Optimal Crop Planting Based on the Monte Carlo-Simulated Annealing Algorithm

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Abstract. This article elucidates the critical importance of sustainable land resource management in safeguarding human well-being amid globalization and urbanization, with a particular emphasis on the optimal utilization of cultivated land resources in the mountainous regions of North China. The study focuses on the cultivated land resources within a specific village in these areas, encompassing open-air cropland, conventional greenhouses, and smart greenhouses. Data is sourced from actual plot classifications and crop cultivation conditions. A Markov chain model is employed to predict planting plot types, integrated with linear programming techniques and Monte Carlo simulated annealing algorithms to maximize total revenue while minimizing unsold waste. The findings demonstrate that a multi-objective optimization model incorporating competitive and cooperative game theory can effectively propose scientifically grounded planting structures that enhance both economic returns and environmental sustainability. This model offers valuable insights for the conservation and utilization of cultivated land resources not only in North China's mountainous regions but also globally, thereby advancing agricultural practices towards greater efficiency, ecological protection, and sustainability.

Keywords: Monte Carlo-Simulated Annealing Algorithm, Crop Planting Optimization, Sustainable Land Management, Markov Chain Model.

1. Introduction

In the context of globalization and urbanization, the sustainable management of land resources [1] has become an urgent issue for ensuring human well-being and the health of the planet. As a fundamental component of food production [2], the effective protection and rational utilization of arable land are critically important for maintaining global food security [3] and fostering ecological balance [4]. This is particularly pertinent in China, where a large population coexists with limited land availability; thus, safeguarding arable land resources is essential. It necessitates the adoption of scientific and strategic planting practices to ensure both stability and sustainable development in food production.

In the mountainous regions of North China, recognized as a key area for agricultural production [5], the complex landscape and varied soil types create limitations on growing certain crops [6], which heightens the difficulties associated with agricultural output especially in 3d scenes [7, 8]. As a result, it has become crucial to refine crop planting methods in these hilly areas while preserving arable land resources and improving farming efficiency. At present, a village features 1,201 mu of open-field cultivated land divided into 34 plots that include flat dry fields, terraces, and hillside areas (all suitable for one season of grain crops [9, 10]), along with irrigated sections (appropriate for either one season of rice or two seasons of vegetables). Furthermore, there are 16 traditional greenhouses (each covering 0.6 mu and ideal for one season of vegetables plus one season of edible fungi) alongside four smart greenhouses (also each at 0.6 mu designed for two vegetable seasons). Each plot allows for the simultaneous cultivation of various crops throughout each growing period. The details are available at https://github.com/yuanasd/Crop-planting-data-of-a-village-in-North-China.

Given the intricate diversity of land types in the mountainous region and the constraints on cultivated land resources [3], this situation inevitably raises a critical issue that requires immediate attention: How can this study ensures the sustainable utilization of farmland in these mountainous areas while simultaneously maximizing agricultural productivity and economic returns to meet the dual demands of increasing food requirements and ecological conservation? This challenge is not only pertinent to the livelihoods and welfare of farmers residing in these regions but also represents a vital aspect of global food security and ecological balance.

Question 1: Assuming that sales volume, costs, per-mu yield, and crop prices remain stable from 2024 to 2030, with crops being planted and sold within the same season. Should total crop output exceed expectations, any surplus will result in waste due to unsold products. In light of this scenario, an optimal planting strategy is essential.

In this study, a Markov chain model is employed to forecast planting plot types [4], which subsequently allows for indirect derivation of planting area. Following this predictive analysis, a linear programming model [5] is developed aimed at maximizing total revenue under conditions where excess crops lead to wastage due to unsold inventory. Python programming techniques are utilized to identify an optimal planting strategy that maximizes overall revenue. Through this research endeavor, it is anticipated that both scientific foundations and practical guidance will be provided for optimizing crop cultivation practices in North China's mountainous regions, thereby facilitating advancements towards more efficient, environmentally sustainable agricultural systems. Concurrently, this study aims to offer valuable insights for protecting and utilizing global cultivated land resources.

2. Problem Formulation

2.1. Problem Definition

The paramount objective of this research endeavor is to furnish solutions to the aforementioned practical issue Research Question 1 by means of establishing models or implementing specific algorithms. Ergo, it becomes imperative to delineate an array of indicators and parameters germane to crop cultivation, attuned to the idiosyncratic conditions and factual circumstances of agricultural practices in North China, thereby facilitating the formulation of simulation models for crop cultivation in this locale.

In the course of exploring the optimal crop planting strategies spanning from 2024 to 2030, this research confronts a cardinal conundrum: Presuming that the anticipated sales volume, cultivation costs, yield per mu, and selling prices remain invariant during 2024-2030 in comparison to 2023, and that all crops sown in each season are vended within the same season. Against this backdrop, when the actual sales surpass the projected sales, resulting in waste due to unsold surplus, an optimal planting strategy must be concocted.

To address this predicament, this research initially conducted data preprocessing and explicitly stipulated the selling prices of each crop. Subsequently, it integrated the relevant variable data and employed the Markov chain prediction technique to determine the types of planting plots, thereby inferring the corresponding planting areas indirectly. Utilizing this data, with maximizing the total revenue as the primary objective and in accordance with the requirements of the topic, constraints were set. Eventually, the planting scheme that maximized the total revenue was determined through the linear programming approach, while taking into account the influence of unsold and spoiled crops. The following illustrates the overarching conceptual framework that guides this research, as depicted in Figure 1.

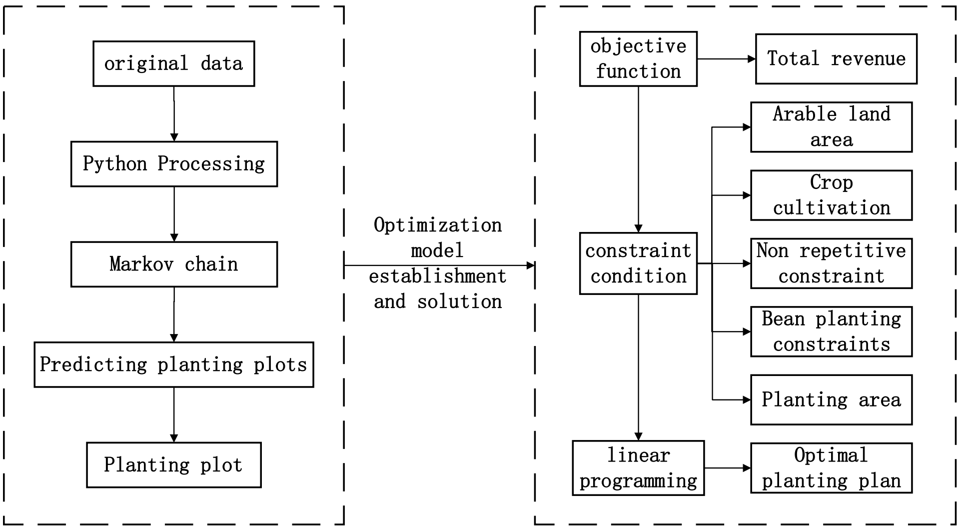

Figure 1. The overall conceptual schema guiding

To ascertain the validity and practicality of the model, the following postulates are delineated as fundamental components for its construction:

(1) Concentrated Cultivation Hypothesis: It is posited that in 2023, crops will be cultivated in a relatively concentrated manner across each plot to alleviate the complexities associated with field management and agricultural practices stemming from dispersed cultivation.

(2) Constant Growth Cycle and Requirements Hypothesis: It is asserted that throughout the analysis period, both growth cycles and cultivation requirements of each crop will remain constant, thereby facilitating model simplification while enhancing prediction stability and reliability.

(3) Feasibility of Proposed Scheme Hypothesis: The proposed optimal planting scheme must demonstrate viability for practical implementation, encompassing elements such as allocation of crop areas, quantities, rotational needs, and distribution patterns.

(4) Stable Market Demand Hypothesis: It is presumed that market demand will maintain relative stability within a defined range to enable effective prediction and optimization of planting strategies across various scenarios.

(5) Sales Ceiling Hypothesis: To avert overproduction issues, a reasonable cap on anticipated sales volumes for crops is instituted to prevent resource wastage or significant price fluctuations.

3. Methodology

3.1. Data Preprocess

In light of the actual situation, the sales prices of crops will fluctuate in accordance with market adjustments. To guarantee the rationality and objectivity of the data, this paper stipulates the formula for the sales price of crops p as follows:

$$p = p_{\min} + \varepsilon(p_{\max} - p_{\min}) \quad \varepsilon \in (0,1) \tag{1}$$

Where $p_{\max}$ is the maximum value of the crop price range, $p_{\min}$ is the minimum value, ε is an arbitrary value within the range of 0 to 1.

3.1.1 Markov Chain Predicts Planting Fields

Upon reviewing the data presented in Attachment 1, it is evident that the planting area information for certain crops (e.g., black beans cultivated on flat dry land) is missing in specific plots. In light of the fact that only historical data from the 2023 planting plots is available, this study employs Markov chains to predict the types of missing planting plots for each crop from 2024 to 2030, thereby indirectly inferring the corresponding planting areas.

In a Markov chain framework, state transitions within the system can be characterized by a transition matrix *P* as follows:

$$P=\begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{pmatrix} \tag{2}$$

Each element P_{ij} within the matrix is referred to as the transition probability of state X_i transitioning to state X_j, represented by $P_{ij} \geq 0$ and $\sum_{j=1}^{n} P_{ij} = 1(i, j = 1, 2, \ldots n)$, and that is, the sum of each row of the matrix is 1.

3.2. The establishment of crop planting optimization model

In this study, it is posited that the sales volume, yield per mu, selling price of agricultural products, and planting costs will remain relatively stable from 2024 to 2030. Furthermore, the anticipated future sales volume for each crop is projected to be 95% of its total output in year *t*. The decision variable $x_{i,j,t}$ represents the area allocated for cultivating the *t* crop on the *j* plot (greenhouse) during year *t*.

3.2.1 Objective Function

Given that the quantity exceeding the anticipated sales cannot be sold to generate revenue, and that cultivation costs must still be incurred, the objective function is established as the maximization of total crop revenue from 2024 to 2030.

$$Max\ S_1 = \sum_i \sum_j \sum_t (p_i * d_{i,t} - c_i * x_{i,j,t}) \tag{3}$$

Where $d_{i,t} = 0.95 * \sum_j x_{i,j,t} * q_i$ is the projected sales quantum of the *i* crop in the *t* year (tons), q_i is the yield per mu of the *i* crop (tons/mu), p_i is the selling price of the *i* crop (yuan/mu), and c_i is the planting outlay related to the *i* crop (yuan/mu).

3.2.2 Constraint conditions

Agricultural practices in the North China region are constrained by environmental factors. Considering the need for sustainable and efficient use of water and land resources within this rural area, the limitations on crop production include restrictions on the available arable land, specific regulations regarding crop planting, requirements against consecutive cropping, and constraints associated with soybean cultivation.

The total planting area of each land parcel must not exceed the actual area, where the area of the land parcel is 1,201 mu and the area of the greenhouse is 12 mu.

$$\sum_j x_{i,j,t} \leq 1213 \tag{4}$$

Flat drylands, terraced fields, and hillside fields can only be used for single-season cultivation of grain crops (excluding rice):

$$\sum_{i=1}^{15} x_{i,j,t} \leq Z_j, \quad j \in (A1 \sim A6, B1 \sim B14, \mathrm{C1{\sim}C6}) \tag{5}$$

Where Z_j is the total area of $A1 \sim A6, B1 \sim B14$ and $C1 \sim C6$ (The same applies to the following formulas and will not be elaborated further).

Irrigated land can only be utilized for single-season rice cultivation:

$$\sum_{i=16} x_{i,j,t} \le Z_j, \quad j \in (D1 \sim D8) \tag{6}$$

The intelligent greenhouse is capable of cultivating two crops annually (with the exception of Chinese cabbage, white radish, and red radish):

$$\sum_{i=17}^{34} x_{i,j,t} \le 2Z_j, \quad j \in (F1 \sim F4) \tag{7}$$

The same crop is prohibited from being grown on the same plot for two consecutive years.

$$a_{i,j,t} + a_{i,j,t+1} \le 1, \quad \forall i, j, t \tag{8}$$

Each plot (greenhouse) is obligated to cultivate beans crops at least once within three years:

$$\sum_{j \in Beans} \sum_{t}^{t+2} a_{i,j,t} \ge 1, \quad \forall i, t = 2024, 2028 \tag{9}$$

3.2.3 Multi-Objective Optimization Solution Based

The Monte Carlo paradigm is a methodology founded upon random sampling and probabilistic statistics, exploited to grapple with convoluted mathematical predicaments and undertake prognoses, estimations, and optimizations. It emulates the uncertainties of the projected sales volume, yield per mu, cultivation costs, and selling prices of crops via recurrent random samplings. The simulated annealing algorithm recapitulates the thermodynamic behaviors of the physical annealing procedure, initiating from a comparatively elevated "temperature" and gradually diminishing it. Furthermore, during the search process, it sanctions random leaps within the solution space with a definite probability to eschew entrapment in local optimal solutions. As the temperature parameter persistently descends, the search purview of the algorithm gradually constricts and ultimately converges to the global optimal solution. The Monte Carlo - Simulated Annealing algorithm amalgamates the virtues of these two approaches for resolving intricate optimization quandaries.
Initialize the temperature as T_0 Employ the Monte Carlo approach to randomly generate an initial solution S_0 and compute the corresponding objective function value $E(x_0)$;

In the vicinity of the current solution S, a new candidate solution S_1, is generated randomly using the Monte Carlo technique, and its associated objective function value $E(x_1)$ is evaluated;
Adhere to the acceptance criteria and calculate the increment.

$$\Delta E = E(x_1) - E(x_0) \tag{10}$$

If $\Delta E < 0$, it implies that the new solution S_1 is more favorable, and the new solution is accepted as the current one.

If $\Delta E > 0$, it suggests that the new solution S_1 is poorer. A certain probability is employed to determine whether to accept the new solution:

$$P = e^{-\frac{\Delta E}{T_1}} \tag{11}$$

Where T_1 is the current temperature.

Set a cooling schedule to update the temperature, where $T = kT_0, 0 < k < 1$, k being the rate of temperature drop.

Suppose the number of iterations is L. At temperature T, iterate L times and repeat the execution of Step2 and Step3;

When the temperature descends beneath the pre-established termination threshold T_{min} or the count of iterations attains the prescribed limit, the search space progressively converges toward a local optimum solution. Subsequently, the Monte Carlo approach is utilized to conduct further exploration within this domain, thereby enhancing the proximity to the global optimum solution. The outcomes of the algorithmic search are subsequently presented.

4. Result Analysis and Discussion

4.1. The results of crop planting optimization model

4.1.1 Markov Prediction Results Analysis

In certain circumstances, some states might lack sufficient transition data. Consequently, certain values within the transition matrix could be vacant. To guarantee the normal usage of the matrix, in this paper, the missing probabilities are filled evenly, namely, the transition probabilities of these states are all set to 0.2.

Take "ordinary greenhouse, sword beans" as an example, the probability matrix is constructed as follows:

$$P=\begin{pmatrix} 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0 & 0 & 1 & 0 & 0 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{pmatrix} \tag{12}$$

Predicting the missing crop planting plot types, some of the results are shown in Table 1:

Table 1. Predicting the Type of Cultivated Land

Crop	Plot	Planting area/acre	Crop	Plot	Planting area/acre
soybean	A4	72	Mung Bean	B4	28
black bean	A4	72	crawling bean	B4	28
adzuki bean	A4	72	wheat	B5	25
Mung Bean	A1	80	wheat	B1	60
crawling bean	A1	80	maize	B7	55

From Table 1, it can be seen that this paper predicts the types of planting plots using a Markov chain, and supplements the planting area with data from Appendix II to provide data support for the subsequent optimization model.

4.2.2 Monte Carlo-Simulated Annealing Algorithm Optimization Results Analysis

As the planting area of crops is a continuous variable, this paper constructs a Monte Carlo - simulated annealing algorithm model. The partial solutions are presented as follows:

Table 2. Planting plan for overstocked cases

Plot	Crops
A1	black bean, adzuki bean, wheat, maize, millet, broomcorn millet, pumpkin, sweet, hulless oat
A2	black bean, Mung Bean, crawling bean, maize, millet, broomcorn millet, pumpkin, sweet, barley
A3	soybean, Mung Bean, crawling bean, wheat, sorghum, buckwheat, pumpkin, sweet, hulless oat
A4	black bean, adzuki bean, Mung Bean, crawling bean, wheat, maize, sorghum, broomcorn millet, sweet
A5	black bean, Mung Bean, crawling bean, wheat, maize, sorghum, broomcorn millet, buckwheat, hulless oat
A6	soybean, black bean, adzuki bean, Mung Bean, crawling bean, wheat, sorghum, buckwheat, barley

Table 2 presents the planting outcomes of crops in Plot A in 2024. According to the analysis of the planting results of Plot A, it was discovered that the crop planting situation in Plot A conformed to the anticipated requirements. All the planted crops were grain crops and excluded rice. This crop planting configuration not only complies with the stipulated planting standards but also indicates from the side that the crop planting plan designed for this time is relatively rational and ideal.

Table 3. Quarterly revenue for unsold excess inventory

Time	Profit/yuan
2024 Season 1	6155828.268
2024 Season 2	3119638.477
2025 Season 1	5664168.621
2025 Season 2	3104269.314
2026 Season 1	6292620.843
2026 Season 2	3113908.575
2027 Season 1	6229314.631
2027 Season 2	3105286.022
2028 Season 1	5573149.871
2028 Season 2	3107127.009
2029 Season 1	5692803.497
2029 Season 2	3122187.362
2030 Season 1	5569575.813
2030 Season 2	3119020.163

Analysis of Table 3 indicates that, overall, revenue in the first quarter of each year tends to be higher. While seasonal fluctuations are present, the revenue remains relatively stable across periods. Specifically, the revenue from excess unsold inventory demonstrates significant seasonal variations, with the first quarter consistently yielding greater returns. This study posits that this period typically aligns with peak market demand, potentially driven by holidays, special events, or heightened seasonal needs—factors contributing to elevated selling prices and sales volumes during this time and consequently resulting in increased revenue. Conversely, demand may diminish in the second quarter, particularly when agricultural supply surges; such a decline in demand is likely to lead to reduced revenues.

4.3.3 Avoid Repeated Planting Analysis

In cases where the same crop is cultivated on the same plot in adjacent years within the planting plan (e.g., black beans are planted on Plot A1 in both 2024 and 2025 in the planting plan when there is an excess of unsold produce), the following explanations are provided in this paper, as depicted in Figure 2:

(a) Plot A1 in 2024 (b) Plot A1 in 2025

Figure 2. A1 Plot crop cultivation in 2024 and 2025

Figure 2 presents a comparison of the planting plans for Plot A1 in the context of agricultural surplus in 2024 and 2025. The contrast of the planting schemes for these two years reveals that although crops such as black beans, red beans, naked oats, and sweet potatoes are continuously cultivated on Plot A1 for two consecutive years, the specific planting areas vary between different years, thereby effectively avoiding the phenomenon of consecutive planting. This differentiated arrangement contributes to reducing the risk of soil degradation, lowering the incidence of pests and diseases, maintaining soil health, and promoting the sustainability of the agricultural ecosystem. Additionally, the rational design of crop rotation optimizes the efficiency of resource utilization, enabling more efficient distribution and absorption of soil moisture and nutrients, thereby further supporting the goals of sustainable agricultural development and enhancing the overall efficiency of land utilization.

Prior to this optimization, the average profit from cultivation in the experimental area in 2023 was 3.8067×10^{6} yuan. Through the introduction of the optimization model based on the Monte Carlo - Simulated Annealing Algorithm, the profit was elevated to 6.1558×10^{6} yuan, attaining remarkable economic growth. The optimization outcomes not only enhanced the income from cultivation but also effectively optimized the allocation of land resources and mitigated the waste resulting from crop surplus. The rational design of crop rotation further decreased the risk of soil degradation, reduced the possibility of pest and disease occurrences, and promoted the sustainable development of the agricultural ecosystem. This research indicates that scientific optimization strategies can facilitate the efficient utilization of resources while augmenting economic benefits, providing a robust guarantee for the sustainable development of the region.

5. Conclusion

This research has constructed a multi-objective optimization model integrating Markov chains and the Monte Carlo simulated annealing algorithm for the systematic analysis of crop planting strategies in the mountainous regions of northern China. The model is designed to maximize the total revenue, enhance the efficiency of cultivated land utilization, and minimize the waste of resources, with the main constraints including cultivated land resources, crop planting restrictions, and market demands. To cope with the possible conflicts among the objectives during the process of seeking the optimal solution, this paper introduces a competition and cooperation game algorithm to determine the optimal crop planting plan. By converting the multi-objective optimization problem into a strategy set, this approach effectively addresses the complex agricultural planting circumstances in the northern mountainous areas and proposes a scientifically and rationally designed planting structure. Future research directions encompass enhancing the dynamic adaptability of the model, such as introducing dynamic programming or machine learning algorithms, integrating remote sensing and meteorological data to achieve precise monitoring and real-time optimization; simultaneously, evaluating the social, economic, and environmental benefits of the model to provide a basis for policy formulation; and further promoting eco-friendly planting strategies, reducing the use of chemical fertilizers and pesticides, and achieving low-carbon agricultural development.

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