside

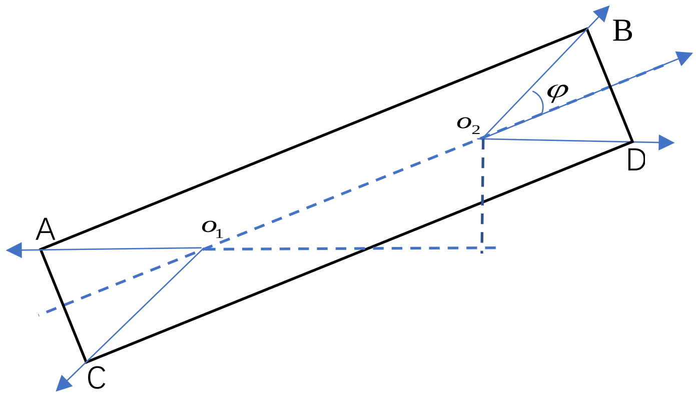

Figure 5. Single Bench Coordinate Analysis

The core issue in studying when collisions occur is to determine the conditions for bench collisions, specifically identifying the time point of the first collision during the movement of the benches during the winding process. Firstly, by observing the positions at the moment of collision, we can deduce that if a vertex O of one rectangle is located inside another rectangle, as illustrated in Figure 3, the sum of the distances from point O to the four sides of the other rectangle is equal to the sum of the length and width of the rectangle. Conversely, if the point is outside the rectangle, as shown in Figure 4, the sum of the distances from point O to the four sides of the rectangle is greater than the sum of its length and width. This criterion can be employed to ascertain whether two rectangles collide.

Once the collision criterion is established, since the handles are consistently positioned on the spiral and their coordinates can be directly calculated, while the four vertices of the rectangle may not lie on the spiral, it is necessary to determine the coordinates of the four vertices of the rectangle to quantitatively analyze whether there is any overlapping area between two rectangles. A separate analysis is conducted for one section of the dragon's body, as depicted in Figure 5, with the same analytical approach applying to the dragon's head.

Assuming the Cartesian coordinates of the front and rear handles are $O_1\left(x\left(i\right), y(i)\right)$ and $O_2\left(x\left(i+1\right), y\left(i+1\right)\right)$, respectively, then the vector between points O_1, O_2 is denoted as $\overrightarrow{O_1O_2} = (x(i+1)-x(i), y(i+1)-y(i))$. By normalizing the vector $\overrightarrow{O_1O_2}$ and multiplying it by the distance from O_2 to side BD, and utilizing the rotation angle $\varphi = arc\tan\left(15/27.5\right)$, the resulting vector can be obtained.

$$\overrightarrow{O_2B} = \frac{27.5\overrightarrow{O_1O_2}}{|\overrightarrow{O_1O_2}|\cos\varphi} \tag{9}$$

Finally, the coordinates of point B can be easily obtained as

$$(x(i)+\frac{27.5(x(i+1)-x(i))}{\sqrt{(x(i+1)-x(i))^2+(y(i+1)-y(i))^2}\cos\varphi}, y(i)+\frac{27.5(y(i+1)-y(i))}{\sqrt{(x(i+1)-x(i))^2+(y(i+1)-y(i))^2}\cos\varphi}) \tag{10}$$

Due to the symmetry within the rectangle, the coordinates of the remaining points can also be determined using the aforementioned method.

After obtaining the positions of the four corners of all benches at each time point, we can now utilize the established model for processing and iteratively calculate the coordinates of the rectangular handles of all benches at each moment. Subsequently, the coordinates of the vertices of the bench rectangles at each moment can be determined. Collision detection is performed at each moment to verify whether any non-adjacent rectangles overlap, i.e., whether any vertex of one rectangle appears on the boundary or within another rectangle (excluding adjacent rectangles). If no collision occurs, the iteration continues until the first collision is detected. At this point, the time t of the first collision is recorded, and the specific location of the collision is output.

3.2. Presentation and Analysis of Collision Model Solution Results

The collision threshold process is shown in Figure 6, Figure 7 ,Figure 8：

Figure 6.60s Bench Distribution Map

By utilizing the front and rear connection points of each bench, a schematic diagram of the position of each bench at the 60th second can be accurately drawn. Figure 6 allows for a relatively precise determination of whether any collisions have occurred among the benches. It can be observed that no collisions have taken place at the 60th second. By changing the time in seconds and employing the bisection method, continuously observing and precisely pinpointing the location of collisions, it can be deduced that the time of collision occurs approximately between 412s and 413s.

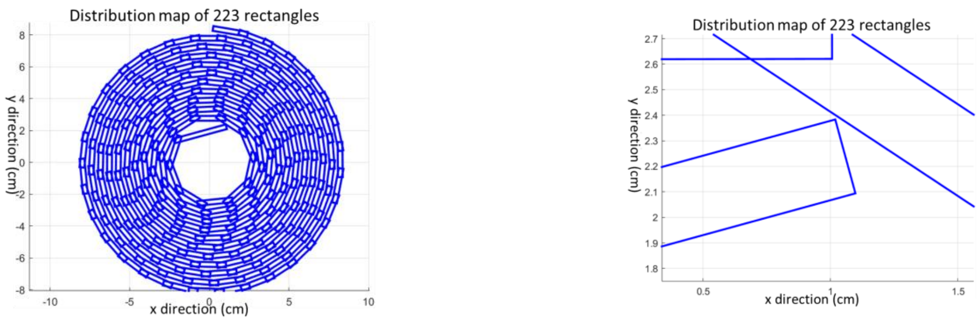

Figure 7. 412s Bench Distribution Map

Observation of the image in Figure 7 reveals that at this point, the benches have not yet collided.

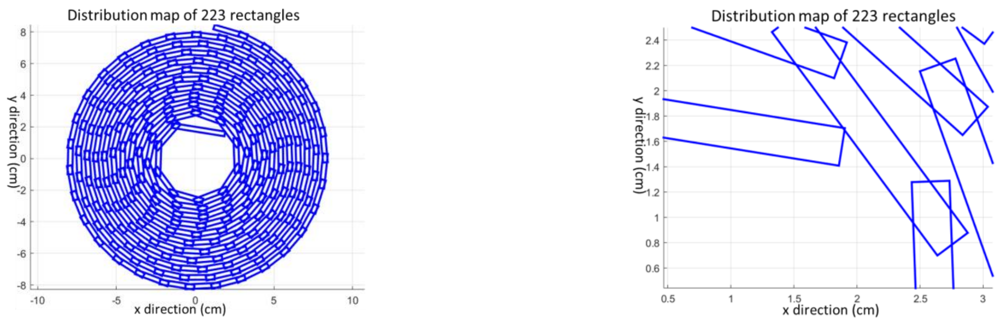

Figure 8.413s Bench Distribution Map

Upon observing the image in Figure 8, it can be discerned that the benches have already collided at this point.

By assessing the collision conditions and utilizing the bisection method, the time of collision can be determined. Based on the collision detection program, the coordinates of the two points on each bench involved in the collision are output, as shown in Table 3.

Table 3. Location of some Nodes

	x Coordinate	y Coordinate
Body 1 (m/s)	-1.216455562	2.050716038
Body 51(m/s)	1.761035875	4.143628668
Body 101 (m/s)	-1.042984858	-5.803967187
Body 151 (m/s)	-1.187613058	-6.896055494
Body 201(m/s)	-7.950193815	-0.722213049
Tail (m/s)	0.249171195	8.381195277

The results are consistent with those displayed in the plots, leading to the conclusion that the collision occurs during the 413-second mark.

Through the aforementioned modeling and calculations, the process of the bench dragon spiraling along an equidistant spiral can be comprehensively described, the motion states of various parts can be analyzed, and the accuracy and reliability of the model calculations can be ensured.

3.3. Establishment and Solution of a Turnaround Space Problem Model

The dragon head requires space to complete its turning maneuver, necessitating an adjustment in the spiral width to enable it to successfully coil in and out within the turning space. This issue presents an optimization challenge. A qualitative analysis reveals that a larger pitch reduces the likelihood of collisions, thereby increasing the possibility of successfully completing the turn. Here, we assume that the turning area is a circular region centered at the origin in polar coordinates. The objective is to determine whether the bench dragon can enter the turning area without any collisions.

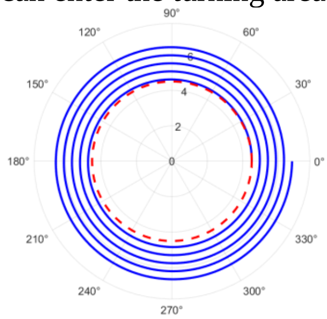

Figure 9. Illustration of The Turning Area

The bisection search method is employed to find the minimum helical pitch. Initially, the turning range is determined, which in this case is assumed to be a circular area centered at the origin of the polar coordinates with a radius of $r = 4.5$, as illustrated in Figure 9. A large range (a,b) for the helical pitch is given, and the helical pitch value in the program is adjusted to fall within this specified range. From Equation (1), the helical coefficient $a = \frac{1}{2\pi}$ can be obtained; and from Equation (2), $\theta_1 = \frac{r}{a}$ can be determined. With these values, the angle of the first point can be calculated, and subsequently, the x_1 and y_1 coordinates of the first point can be derived.

$$x_1 = r * \cos(\theta_1) \tag{11}$$

$$y_1 = r * \sin(\theta_1) \tag{12}$$

Based on the bench dragon model utilizing the Archimedean spiral, we can determine the specific time and position when the bench dragon enters the designated area. Simultaneously, using the collision model and iterating through the bench dragon model based on the Archimedean spiral, we can ascertain the precise time and location of the first collision. At this point, it is necessary to assess whether the dragon head has entered the assumed turning range with a specified diameter at the moment of collision. If the location of the first collision is found to be within the turning range, the pitch should be reduced; otherwise, it should be increased, thereby updating the boundary values and narrowing the range. After repeated iterations, if $|a-b|< 0.00001$, and the collision occurring before entering the turning space with a pitch of a, but not with a pitch of b, it can be concluded that the pitch of b is the minimum pitch. The optimal value for the minimum pitch is determined to be approximately 0.454426 meters.

4. Conclusions

This paper presents a qualitative analysis of bench dragon dance trajectories through mathematical modeling, addressing trajectory depiction, position calculation, collision detection, and velocity differences. The methodology employs vector methods and parametric equations, ensuring clarity in mathematical expressions and facilitating the solving process. By defining model parameters and processing data meticulously, the model's accuracy and reliability are enhanced, effectively tackling complex issues in the bench dragon's spiraling motion.

The current model simplifies calculations of physical quantities like connecting forces and frictions between benches, introducing errors. It assumes constant dragon head speed and spiral pitch, ignoring external factors' influence. Future research could use experimental validation or sophisticated mechanical analysis to refine coefficients. Incorporating advanced dynamical models and practical factors will enhance prediction accuracy. The model's scalability allows application to various dragon dance props and other spiraling objects, showing broad potential.

Culture is deeply rooted in the soul of a nation, constituting its inner core values and serving as the spiritual pillar driving national progress and social development. This cultural activity bears witness to the continuous transmission of Chinese civilization, connects profound national sentiments, and is an important part of China's excellent traditional culture. This paper provides new ideas for the inheritance and innovative performance forms of bench dragon culture, contributing to enhancing people's sense of identity and pride in local culture and promoting the revitalization of rural industries and culture.

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