Research on Production Process Optimization Based on Multi stage Decision Model

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Abstract. This study developed a multi-stage decision model to optimize detection and disassembly strategies for minimizing costs and maximizing profits in the production of a popular electronic product. The product, assembled from two key components, is deemed defective if either component is faulty, though other factors may also cause defects. For defective products, the company can choose to either scrap them directly or disassemble them to recover parts, which incurs additional costs. Using sampling inspection methods, the study estimated the defect rate in production and optimized inspection plans for components and finished products. The model simulated various strategy combinations, calculating and comparing the total cost and profit of each to identify the optimal solution. Additionally, the study examined the impact of sampling errors on defect rate estimation and made dynamic adjustments to detection and disassembly strategies to enhance production efficiency and quality management. The findings provide important theoretical and practical guidance for quality control in enterprise production decision-making.

Keywords: Process optimization, Sampling and testing model, Monte Carlo simulation, Multi-stage decision-making model.

1. Introduction

This study addresses quality control and cost optimization in industrial production management, focusing on decision-making in spare parts procurement, product assembly, and finished product inspection within electronic product manufacturing. It utilizes statistical and operations research methods to develop a multi-stage decision model aimed at reducing production costs and improving product quality. By optimizing inspection and processing strategies under complex conditions—considering factors like defect rates, inspection, and assembly costs—this approach seeks to minimize overall costs, maximize profit, and enhance operational efficiency for enterprises.

In existing research, scholars from various fields have explored the issues of production process optimization and decision-making from multiple perspectives. Ying Wang studied resource allocation optimization using linear programming to help enterprises make scientific production decisions, minimizing costs and maximizing profits[1]. Jiaxin Li examined the application of big data technology in the strategic decision-making and production operations of manufacturing enterprises, improving the efficiency of supply chain management and production processes through big data[2]. Chunwei Zhou and colleagues studied the application of multi-modal production and AI decision-making, achieving the optimization of intelligent manufacturing and economic goals through AI decision-making systems[3]. Qiqiang Li researched intelligent decision-making and scheduling in production processes, proposing a hybrid method that combines AI technology and mathematical programming to optimize multi-objective and multi-constraint production scheduling problems[4].

This paper establishes a decision model for assessing defective rates in supplier parts using binomial and normal distributions, applying the central limit theorem for larger samples and Monte Carlo simulations to optimize sampling costs. It then constructs a multi-stage decision optimization model that integrates inspection and rework strategies, defining variables to simulate defect rates, and establishing cost-profit functions. By comparing total costs and profits across various strategies, the model identifies the optimal approach. Finally, the model extends to multi-process, multi-part production, traversing scenarios to select the profit-maximizing strategy combination.

2. Literature review

In existing research, several scholars have explored the issues of production process optimization and enterprise decision-making from the perspectives of supply chain management, intelligent manufacturing, and dynamic decision-making. Jia Yonglan studied the specific impact of different cost measurement methods on enterprise decision-making, analyzing the roles of various measurement methods, such as actual cost, standard cost, and variable cost, in production and pricing decisions[5]. Xue Feng proposed a forward dynamic programming algorithm to optimize production and inventory holding costs for the dynamic batch decision-making problem of perishable goods under production capacity constraints[6]. Zengxin Wei developed a joint decision-making model for producers and retailers in the context of supply chain management, proving that joint decision-making can bring greater economic profits to the supply chain[7]. Juan Wang discussed cost control methods in manufacturing companies based on supply chain management, proposing the application of target costing and the supply chain balanced scorecard[8].

Compared with these studies, the innovation of this paper lies in the construction of a multi-stage decision optimization model, which systematically integrates statistics, operations research, and big data algorithms for comprehensive quality control and cost management, especially for the multi-steps involved in the manufacturing process of electronic products, such as parts inspection and defective products processing. This paper not only combines the sampling inspection method with dynamic optimization algorithms to enhance the adaptability of the model in dynamic and complex environments, but also optimizes the decision-making process in multi-process and multi-spare-parts production scenarios to provide more accurate and practical decision support.

3. Research Models and Data

This study constructs a multi-stage decision optimization model for the inspection and processing strategies of spare parts procurement, semi-finished products and finished products in the production process of an enterprise, aiming to minimize costs and maximize profits by optimizing the inspection and disassembly strategies. Specifically, the model is divided into three main parts:

3.1. Statistical modeling based on sample testing

In sample testing, the binomial distribution can be used to efficiently estimate the overall defective rate and set the reliability level of the test through a confidence interval[9]. The reliability level of the test is set by a confidence interval. Assuming that n samples are taken from a batch of spare parts, the number of defective products in the samples X obeys the binomial distribution, i.e., $X \sim B(n,p)$, and the expectation $E[X]$ and variance $Var(X)$ of the binomial distribution $B(n,p)$ are $E[X] = np$ and $Var(X) = np(1-p)$, respectively. Through statistical sampling, enterprises can infer the overall quality with a smaller sample size, thus reducing inspection costs and optimizing the allocation of resources[10].

3.1.1 Exact decision model for binomial distribution

First, from the bilateral test, determine the original hypothesis (H_0): the defective rate of spare parts $p = p_0 = 0.1$; alternative hypothesis (H_1): the defective rate of spare parts $p \neq p_0 \neq 0.1$. In the case of a small sample, use the cumulative probability distribution function of the binomial distribution $(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ to calculate the probability of the number of defective parts in a given sample and determine whether to reject the original hypothesis. Use the cumulative probability distribution function of the binomial distribution $P(X \leq k)$ to calculate the probability of rejection given the number of defective parts and determine whether to reject the original hypothesis by using the critical values of $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$.

3.1.2 Simplified model of approximate normal distribution

First, from the one-sided test, the original hypothesis H_0: the defective rate of spare parts does not exceed the nominal value claimed by the supplier p_0, i.e.$p \leq p_0$; Alternative hypothesis H_1: the defective rate of spare parts exceeds the nominal value p_0, i.e.$p > p_0$. According to the Central Limit Theorem, the binomial distribution can be approximated by the normal distribution when n is large enough. Use the standardized formula of normal distribution to calculate the sample size, the number of defective $X \sim N(np,np(1\text{-}p))$, In order to standardize the binomial distribution to the standard normal distribution $N(0,1)$, the standardized variable $Z = \frac{X-np}{\sqrt{np(1-p)}}$ is introduced, in order to facilitate the detection, this paper uses the standardized sample defective rate $\hat{p}$, which can be expressed as $\hat{p} = \frac{X}{n}$. The standardized formula of binomial distribution is used for the sample defective rate, where $\hat{p}$ is the defective rate in the sample. The sample reject rate from the binomial distribution is converted to a normal distribution by deriving the normal distribution from $\hat{p}$:$\hat{p} \sim N(p, \frac{p(1-p)}{n})$. After standardization, the test statistic $Z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_{0(1-p_0)}}{n}}}$ can be determined. Using the standard normal distribution table, the critical value Z_α is determined for a given confidence level α: at the 95% confidence level, the rejection condition is $Z > 1.645$ and at the 90% confidence level, the acceptance condition is $Z < 1.28$. Confidence intervals are constructed based on the sample reject rate and standard error: if $Z > Z_\alpha$, the hypothesis is rejected; if $Z \leq Z_\alpha$, the hypothesis is not rejected. In order to ensure the accuracy of the test within the given confidence level α and error range E, the minimum sample size n needs to be calculated, and the error range E represents the error between the actual defective rate and the estimated defective rate, and the required sample size is: $n = \frac{Z^2 p_0(1-p_0)}{E^2}$.

3.2. Multi-stage decision optimization model

To minimize costs and maximize profits, this paper proposes a multi-stage decision optimization model for optimizing inspection, disassembly, and disposal strategies, considering defect rate, decision variables, costs, and profits.

3.2.1 Calculation of defective rate

Assuming that the defective rate of part 1 is p_1, the defective rate of part 2 is p_2, the defective rate of the finished product is p_3, and the total defective rate of the finished product is p_{total}, the defective rate calculation is divided into the following cases:

(1) When no parts and finished products are detected:

$$p_{total} = 1 - (1 - p_1) \times (1 - p_2) \times (1 - p_3) \tag{1}$$

(2) When only part 1 is detected: detecting only part 1 and discarding substandard part 1 without detecting part 2 and the finished product, the total defective rate is:

$$p_{total} = p_1 + (1 - p_1) \times p_2 \times p_3 \tag{2}$$

(3) When only part 2 is detected: Only test parts 2 and discard failed parts 2, without testing parts 1 and finished products, the total defective rate is:

$$p_{total} = p_2 + (1 - p_2) \times p_1 \times p_3 \tag{3}$$

Simultaneous testing of parts 1 and 2: If substandard parts 1 and 2 are tested and discarded at the same time, the defective rate of the finished product is affected only by its own defective rate:$p_{total} = p_3$.

3.2.2 Decision variable setting

The decision making process of a firm involves 4 key nodes, each with different decision making options. In this paper, we use 0/1 variables to denote the decision state of each node, i.e., $V =$

$[x_1, x_2, x_3, x_4]$ where: x_1: Whether to detect Spare Part 1 (if so, then $x_1 = 1$; otherwise $x_1 = 0$). x_2: Whether to test part 2 (if so, then $x_2 = 1$; otherwise $x_2 = 0$). x_3: Whether to test the finished product (if so, $x_3 = 1$; otherwise $x_3 = 0$). x_4: Whether to disassemble the failed finished product (if disassemble, then $x_4 = 1$; otherwise $x_4 = 0$). Based on these variables, this paper defines 16 possible decision combinations to analyze the effects and costs of different strategies.

3.2.3 Costing

The total costs of a business include the following:

(1) Cost of purchasing spare parts:

The cost of purchasing spare part 1 and spare part 2 is:

$$C_{purchase} = s_1 \cdot c_1 + s_2 \cdot c_2 \tag{4}$$

Where s_1 and s_2 are the quantities of Parts 1 and 2, respectively, and c_1 and c_2 are their unit prices.

(2) Testing costs:

The cost of inspection for spare parts and finished goods is:

$$C_{inspection} = s_1 \cdot d_1 \cdot x_1 + s_2 \cdot d_2 \cdot x_2 + n \cdot d_3 \cdot x_3 \tag{5}$$

Where d_1, d_2, d_3 is the cost of inspection for spare parts 1, 2 and finished goods respectively and n is the number of finished goods.

(3) Assembly cost:

The assembly cost of the finished product is:

$$C_{assemble} = n \cdot a \tag{6}$$

Where a is the assembly cost per unit of finished product.

(4) Dismantling cost:

The cost of dismantling substandard finished products is:

$$C_{disassembly} = m \cdot d_0 \cdot x_4 \tag{7}$$

where d_0 is the dismantling cost of each substandard finished product and m is the number of substandard finished products.

(5) Exchange loss:

When the unqualified finished product into the market when the exchange or return loss is generated:

$$L_{exchange} = p_{total} \cdot e \cdot n \tag{8}$$

where, e for the exchange loss,p_{total} for the finished product of the defective rate.

3.2.4 Profit calculation

The total profit of a business is calculated by the difference between the proceeds of sales and total costs:

(1) Gross proceeds:

The proceeds from sales are:

$$R = n \cdot q \tag{9}$$

where q is the revenue from the sale of each qualified finished product.

(2) Total cost:

The total cost includes the cost of all production stages:

$$C_{total} = C_{purchase} + C_{inspection} + C_{assemble} + C_{disassemble} + L_{exchange} \tag{10}$$

(3) Profit:

The final profit is:

$$\mathrm{Profit} = \mathrm{R} - \mathrm{C}_{\mathrm{total}} \tag{11}$$

3.3. Profit calculation Multi-process optimization models for complex production scenarios

In this paper, a multi-stage decision optimization model is constructed for the problems of spare parts procurement, product assembly and quality control in the production process of an enterprise, aiming at cost minimization and profit maximization by optimizing the inspection and disassembly strategies.

3.3.1 Cost and benefit modeling

In this paper, a total cost model is first constructed to assess the impact of each decision stage on the total cost. The total cost consists of the following components:

$$\mathrm{E}_{\text{total cost}} = \mathrm{C}_{\text{spare parts}} + \mathrm{C}_{\text{detection}} + \mathrm{C}_{\text{assemble}} + \mathrm{C}_{\text{disassemble}} + \mathrm{C}_{\text{exchange}} \tag{12}$$

Where: **Cost of purchasing spare parts**: $\mathrm{C}_{\text{spare parts}} = \sum_{\mathrm{i}} \mathrm{s}_{\mathrm{i}} \cdot \mathrm{c}_{\mathrm{i}}$, represents the cost of purchasing spare parts 1 and 2. **Inspection cost**: $\mathrm{C}_{\text{detection}} = \sum_{\mathrm{i}}(\mathrm{s}_{\mathrm{i}} \cdot \mathrm{d}_{\mathrm{i}} \cdot \mathrm{f}_{\mathrm{i}}) + \sum_{\mathrm{i}}(\mathrm{N}_{\mathrm{i}} \cdot \mathrm{D}_{\mathrm{i}} \cdot \mathrm{F}_{\mathrm{i}}) + \mathrm{n} \cdot \mathrm{D} \cdot \mathrm{F}$, which includes the cost of inspection of spare parts, semi-finished products and finished products. **Assembly cost**:$\mathrm{C}_{\text{assemble}} = \sum_{\mathrm{i}}(\mathrm{n} \cdot \mathrm{a}_{\mathrm{i}})$ consists of the assembly cost of all parts and semi-finished products. **Dismantling Costs**: $C_{assemble} = \sum_{i}(m_i \cdot k_i \cdot J_i)$, includes the cost of dismantling and re-testing and reassembling of non-conforming products. **Cost of exchange**: $\mathrm{C}_{\text{exchange}} = \mathrm{O} \cdot \mathrm{e}$ involves the loss of exchange or return due to the number of defective products that reach the market. Total sales are:$\mathrm{W}_{\text{total sales}} = \mathrm{N}_{\text{Number of finished products}} \cdot \mathrm{W}_{\text{Sales unit price}}$. Total profit is calculated at $\mathrm{W}_{\text{total profit}} = \mathrm{W}_{\text{total sales}} - \mathrm{E}_{\text{total cost}}$.

3.3.2 Calculation of defective rate

The model calculates the defective rate of spare parts, semi-finished products and finished products by means of a recursive relation: **defective rate of semi-finished products:** $\mathrm{N}_{\mathrm{i}} = 1 - (1 - \mathrm{p}_{\mathrm{i}}) \cdot (1 - \mathrm{f}_{\mathrm{i}})$ where p_{i} is the defective rate of spare parts and f_{i} indicates whether or not to detect them (taking the value 1 or 0). **Finished product defective rate:** $\mathrm{M} = 1 - (1 - \mathrm{N}_{\mathrm{i}}) \cdot (1 - \mathrm{F}_{\mathrm{i}})$where$\mathrm{N}_{\mathrm{i}}$ is the defective rate of semi-finished products and F_{i} indicates whether or not to detect semi-finished products. In the production process, the costs and benefits under different combinations of strategies are evaluated by setting decision variables. Recursive algorithms and Monte Carlo simulations are used to validate the results of different strategies, providing a practical tool for optimizing decision making in complex production environments.

4. Analysis of empirical results

4.1. Analysis of sampling test model results

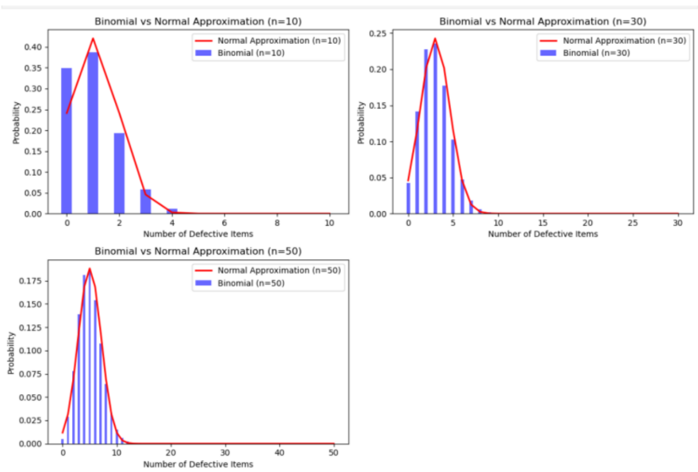

Figure 1 Visualization of distribution under different sample sizes

Python is used to plot binomial and normal distribution curves for sample sizes of 10, 30, and 50, as shown in Figure 1, to illustrate their convergence. The visual comparison shows the binomial distribution converging to the normal distribution as sample size increases. For accuracy, the model uses the binomial distribution for small samples ($n \leq 30$) and the normal distribution for larger samples ($n > 30$) based on the central limit theorem. At a 95% confidence level, hypothesis testing shows the defective rate is not significantly above the nominal value (critical values: 0 to 7); at 90%, the same result is observed (critical values: 1 to 6).

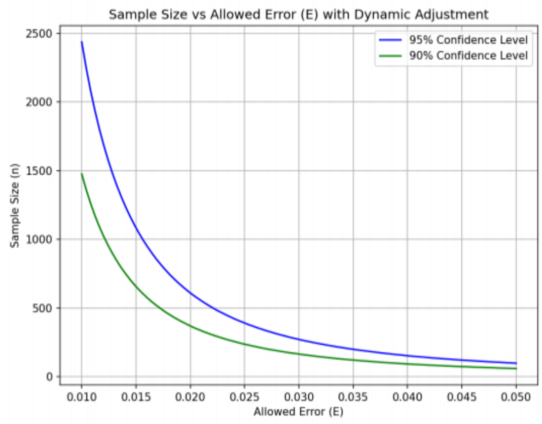

Figure 2 Visualization of dynamic changes in E between 0.01 and 0.05

In the approximate normal distribution model, in order to control the margin of error at a given confidence level E, it is necessary to determine the minimum sample size n. When the margin of error E = 0.05, 98 parts need to be sampled at the 95% confidence level, and 59 at the 90% confidence level; when the margin of error E = 0.02, 609 parts need to be sampled at the 95% confidence level, and 369 parts need to be sampled at the 90% confidence level. When the margin of error E=0.02, 609 parts need to be extracted at 95% confidence level and 369 at 90% confidence level. The results show that the required sample size increases significantly as the error margin decreases and the confidence level increases to ensure the accuracy of the test results.

By solving the model under the above two different conditions, two specific sampling schemes are given according to different confidence levels and error ranges. As shown in Figure 2 above, By dynamically adjusting the range of the allowable error E, the required sample size under different errors is calculated, and the curves visualizing the variation of the sample size n with the allowable error E between 0.01 and 0.05 are shown in the figure at two different confidence levels (95% and 90%):

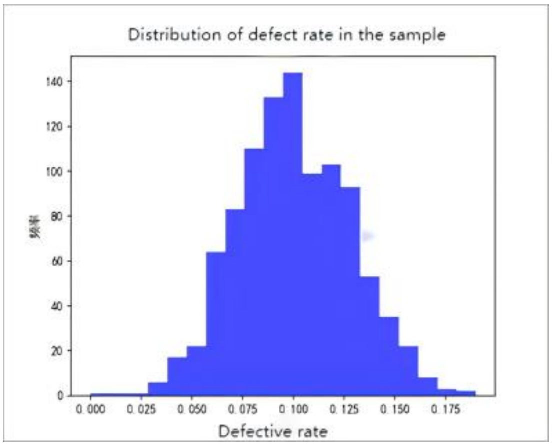

Figure 3 Distribution diagram of defective parts rate under Monte Carlo simulation sampling detection

As shown in Figure 3, Simulating different sample sizes and defect rates, the study evaluates the sampling scheme's effectiveness using the statistic Z to decide batch acceptance. In 1000 trials, each with 100 samples and a 10% defect rate, defective samples appear with a 10% probability. After each trial, the sample defect rate and Z-value are calculated.When the Z value is greater than 1.645, the batch is rejected at 95% confidence; when the Z value is less than 1.28, it is accepted at 90% confidence. Results show a 6.40% rejection rate at 95% confidence and an 88.00% acceptance rate at 90%, with defect rates approximating a normal distribution around 10%. Monte Carlo simulation enables firms to set precise quality standards and strategies.

4.2. Multi-stage decision optimization model

As shown in Table.1, The article constructs a multi-stage decision model for six scenarios, assuming a purchase of 1,000 parts, and uses Python to calculate costs and profits across 16 decision scenarios. By analyzing each scenario, the optimal combination generally involves testing parts 1 and 2, omitting finished product testing, and disassembling defective products.

Table. 1 The optimal decision plan under six different scenarios

state of affairs	Inspection of Part 1	Inspection of Part 2	Inspection of finished products	Disassembly of substandard products
Situation 1	sensing	sensing	untested	disassemble
Situation 2	sensing	sensing	untested	disassemble
Situation 3	sensing	sensing	untested	disassemble
Situation 4	sensing	sensing	untested	disassemble
Situation 5	untested	sensing	untested	disassemble
Situation 6	sensing	sensing	untested	non-disassembled

In order to more visually analyze and compare the performance of each decision-making group in different scenarios, the costs and profits are integrated into bar charts, as shown in Figure 4 below. The impact of different strategy combinations on cost and profit can be clearly seen through the charts.

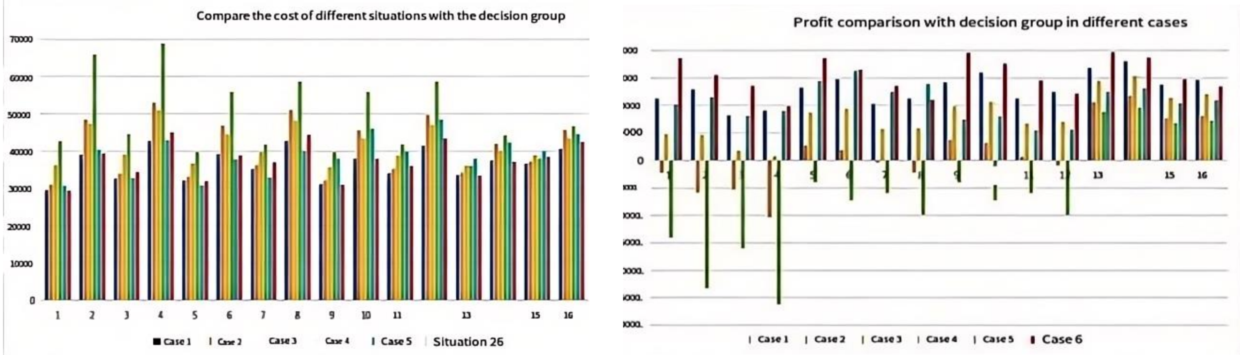

Figure 4 Cost and Profit Results Bar Chart

This paper analyzes the costs and profits of 16 decision groups across scenarios, finding that reduced inspection and disassembly lower costs and increase profits at low defect rates, while increased inspection minimizes losses at high defect rates. Groups 5, 9, and 15 demonstrate optimal cost control and profits, while groups 4 and 8 incur high costs and low profits due to excessive operations. Companies are advised to adjust inspection and disassembly strategies based on market demand and defect rates for optimal efficiency.

4.3. Multi-process optimization model for complex production scenarios

Assuming 1,000 units of each part are bought, the given parameters are substituted into the model, and a Python program is used to calculate costs and profits across 216 decision scenarios for each situation. (Full results are provided in the supporting material due to the large number of scenarios.)

By comparing the profits of all the decision scenarios, the most profitable decision scenario can be found, i.e., the decision scenario in the case of the question. The results are shown in the Table.2 below:

Multi-process decision-making scheme for complex production scenarios

Table. 2 The highest profit decision plan table

decision group	Cost ($)	Profit ($)
(0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1)	95883.3922	42943.039

Research shows that without finished product testing and dismantling (decision group (0,0,0)), total costs reach 124,702.82 yuan, resulting in a serious profit loss of -68,216.91 yuan. Implementing testing and dismantling strategies (decision group (1,1,1)) reduces costs to 98, 175.70 yuan and profit loss to -41,689.80 yuan, saving about 26,527 yuan. This underscores the cost-saving and profit-enhancing impact of finished product testing and disassembly. Additionally, dismantling defective products alone reduces costs by the same amount, as it enables material recycling. Analysis indicates that finished product testing significantly reduces total costs and market losses, proving more impactful than parts or semi-finished product testing. After evaluating 216 options, the study identified the optimal decision group (0,0,0,0,0,0,0,1,1,1,1,0,0,1,1,1,1,0,1), achieving a total cost of 95,883.39 yuan and a profit of 42,943.04 yuan, thereby maximizing profits and effectively controlling production costs.

5. Conclusions

5.1. Findings

For the sampling and testing model, the study estimates the defective rate of parts through a sampling model based on the binomial and approximate normal distribution, using binomial distribution for small samples and normal distribution for larger samples via the central limit theorem. Monte Carlo simulations validate the minimum sample size needed at various confidence levels to optimize inspection costs and decision accuracy.

For the multi-stage decision optimization model, the study integrates inspection and disassembly strategies, simulating defective rates under various strategies and establishing cost-profit functions by incorporating purchase, inspection, assembly, and disassembly costs, along with market loss. Sixteen strategy combinations are analyzed to determine the optimal inspection and disassembly scheme for cost minimization and profit maximization.

For complex production scenarios, the model extends to multi-process cases, adding variables for inspection and disassembly strategies across processes. Recursive algorithms and Monte Carlo simulations optimize each decision step, traversing all scenarios to calculate costs and sales, identifying the profit-maximizing strategy combination. The model thus balances cost and quality control, aiding companies in strategy formulation under complex conditions.

5.2. Policy recommendations

Optimization of testing policy: Enterprises should adapt testing strategies to production conditions and quality needs. For high defect rates, they should increase testing frequency or standards to ensure market quality; with low defect rates, testing intensity can be reduced to lower costs.

Introducing a dynamic adjustment mechanism: Enterprises implement a dynamic adjustment mechanism to modify inspection and dismantling strategies in real time based on inspection results and market feedback. Monte Carlo simulation supports ongoing evaluation, enabling timely adjustments to maintain optimal cost-quality balance.

Enhance multi-process optimization management: In complex production scenarios, enterprises should consider each process's defect rate and cost to develop full-process quality control strategies. Multi-stage decision models and optimization algorithms help maximize benefits, reduce unnecessary costs, and enhance operational efficiency.

Enhance the level of information technology application: Enterprises should enhance the use of big data analysis and Monte Carlo simulation to improve data analysis and decision-making support, enabling precise problem identification and strategy optimization to strengthen the effectiveness of production decisions.

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