Research on bench dragon motion based on Runge-Kutta algorithm

Xiaoou Bai[1, #], Kangrong Shi[2, *, #], Zihao Bai[1], Haohang Wen[3]

[1]School of Mathematics, Statistics and Mechanics, Beijing University of Technology, Beijing, China, 100124

[2]School of Information Science and Technology, Beijing University of Technology, Beijing, China, 100124

[3]School of Electronics and Information Engineering, Harbin Institude of Technology, Harbin, China, 150001

*Corresponding author: 22026105@emails.bjut.edu.cn

#These authors contributed equally.

Abstract. "The Bench Dragon" is a distinctive folk art form originating from the Fujian and Zhejiang regions of China. It plays a pivotal role in cultural preservation and serves as a vital conduit for cultural exchange, both within China and globally. A well-executed "Bench Dragon" performance significantly bolsters national cultural identity and is a prominent display of China's cultural allure. The study focuses on enhancing the performance's fluidity and safety by precisely determining the real-time location of the dragon dance team and anticipating potential collisions. The research employs a mathematical approach to model the dragon's movement as a spiral curve, utilizing differential equations and the Runge-Kutta algorithm to describe the position of the dragon head's front handle at any moment. This methodological innovation offers a scientific basis for the performance, optimizing it through mathematical modeling to improve both its visual appeal and safety. Additionally, the cosine iteration method is applied to calculate the positions of the dragon body and tail handles, based on the known position of the head's front handle, ensuring the performance's synchronicity. The study also delves into the analysis of potential collisions using the SAT algorithm, which, after simplifying the judgment criteria, accurately forecasts the timing of collisions. This analysis is crucial for ensuring the performance's safety and smooth progression. The integration of mathematical and physical sciences with the "Bench Dragon" tradition not only enriches its research domain but also provides novel scientific methodologies and technical support for the continuation and evolution of this traditional folk art. This interdisciplinary approach underscores the potential for blending traditional practices with modern scientific techniques to foster cultural heritage while ensuring performances are conducted with enhanced safety and aesthetic quality.

Keywords: Differential Equation, Runge-Kutta Algorithm, SAT Crash Test, Cosine iterative method.

1. Introduction

Since the beginning of the 21st century, scholars have shown strong interest in the traditional Chinese folk activity of bench dragon, especially in the discussion of educational significance[1,2]. However, although the research of bench dragons has achieved certain results, most of the research is still concentrated in the field of humanities, and there are relatively few interdisciplinary studies on bench dragons and natural sciences such as mathematics and physics. This kind of interdisciplinary research can not only provide a new perspective for the performance of the bench dragon, but also optimize the performance effect through scientific methods, and improve its ornamentation and safety.

For example, based on the Archimedes spiral theory and mathematical modeling methods, some studies have studied the movement process of the bench dragon in depth, aiming to improve its ornamentation and safety. Through the accurate calculation and optimization of the movement of the bench dragon dance team along the Archimedes spiral, a number of key problems were solved, and the application of mathematical principles in folk activities was discussed. In addition, there are also studies to analyze and solve the movement trajectory and speed optimization problem of the bench dragon dance team through mathematical modeling, which provides theoretical support for optimizing the movement path of the "bench dragon" dragon dance team and improving the performance effect. These studies show that the combination of bench dragon with mathematics and physics can not only enrich the research content of bench dragon, but also provide new scientific methods and technical support for the inheritance and development of traditional folk activities.

2. Method

2.1. Differential equation model

A differential equation is the relationship between an unknown variable and its derivative, and its core is to describe the rate of change of a quantity in a system [3]. As shown in Equation (1):

$$\frac{dy}{dt} = f(y, t) \tag{1}$$

Equation (1) is a differential equation that describes the rate of change of y versus t in relation to y, t. The advantage of using differential equations is that they can accurately describe and predict changes in the state of a system over time, which is essential for understanding the behavior of complex systems. For the study of a problem such as a "bench dragon", establishing a differential equation can describe its exact position at any moment [4].

2.2. Runge-Kutta

After establishing the differential equation, it is necessary to find a suitable method to solve it. There are many ways to solve differential equations, and for very few differential equations, accurate analytical solutions can be found; However, for most equations, when analytical solutions are difficult to obtain or do not exist [5], the differential equations can only be approximated by numerical methods such as the Runge-Kutta method [6].

Suppose there is a first-order differential equation as shown in equation (2):

$$\frac{dy}{dt} = f(y, t), y(t_0) = y_0 \tag{2}$$

where $f(y, t)$ is a known function about y, t, and $y(t_0) = y_0$ is the initial value condition of the equation. Determine a step h to advance on the timeline, and for each step i (i = 1, 2, · · · · , n), iterate from t_0 to tn as follows, as shown in equation (3):

$$\begin{cases} k_1 = h * f(t_i, y_i) \\ k_2 = h * f(t_i + \frac{h}{2}, y_i + \frac{k_1}{2}) \\ k_3 = h * f(t_i + \frac{h}{2}, y_i + \frac{k_2}{2}) \\ k_4 = h * f(t_i + \frac{h}{2}, y_i + \frac{k_3}{2}) \\ y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\ t_{i+1} = t_i + h \end{cases} \tag{3}$$

Update the values of y_i, t_i in the order of the above formula until all iterations are completed and the numerical solution of the differential equation is found [7].

2.3. Cosine iterative method

For a point on the helix where the coordinates are determined $(r_{n,t}, \theta_{n,t})$ and a point $(r_{n+1,t}, \theta_{n+1,t})$ on the helix that is far from that point $L_{n,t}$ (suppose that the polar diameter is greater), the angle between the two points needs to be solved by the cosine theorem. As shown in equation (4):

$$\begin{cases} r_{n,t}^2 + r_{n+1,t}^2 - 2 * r_{n,t} * r_{n+1,t} * cos(\theta_{n+1,t} - \theta_{n,t}) = L_{n,t}^2 \\ r_{n+1,t} = a + b * \theta_{n+1,t} \end{cases} \tag{4}$$

Theoretically, $r_{n+1,t}$, $\theta_{n+1,t}$ can be found, but the solution of this transcendental equation cannot be easily obtained. Therefore, this paper considers using the iterative method to solve $r_{n+1,t}$, $\theta_{n+1,t}$. As shown in Figure 1, the initial iterative radius is $r_{n,t}{}^{(1)}$, draw a circle with $r_{n,t}{}^{(1)}$ as the radius, find the point with $L_{n,t}$ at a distance from the point $(r_{n,t}, \theta_{n,t})$, connect the point to the center of the helix, and extend the intersection of the helix to the purple point on the way, so that there is a new iteration point, the polar diameter and polar angle are $r_{n,t}{}^{(2)}$, $\theta_{n,t}{}^{(2)}$.

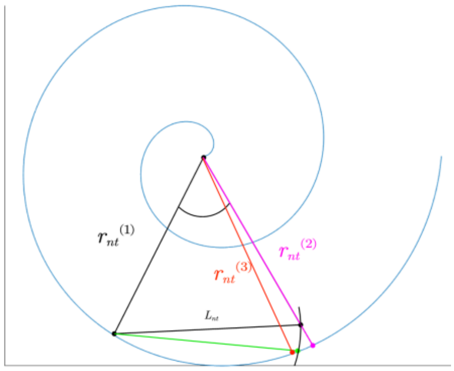

Figure 1: Schematic diagram of the cosine iterative method

Find the triangle with $r_{n,t}{}^{(2)}$,$L_{n,t}$, $r_{n+1,t}$ as the side lengths again, so that the red dot with the polar diameter and angle of $r_{n,t}{}^{(3)}$, $\theta_{n,t}{}^{(3)}$ is found, and so on, until the distance from the point $(r_{n,t}, \theta_{n,t})$ is less than $L_{n,t} \pm 10^{-5}$, so that $(r_{n+1,t}, \theta_{n+1,t})$ is obtained.

2.4. SAT Crash Test

The SAT Crash Test, also known as the separation axis theorem, is an algorithm used to determine whether two convex shapes intersect. If two convex shapes intersect, then they are considered to have collided. Let's take a rectangle as an example to introduce the principle of this algorithm [8].

As shown in Figures 2 and 3, the difference between the two rectangular collisions and no collisions under the SAT algorithm is shown. The normal vector **n** of any edge of the target rectangle and the projection notation lengths of the target rectangle and the detection rectangle on **n** are l_1 and l_2, respectively. Then, find the projection points of the eight vertices to the normal, and note that the maximum distance between the projection points is l_{max}. If l_{max} satisfies:

$$l_{max} > l_1 + l_2 \tag{5}$$

Then it means that there is no overlapping part of the two projections. If no such relationship is found, there is no gap and a collision is assumed. Due to the symmetry of the rectangle, two opposite edges can share a normal vector, so this paper only needs to be tested twice at most [9].

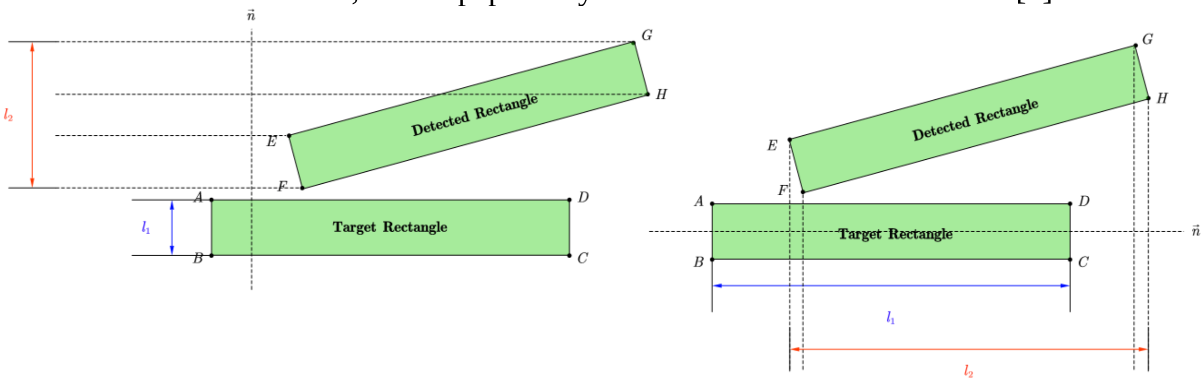

Figure 2: SAT Crash Test (No Collision)

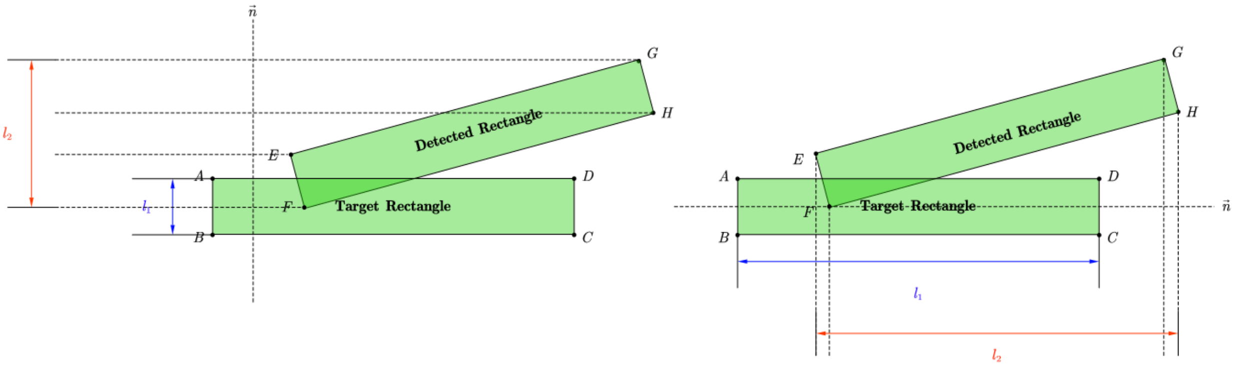

Figure 3: SAT Crash Test (Already collided)

3. Model building and solving

3.1. Basic assumptions

A "bench dragon" consists of 1 dragon head, 221 dragon bodies, 1 dragon tail, these 223 benches and a number of handles. Among them, the board length of the dragon head is 341 cm, the plate length of the body and tail of the dragon is 220 cm, and the width of all benches is 30 cm. Each section of the bench has two holes with a diameter of 5.5 cm and the center of the hole 27.5 cm from the nearest head. A schematic diagram of the dragon body and dragon head is shown in Figure 4.

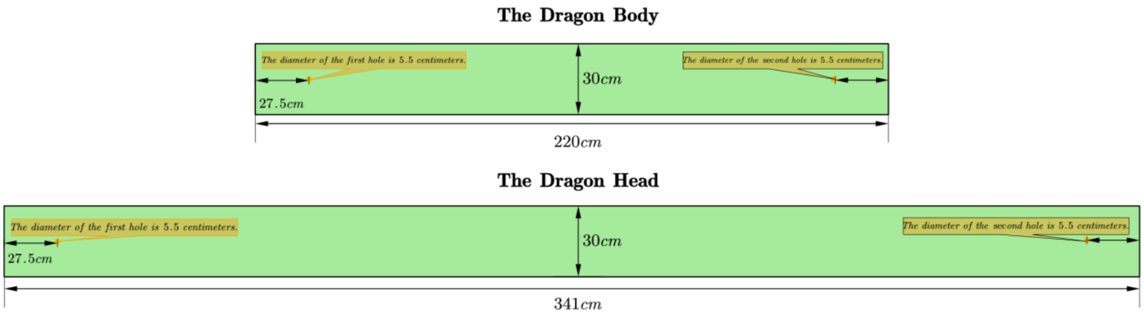

Figure 4: Schematic diagram of the dragon's body and dragon's head

After specifying the parameters of the dragon head and the dragon body, it is also necessary to set a route for the dragon dance team. In order to conform to the actual situation of the "bench dragon" coiled from the outside to the inside, this paper assumes that the dragon dance team coils clockwise along an equidistant spiral with a pitch of 55 cm, the center of each handle is located on the spiral at all times, and the travel speed (linear velocity) of the front handle of the dragon head is always maintained at 1 m/s. Initially, the faucet is located at point A on the 16th turn of the spiral. The schematic diagram is shown in Figure 5.

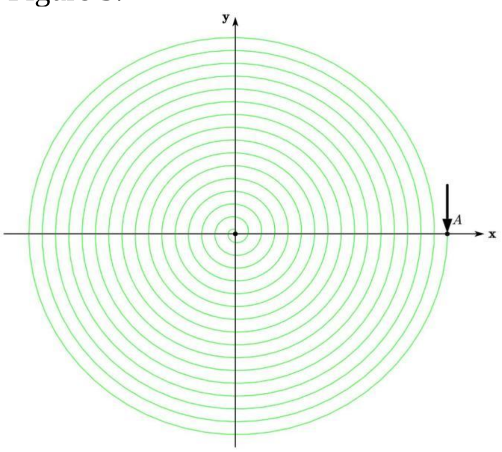

Figure 5: Schematic diagram of the movement trajectory of the bench dragon

3.2. Establishment of spiral polar coordinates

The polar coordinates of the equidistant helix are expressed by equation (6):

$$r = a + b * \theta \tag{6}$$

where r denotes the radius of curvature of the helix in meters, which is related to the angle θ, the parameter b that controls the pitch, and the initial radius a. In the previous hypothetical scenario, the initial radius a = 0 m, and the parameter $b = \frac{0.55}{2*\pi}$ that controls the pitch. Therefore, the helix equation in this paper is equation (7):

$$r = \frac{0.55}{2*\pi} * \theta, \theta \in [\,0, 32\pi] \tag{7}$$

3.3. Determine the position of the front handle

Based on the above description, this paper takes the distance dl traveled in a very short dt time as equation (8):

$$dl = v_h * dt = 1 * dt \tag{8}$$

Where v_h is the line velocity of the front handle, and it is 1 m/s.

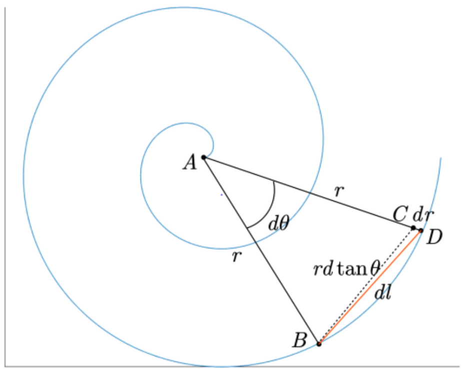

Figure 6: Schematic diagram of the velocity microelement

As shown in Figure 6, the relationship between dl, $d\theta$, and dr is obtained by approximation, where dr is the decrease in the polar diameter of the $d\theta$ time. Because $d\theta$ is small enough, there is shown as equation (9):

$$\theta = tan\theta \tag{9}$$

On the other hand, in the isosceles triangle ABC, it can be seen as a right triangle with AC, CB as the right angled side, and the length of the dashed line is shown as equation (10):

$$r * dtan\theta = r * d\theta \tag{10}$$

According to the right angle relationship. Then, according to the Pythagorean theorem, the equation (11) is obtained in this paper:

$$(dl)^2 = (r * d\theta)^2 + (dr)^2 = [(a + b * \theta) * d\theta]^2 + [d(a + b * \theta)]^2 \tag{11}$$

Deriving the time on both sides of the above equation gives the linear velocity v_h as opposed to the angular velocity w is shown as equation (12):

$$\begin{cases} \frac{d\theta}{dt} = \omega = \frac{v_h}{\sqrt{b^2 + (a + b*\theta)^2}} = \frac{1}{\sqrt{b^2 + (a + b*\theta)^2}} = f(\theta, t) \\ \theta(0) = 32\pi \end{cases} \tag{12}$$

This results in a first-order differential equation that can be solved to know the relationship between θ and t, that is, the position of the faucet at any given time.

3.4. Solve with Runge-Kutta algorithm

For the above differential equations, this paper uses a MATLAB program to solve them using the fourth-order Runge-Kuta algorithm, and converts the polar coordinates of the dragon head (r_t,θ_t) at time t into a Cartesian coordinate system (x_t, y_t), as shown in equation (13):

$$\begin{cases} x_t = (a + b * \theta_t) * cos(\theta_t) \\ y_t = (a + b * \theta_t) * sin(\theta_t) \end{cases} \tag{13}$$

3.5. Determine the position of the dragon's body

The position of the front handle and the rear handle of each dragon body (including the tail) is calculated below. At time t, the pole diameter of the front handle of the nth dragon body is $r_{n,t}$, and the corresponding pole angle is $\theta_{n,t}$. Since all the handles of the bench must be on the spiral, according to the position relationship in Figure 7, this paper can use the cosine theorem to determine the pole diameter and pole of the front handle of the nth dragon body (including the tail of the dragon). The horns are known to introduce the n+1 front handle pole diameter and polar angle of the dragon body, n∈[1, 223].

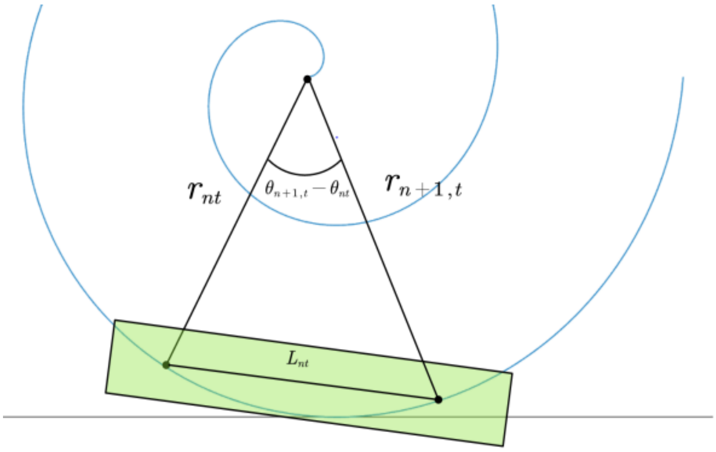

Figure 7: Schematic diagram of the dragon body’s solution

The position of the front handle of the dragon head at time t has been calculated earlier, then the position of the first front handle of the dragon body (r_t,θ_t) at time t is shown as equation (14) :

$$\begin{cases} r_{n,t}^2 + r_{n+1,t}^2 - 2 * r_{n,t}^2 * r_{n+1,t}^2 * cos(\theta_{n,t} - \theta_{n+1,t}) = L_{n,t}^2 \\ r_{n,t} = a + b * \theta_{n,t} \\ r_{n+1,t} = a + b * \theta_{n+1,t} \\ L_{n,t} = 2.86m \end{cases} \tag{14}$$

In the same way, in this paper, the front handle of the first dragon body pushes out the front handle of the second dragon body, until the last handle of the dragon tail is pushed, and the position of all dragon body (including dragon tail) handles is found, and only $L_{n,t}$ needs to be changed to the distance between the handles between the dragon bodies, which is 1.65 m. The cosine iterative method is used to solve this equation, and then it is converted into coordinates in the Cartesian coordinate system. Got the position of all dragon bodies and tails at any time t.

3.6. Determine the position of the dragon's body

Knowing the distance between the two handles, the position coordinates in the Cartesian coordinate system, and the vertical condition of the rectangle, using the knowledge of plane analytic geometry, the coordinates of the two vertices closest to the handle on a bench can be found. Figure 8 takes the handle of a bench dragon (x_2,y_2) as an example to make a specific calculation [10].

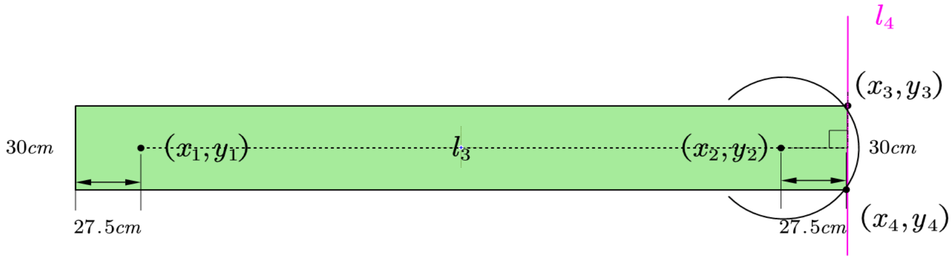

Figure 8: Schematic diagram of vertex coordinates calculated

First, know the coordinates of the handle (x_1,y_1) and (x_2,y_2), connect the two points, and extend to the edge of (x_3,y_3) and (x_4,y_4), and the line is l_3, and the line where (x_3,y_3) and (x_4,y_4) are located is l_4. According to the properties of the rectangle, the direction vectors of the line l_3 and the line l_4 are perpendicular and the distance from (x_2,y_2) to the line is 27.5 cm. Combining these two equations gives two linear equations for l_4, which are equation (15):

$$\begin{cases} \mathrm{x}+\frac{\mathrm{y}_2-\mathrm{y}_1}{\mathrm{x}_2-\mathrm{x}_1}*\mathrm{y}\pm|l_c|=0 \\ |l_c|=\left|-0.275\sqrt{1+(\frac{\mathrm{y}_2-\mathrm{y}_1}{\mathrm{x}_2-\mathrm{x}_1})^2}+x_2+\frac{\mathrm{y}_2-\mathrm{y}_1}{\mathrm{x}_2-\mathrm{x}_1}*\mathrm{y}_2\right| \end{cases} \tag{15}$$

Equation (15) solves two straight lines, one to the left of (x_2,y_2) and one to the right of (x_2,y_2). In order to filter out l_4, the distance from (x_1,y_1) to the line is required to determine the precise position of the line l_4. Take l_c = $|l_c|$, when it is judged that the distance from (x_1,y_1) to the straight line is less than 1.65 meters (2.86 meters for the head), then l_c = -$|l_c|$. Otherwise, l_c = $|l_c|$.

From the geometric relation, the distances from (x_3,y_3), (x_4,y_4) to (x_2,y_2) are equal, so the coordinates of (x_3,y_3) and (x_4,y_4) can be obtained by the equation of the straight line l4 and the system of circular binary equations with the center of (x_2,y_2) and R as the radius ($R^2 = 0.275^2 + 0.55^2$), and the coupling of the binary equations is immediately equation (16):

$$\begin{cases} \mathrm{x}+\frac{\mathrm{y}_2-\mathrm{y}_1}{\mathrm{x}_2-\mathrm{x}_1}*\mathrm{y}+|l_c|=0 \\ (x-x_2)^2+(y-y_2)^2=R^2 \end{cases} \tag{16}$$

Starting from the faucet, each handle is shared by two benches, except for the front handle of the faucet and the rear handle of the dragon's tail. According to the above method, the coordinates of the two vertices closest to the handle can be found under a certain bench, so that the coordinates of the four vertices of all benches can be found by using the coordinates of all the handles.

3.7. Determine the position of the dragon's body

In this paper, each rectangle in the spiral is tested for collision, and after the target rectangle is selected, all other rectangles (except for the rectangle adjacent to the target rectangle) are used as test rectangles, and the collision situation at time t is judged by the SAT algorithm.

3.8. Optimize the number of test rectangles

Of course, the above test method is a bit computationally large, through observation, this paper found that the position of each dragon body and dragon tail on the spiral is roughly parallel, and the dragon head is the longest of all the benches, so the most likely to collide is the dragon head. The only need is to use the coordinates of the four vertices of the dragon head to perform the SAT test on the dragon head and the nearest circle of the dragon body that surrounds the dragon head.

For adjacent laps, the control method starts from the angle $\theta_{h,t}$ of the front handle of the faucet, and stops after the angle range increases by 2π. Therefore, there are only those rectangles with angles between $\theta_{h,t}$ and $\theta_{h,t}+2\pi$, which is much less computationally than the original, but whether such a judgment can be used still needs to be rigorously proven.

3.9. Improved collision checking

Based on the idea of reducing the test rectangle above, if the distance between the four vertices of the dragon head and the straight line where all the handles of the dragon body are located is greater than 15 cm, it is also possible to describe the situation where no collision is obtained. This method can be cross-validated with the SAT crash test to get a more reasonable crash time. Figure 9 shows a schematic diagram of Improved way to calculate rectangular collisions for inspection:

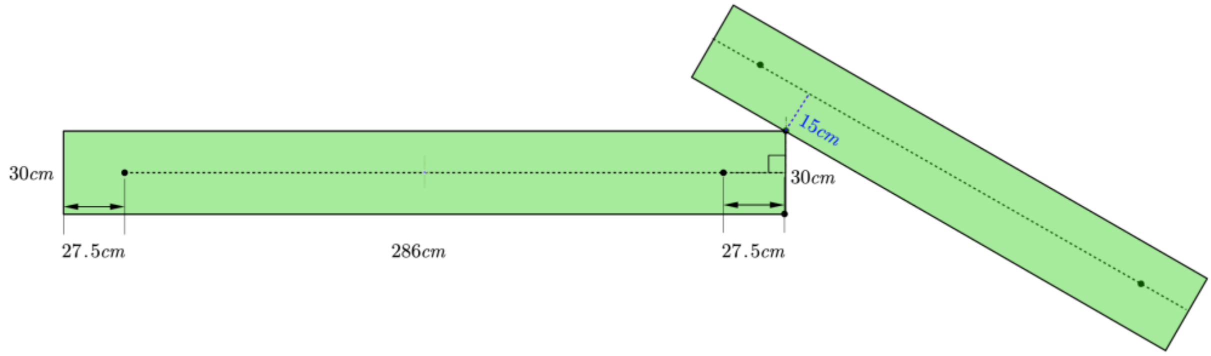

Figure 9: Schematic diagram of Improved way to calculate rectangular collisions for inspection

4. Result

Table.1 shows the horizontal and vertical coordinates of the dragon's head, the 1st, 51st, 101st, 151st, and 201st dragon's body and tail at seconds 0, 60, 120, 180, 240, and 300 seconds, respectively.

Table.1. Specify the time location result

	0s	60s	120s	180s	240s	300s
dragon's head(x)	8.800000	6.006341	-3.223928	-4.746743	4.935803	1.147869
dragon's head(y)	0.000000	-5.559985	-6.798006	-4.876211	-3.406363	4.935781
1st body(x)	8.352500	7.609093	-0.399119	-6.382485	5.992714	-1.713823
1st body(y)	2.860939	-3.099459	-7.547329	2.467101	-0.747375	4.821879
51st body(x)	-0.571198	-9.073375	-7.407890	-1.138761	7.116666	-4.250343
51st body(y)	-1.107719	0.105189	4.128192	7.767208	0.782807	-4.783368
101st body(x)	7.235129	8.806276	8.567815	6.960241	1.326690	-6.695846
101st body(y)	-7.442951	-4.433153	-3.654627	-5.291206	-8.017833	-3.300480
151st body(x)	7.165223	0.109593	-4.858381	-6.554109	-5.292141	-0.941782
151st body(y)	8.446033	10.586026	8.833633	6.951001	7.270466	8.344359
201st body(x)	-5.761030	-11.246055	-7.452665	-0.708111	2.557527	2.885727
201st body(y)	10.220588	0.724502	-7.810706	-10.282126	-9.445524	-8.778141
dragon's tail(x)	5.773922	11.507128	4.862480	-3.335725	-7.682745	-8.695006
dragon's tail(y)	-10.526909	1.085720	9.975842	10.077765	6.576379	4.038106

Using the SAT test, the time to collision is approximately 428 seconds, at which point the position of the bench dragon's handle is shown in Figure 10.

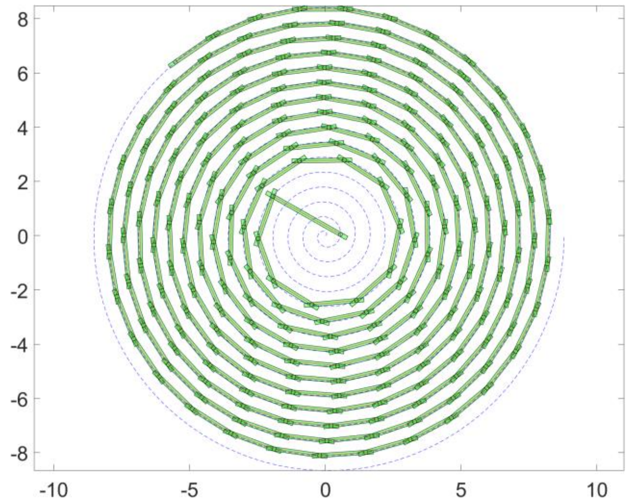

Figure 10: Schematic diagram of a collision at 428 seconds

5. Conclusions

Through accurate mathematical modeling and calculation, this paper obtains the key dynamic information of the bench dragon during the performance. Firstly, based on the data assumptions, this paper establishes a differential equation model, which can simulate the trajectory of the bench dragon during the performance. These differential equations are then solved by the lattice-Kutta algorithm to predict the position of the bench dragon at any given point in time. The calculations showed that without any intervention, the bench dragon would inevitably collide at 428 seconds, which is a critical safety warning time.

Further analysis of the location of the collision shows that the collision point is located in the head of the dragon. This finding is of great significance for optimizing the design and performance choreography of the bench dragon. By reducing the length of the dragon's head, the risk of collision can be reduced, which improves the safety and fluidity of the performance. Such adjustments not only protect the safety of the performers, but also enhance the viewing experience of the audience.

In addition, the research in this paper also confirms the effectiveness of differential equations and iterative methods in describing and solving bench dragon motion problems. These mathematical tools can not only help us understand the movement of the bench dragon, but also accurately calculate the position of the bench dragon at each moment, which provides strong theoretical support for the planning and execution of the performance. Through these methods,

This paper can predict and optimize the performance path of the bench dragon to ensure the smooth progress of the performance and reduce the potential risk caused by collisions. These research results have important practical value for the modern inheritance and innovation of traditional culture.

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