The "Bench Dragon" Motion Trajectory Model Based on the Spiral Equation

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Abstract. Dragon dancing, a vital aspect of traditional Chinese folk art, has evolved through the integration of modern technology and mathematical models. This study investigates the motion characteristics of dragon dance teams, addressing five critical questions about their complex trajectories. Initially, the trajectory equation of the equidistant spiral was established, and the Fourth-Order Runge-Kutta algorithm, alongside the bisection method, facilitated a high-precision analysis of the team's motion over 300 seconds, calculating key positions and velocities. To determine the termination moment of the spiral entry, a bench collision model was developed to ensure safety under varying spatial conditions. Additionally, optimizing the minimum pitch improved the performance's smoothness, while modeling arc radius as a decision variable reduced the area and speed needed for the dragon's movement. A scheme to enhance the speed of the dragon's head was also proposed, enriching the visual appeal of the performance. This research offers fresh insights into dragon dance motion patterns and promotes innovation in traditional culture.

Keywords: Fourth-Order Runge-Kutta, Bisection Method, Optimization, Spiral Equation.

1. Introduction

Dragon dance, as an important embodiment of Chinese traditional culture, not only carries rich history and folklore but also revitalizes in modern society. The performance forms of dragon dance are diverse, and the underlying movement laws and coordination are crucial for enhancing its aesthetic appeal [1]. With the advancement of technology, particularly in computer simulation, optimizing traditional dragon dance using modern scientific methods has become a focus of researchers [2-3]. The equidistant spiral is a curve with unique geometric features, widely used in both artistic expression and scientific research [4]. Its motion trajectory, formed by the combination of uniform linear and circular motions, effectively describes the dynamic behavior of the dragon dance team [5]. By establishing a motion model based on the equidistant spiral, this article can systematically analyze the movement laws of the dragon dance team in complex environments, providing theoretical support for enhancing the safety and aesthetics of the performance [6].

This research focuses on the movement patterns of the "benched dragon" and poses the following key questions:

(1) Motion Trajectory Model: How to construct the motion model of the dragon dance team and accurately calculate the positions and velocities of the entire team (including the dragon head, body, and tails) from the initial moment to 300 seconds? To achieve this, this article convert Cartesian coordinates to polar coordinates, establish the trajectory equation of the equidistant spiral, and implement numerical solutions using an improved fourth-order Runge-Kutta method [7-8].

(2) Collision Detection Model: During the movement of the dragon dance team, how to determine the termination moment and ensure no collisions occur? This article define the collision detection criteria for the dragon dance team and calculate potential collisions adjacent benches using rotation matrices and the axis separation theorem, thereby determining the termination signal for the team's movement [9-10].

(3) Pitch Optimization: Given a specified turning space diameter, how to set the pitch parameters to minimize the pitch when the dragon head handle stops at the boundary of the turning space? By solving the equations of the dragon team's motion spiral alongside the geometric conditions of the turning space, this article successfully minimized the pitch, thereby enhancing the overall efficiency of the dragon dance [11-12].

(4) Turning Path Optimization: How to optimize the circular arc length of the turning path while maintaining tangential contact parts to reduce the team's travel resistance? This article employed a simulated annealing algorithm to design an optimization model, where the radii of two circular arcs serve as decision variables, aiming to minimize the total arc length, significantly improving the fluidity and aesthetic quality of the performance [13-14].

(5) Maximum Speed Optimization: Under the premise of ensuring safety, how to determine the maximum traveling speed of the dragon head, ensuring that the speeds of all parts do not exceed 2 m/s? This article appropriately adjusted the speed of the dragon head to enhance the overall aesthetic appeal and safety of the dragon dance team [15-16].

Through systematic analysis and optimization of these questions, this study aims to provide scientific theoretical support and practical guidance for traditional dragon dance performances, promoting the modernization of traditional culture and enhancing its influence in international cultural exchanges.

2. Application Method Description

2.1. Establishment of the Benched Dragon Motion Trajectory Using an Improved Fourth-Order Runge-Kutta Method

First, it is necessary to determine which coordinate system to use. In this case, since the 'benched dragon' coils inward in the form of an equidistant spiral, this article consider using a polar coordinate system to represent the positions of the dragon head, body, and tail. The established polar coordinate system is shown in Figure 1 below:

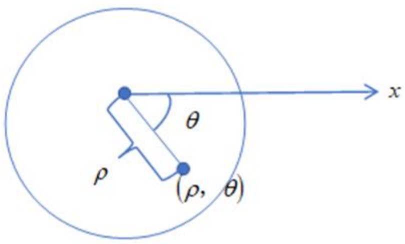

Figure 1. Establishment of the Polar Coordinate System

(1) Determine the Equidistant Spiral Equation

Since this article need to consider the positions of the dragon dance team both before and after coiling in, which involves a large angular span, this article simplify calculations by defining the angle in the above polar coordinates as the rotation angle starting from the axis, with a range of $[0, 32\pi]$.

In this problem, this article take into account the practical situation and consider that the shape formed by the various parts of the dragon dance team before coiling in along the given equidistant spiral is an expansion of the given equidistant spiral. This is shown in Figure 2:

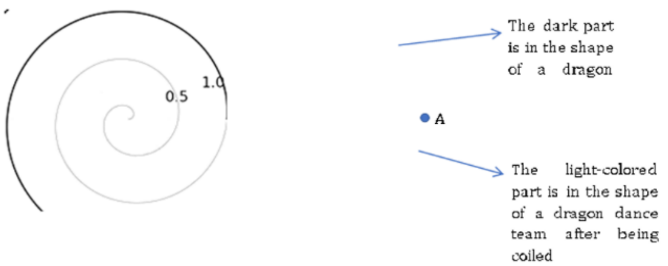

Figure 2. Schematic diagram of the position of the dragon dance team before and after coiling

Since this article need to consider the equidistant spiral equations for the dragon dance team both before and after coiling in, let the equidistant spiral equation be defined as follows:

$$\rho = a + b\theta \tag{1}$$

Where a, b is a constant that relates to the properties of the equidistant spiral.

• The equidistant spiral equation before coiling in: Taking point A as the starting position, the value increases outward. Based on the given data, with a pitch of 0.55 m and point A located at the 16th turn, the spiral equation is defined as follows:

$$\rho = 8.8 + \frac{0.55}{2\pi}\theta \tag{2}$$

• The equidistant spiral equation after coiling in: Taking point A as the starting position, the value decreases inward. The spiral equation is defined as follows:

$$\rho = 8.8 - \frac{0.55}{2\pi}\theta \tag{3}$$

(2) Determine the Position Change Pattern of the Dragon Head Handle Over Time

Let the distance traveled by the dragon head handle be denoted as L. Since its speed is consistently 1, by integrating the motion angle, this article obtain the following formula:

$$L = \int_0^{\theta} \sqrt{\left(8.8 - \frac{0.55}{2\pi}\right)^2 + \left(\frac{0.55}{2\pi}\right)^2}\, d\theta \tag{4}$$

$$\frac{d\theta}{dt} = \frac{1}{\sqrt{\left(8.8 - \frac{0.55}{2\pi}\theta\right)^2 + \left(\frac{0.55}{2\pi}\theta\right)^2}} \tag{5}$$

Thus, this article can establish the relationship the rotational angle of the dragon head handle and time through integration.

(3) Use the Least Squares Method to Determine the Iterative Relationship the Dragon Head, Body, and Tail

Let the coordinates of the front handle of the i bench be. The relationship this article its horizontal and vertical coordinates can be expressed as follows, shown in Figure 3.

$$\rho_i = a + b\theta_i \tag{6}$$

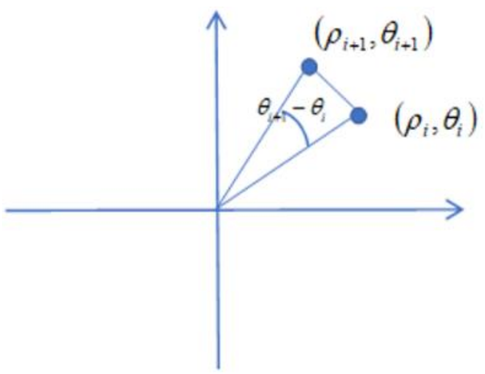

Figure 3. Position Relationship the Front Handles of Adjacent Benches

From the above diagram, the positional relationship the front handle of the \(i\)-th bench and that of the next bench can be derived using the cosine theorem as follows:

$$l_i^2 = \rho_i^2 + \rho_{i+1}^2 + 2\rho_i\rho_{i+1}\cos(\theta_{i+1} - \theta_i)\,,\quad i = 1,2......222 \tag{7}$$

Where:

$$l_i = \begin{cases} 2.86, i = 1 \\ 1.65, i = 2.3......222 \end{cases}$$

$\rho = a + b\theta$ substituting into the above formula yields:

$$l_i^2 = (a + b\theta_i)^2 + (a + b\theta_{i+1})^2 + 2(a + b\theta_i)(a + b\theta_{i+1})\cos(\theta_{i+1} - \theta_i), \quad i = 1,2......222 \tag{8}$$

For a known coordinate of the front handle of the \(i\)-th bench, this article can use the bisection method or Newton's iterative method to traverse to the coordinates of the next bench's front handle, stopping when the following least squares criterion is met:

$$\min\ (l_{theory} - l_{ture})^2, \quad i = 1,2......222 \tag{9}$$

(4) Establish the Velocity Model for Each Handle

This article set the time step to 0.001 seconds. For the front handle of the i the bench at the k second, this article calculate the speed by using the distance its position at the k second and its position at k+ 0.001 seconds. The formula is as follows:

$$v_{ik} = \frac{\sqrt{\Delta x^2 + \Delta y^2}}{\Delta h}, \quad i = 1,2......222 \tag{10}$$

Where h is the chosen time step.

2.2. Establishment of the Collision Model Based on the Bisection Method and Axis Separation Theorem

(1) Traverse to Obtain the Relationship the Positions of All Bench Handles

Taking point A as the initial point, this article can determine the position coordinates of the next handle using the distance it and the next bench handle, as this articlell as the equation of the equidistant spiral. Let the polar coordinates of the first handle be $(\rho_i, \ \theta_i)$, then this article can derive the following recursive relationship:

$$\begin{cases} \rho_i^2 + \rho_{i+1}^2 - 2\rho_i\rho_{i+1}\cos(\theta_{i+1} - \theta_i) = l_i^2 \\ \rho_i = 8.8 - \dfrac{0.55}{2\pi}\theta_i \end{cases}, \quad i = 1,2......222 \tag{11}$$

Where: $l_i = \begin{cases} 2.86, i = 1 \\ 1.65, i = 2.3......222 \end{cases}$

For a given time, by using the specified position of the dragon head handle and applying the bisection method, this article can obtain the approximate position coordinates of each handle.

(2) Use the Rotation Matrix and Translation to Determine the Coordinates of the Four Vertices of Each Bench

When determining the coordinates of the four vertices of each bench, this article first fix the center of each bench at the origin. Then, this article use matrix rotation and translation to align the bench with its actual position. The specific process is illustrated in the following diagram shown in Figure 4:

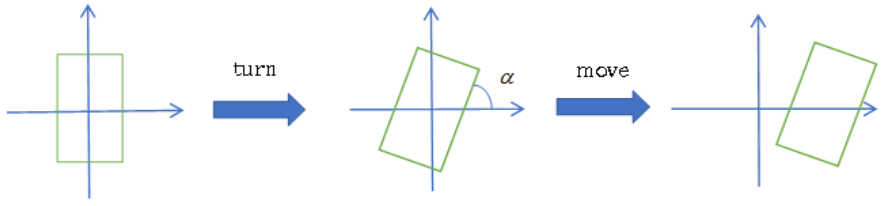

Figure 4. Diagram Illustrating the Bench Transformation

(1) Rotation of the Bench: For a given bench, let the Cartesian coordinates of its two handles be (x_i, y_i)、(x_{i+1}, y_{i+1}), after rotation, the angle bet this article en the bench's length and the x is α_i. The rotation matrix for this handle is given by:

$$\begin{bmatrix} \sin\alpha_i & \cos\alpha_i \\ -\cos\alpha_i & \sin\alpha_i \end{bmatrix}, \quad (i = 1 \sim 223) \tag{12}$$

Where $\alpha_i = \arctan(\frac{y_{i+1} - y_i}{x_{i+1} - x_i})$.

(2) Translation of the Bench: For the i bench, let the initial coordinate of one vertex be $d_{i1}(x_{i1}, y_{i1})$. After rotation and translation, the new horizontal and vertical coordinates of this vertex are expressed as follows:

$$\begin{cases} x'_{i1} = \sin\alpha_i x_{i1} + \cos\alpha_i y_{i1} + \dfrac{x_{i1} + x_{i2}}{2} \\ y'_{i1} = -\cos\alpha_i x_{i1} + \sin\alpha_i y_{i1} + \dfrac{y_{i1} + y_{i2}}{2} \end{cases}, \quad (i = 1 \sim 223) \tag{13}$$

2.3. Use Simulated Annealing Algorithm to Find the Shortest Turnaround Path

The simulated annealing algorithm is a global optimization method that mimics the physical annealing process to find the global optimum. First, this article plot the background of the coiling in and coiling out spiral lines: the code generates two symmetric spirals representing the paths for coiling in and out, serving as a reference for path planning. Next, this article set the radius of the first segment of the arc to be larger (R1=10) and the radius of the second segment to be smaller (R2=R1/2). These two arcs form an S-shaped curve for the turnaround. By calculating the lengths of both arcs, this article output each arc's length and their total length to evaluate the turnaround path length, and then This article reduce the radius of the first arc (R1_optimized=8) to optimize the path, making the turnaround shorter.

Simulated Annealing Algorithm:

(1) Objective Function: $\mathrm{goal}(R_1 + R_2) = \frac{2\pi}{4}(R_1 + R_2)$

(2) Acceptance Criterion: $new\_lenth < best\_lenth$

(3) Cooling Schedule: $\mathrm{Current\_}T = \mathrm{Current\_T} * \mathrm{cooling\_rate}$

(4) Termination Condition: $Current_T < T\_final$

3. Results

3.1. Bench Dragon Position Model Solving

(1) Model Solving for the Position of the Dragon Head Handle.

Here, this article use the fourth-order Runge-Kutta method to numerically solve for the position of the dragon head handle. The basic form of the differential equation for the fourth-order Runge-Kutta method is as follows:

$$\begin{cases} y' = f(x, y) \\ y(x_0) = y_0 \end{cases} \tag{14}$$

Where y_0 is the initial condition and $f(x, y)$ is a known function of x, y.

For the basic form above, the recursive formula is as follows:

$$\begin{aligned} & y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 3k_3 + k_4) \\ & k_1 = f(x_n, y_n) \\ & k_2 = f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_1\right) \\ & k_3 = f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_2\right) \\ & k_4 = f(x_n + h, y_n + hk_3) \end{aligned} \tag{15}$$

In this problem, the basic form is:

$$\begin{cases} \theta' = \dfrac{1}{\sqrt{(a + b\theta)^2 + (b\theta)^2}} \\ \theta(t_0 = 0) = 0 \end{cases} \tag{16}$$

Where the time range is [0, 300s], with a step size of 0.001s. Thus, this article can obtain the positions of the dragon head handle at all time points within the time interval from 0 to 300 seconds.

(2) Iterative solution of dragon body and dragon tail handle

From the above steps, the position of the front handle of the faucet at each time point has been obtained. At each time point, based on the position of the front handle of the faucet, the least squares method in the iterative model of the faucet, body, and tail $\min\,(l_{theory} - l_{ture})^2$ is used to calculate all dragons. Traverse the body and dragon tail handles to obtain the positions of the dragon body and dragon tail handles at each moment. Here this article use the method to set the $1 \times e^{-8}$ tolerance to calculated.

(3) Solution of the speed of the dragon body and dragon tail handle at various time points

From the above speed calculation formula at each time point and the positions of the dragon body and dragon tail handles at each time point, the speed of the dragon body and dragon tail at each time point can be obtained. Result as show in table 1 and table 2.

Table 1. Position results of various parts of the dragon dance team

	0s	60s	120s	180s	240s	300s
	8.800000	5.799209	-4.084887	-2.963609	2.594494	4.420274
	0.000000	-5.771092	-6.304479	6.094780	-5.356743	2.320429
Faucet x(m)	8.363824	7.456758	-1.445473	-5.237118	4.821221	2.459489
Faucet y(m)	2.826544	-3.440399	-7.405883	4.359627	-3.561949	4.402476
Chapter 1 x(m)	-9.518732	-8.686317	-5.543150	2.890455	5.980011	-6.301346
Chapter 1 y(m)	1.341137	2.540108	6.377946	7.249289	-3.827758	0.465829
Chapter 51 x(m)	2.913983	5.687116	5.361939	1.898794	-4.917371	-6.237722
Chapter 51 y(m)	-9.918311	-8.001384	-7.557638	-8.471614	-6.379874	3.936008
Chapter 101 x(m)	10.861726	6.682312	2.388757	1.005154	2.965378	7.040740
Chapter 101 y(m)	1.828753	8.134544	9.727411	9.424751	8.399721	4.393013
Chapter 151 x(m)	4.555102	-6.619664	-10.627211	-9.287720	-7.457151	-7.458662
Chapter 151 y(m)	10.725118	9.025570	1.359847	-4.246673	-6.180726	-5.263384
	-5.305444	7.364557	10.974348	7.383896	3.241051	1.785033
	-10.676584	-8.797992	0.843473	7.492370	9.469336	9.301164

Table 2. Speed results of various parts of the dragon dance team

	0s	60s	120s	180s	240s	300s
Faucet (m/s)	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
Chapter 1 (m/s)	0.999971	0.999961	0.999946	0.999917	0.999859	0.999709
Chapter 51 (m/s)	0.999743	0.999660	0.999538	0.999331	0.998941	0.998065
Chapter 101 (m/s)	0.999576	0.999451	0.999270	0.998971	0.998435	0.997302
Chapter 151 (m/s)	0.999449	0.999297	0.999079	0.998735	0.998114	0.996861
Chapter 201 (m/s)	0.999349	0.999178	0.998936	0.998551	0.997892	0.996573
	0.999312	0.999134	0.998884	0.998489	0.997815	0.996477

3.2. Solution to Bench Dragon Collision Model

(1) Solution to ideal situation

Here This article use the Separating Axis Theorem to determine whether two benches overlap. The Separating Axis Theorem is an effective method for determining whether two convex polygons intersect. For the i bench, record its four vertices as $d_{i1}, d_{i2}, d_{i3}, d_{i4}$, and obtain the vectors of its rectangular length and width respectively:

$$\overrightarrow{d_{i1}d_{i2}} = (x_{i2} - x_{i1}, y_{i2} - y_{i1}), \overrightarrow{d_{i2}d_{i3}} = (x_{i3} - x_{i2}, y_{i3} - y_{i2}) \tag{17}$$

Similarly j The separation axis of a bench is: $\vec{N}_3, \vec{N}_4$ General i bench and j Calculate the projection of the four vertices of a bench on the four separation axes, and obtain its projection interval, which is recorded as $[A_{\min}, A_{\max}]$, $[B_{\min}, B_{\max}]$, If the projection interval satisfies the following relationship, it means that there is no overlap this article the two benches:

$$A_{\min} > B_{\max} \text{ or } B_{\min} > A_{\max} \tag{18}$$

This article found that there was no collision this article the dancing dragons during 300s, so this article started from 300s to 1×10^{-5} s traverse for the step size, and use the above method to determine whether there is overlap. If overlap is found, stop and output the movement time of the dragon team. Through calculation, 412s was obtained as the end time of the Dragon team‘s entry.

Result as show in table 3.

Table 3. Results

	Abscissa x(m)	vertical y(m)	speed(m/s)
faucet	1.148135	1.983191	1.000000
Chapter 1	-1.698251	1.704468	0.991600
Chapter51Body	1.212276	4.347856	0.976949
101 Body	-0.464642	-5.887299	0.974643
51 Body	1.040116	-6.948087	0.973729
201 Body	-7.882550	1.9755719	0.973206
Queen of Tail	0.884896	8.331376	0.973032

3.3. Analysis of bench dragon path optimization results

The first arc length is 15.708 meters, and the second arc length is 7.854 meters, totaling 23.5619 meters. After optimization, the first arc length becomes 12.5664 meters, the second arc length 6.2832 meters, with a new total of 18.8496 meters.

The accompanying figure illustrates path changes, showing the spiral background, the original U-turn path, and the optimized U-turn path, using different colors to distinguish the arcs.

In this analysis, this article focus on the coordinates of the front handle of the faucet under the specific condition where the two arcs are tangent to the U-turn space, with R1=3 and R2=1.5. By applying the polar coordinate formulas for both arcs and their corresponding spirals, the tangent points A and B of the two arcs can be determined.

$$\begin{cases} \rho = R_1 \\ \rho = -1.7/(2 * \pi)\theta \end{cases} \qquad \begin{cases} \rho = R_2 \\ \rho = 1.7/(2 * \pi)\theta \end{cases} \tag{19}$$

Solved A (4.5,60/17*(2π)), B (4.5,30/17*(2π))

After converting the coordinates of A and B into Cartesian coordinates, the coordinates of A and B are (-4.4235, 0.2754), (4.4235, -0.8263). Then This article can find the center coordinates O1 (-1.4745, 0.2754) and O2 (2.9745, -0.5509) of the large arc and the small arc.

-100 seconds~0 seconds:

The spiral coordinates are $y = -1.7/(2 * \pi)\theta \qquad \theta \in (0,32\pi)$

Bringing the 0-second faucet coordinate A (4.5,60/17*(2π)) as the initial value into the first question model to solve the problem, This article can get the coordinates of the faucet on the coiling spiral from -100 seconds to 0 seconds and the dragon dance team per second. position and speed.

2.0~3πseconds:

The displacement speed of the faucet is 1m/s, and the length of the great arc is 3π, so bet This articleen 0 and 3π seconds, the front handle of the faucet is on the great arc, and at 0 seconds, the front handle of the faucet is at point A. The distance the faucet moves on the great arc in t seconds is t meters, and the angle of movement is θ, because the ratio of the distance the faucet moves on the circle to the circumference of the circle is equal to the ratio of the angle θ and 2π of the movement.

$$\frac{t}{2R_1\pi} = \frac{\theta}{2\pi} \tag{20}$$

This article can get $\theta = \frac{t}{r}, R_1 = 3$.

The equation of the coordinates of the faucet on the great circle with respect to time t

$$\begin{cases} x = R_1 * \cos\left(\frac{t}{R_1}\right) + x_{o_1} \\ y = R_1 * \sin\left(\frac{t}{R_1}\right) + y_{o_1} \end{cases} \tag{21}$$

Bringing in time t, This article can get the coordinates of the faucet's front handle on the great arc in 0~3π seconds.

3π~9/2πseconds:

Similar to 0~3π seconds, this article can derive the equation of the coordinates of the faucet on the small circle with respect to time t

$$\begin{cases} x = R_2 * \cos\left(\frac{t-3\pi}{R_2}\right) + x_{o_2} \\ y = R_2 * \sin\left(\frac{t-3\pi}{R_2}\right) + y_{o_2} \end{cases} \tag{22}$$

9/2π~100secnonds:

The spiral coordinates are $y = 1.7/(2 * \pi)\theta \quad \theta \in (0,32\pi)$.

Bringing the 0-second faucet coordinate B (4.5, 45/17*(2π)) as the initial value into the first question model to solve the problem, this article can get the coordinates of the faucet on the coiling spiral from -100 seconds to 0 seconds.

4. Conclusion

This study offers a comprehensive understanding of the "bench dragon" movement model, uncovering the dynamic characteristics of dragon dance teams in complex environments through the analysis of the equidistant spiral trajectory. The application of the fourth-order Runge-Kutta method showcases its efficiency and accuracy in solving trajectory equations lacking analytical solutions. Additionally, the axis-separation theorem effectively assesses collision occurrences, providing essential safety guarantees for the model. These findings lay a solid mathematical foundation for practical dragon dance performances.

The fourth-order Runge-Kutta method excels in addressing complex dynamic system problems, delivering accurate numerical solutions for differential equations. Meanwhile, the axis-separation theorem simplifies collision detection, reducing computational costs and enabling rapid assessments in dynamic scenarios. The simulated annealing algorithm proves effective for optimizing turning paths under multiple constraints. Numerical solving methods may be sensitive to initial conditions, leading to local optima that affect overall performance. The optimization outcomes of the simulated annealing algorithm often rely on parameter selection, necessitating careful adjustments. Additionally, constructing complex models and performing multiple iterations may prolong computation times, impacting efficiency.

This research serves as a methodological reference for future studies on dragon dance performances and other dynamic systems. Future investigations could integrate advanced optimization algorithms, such as genetic algorithms or particle swarm optimization, to enhance model performance. Exploring real-world constraints will further improve applicability. Ultimately, simulating the movement of dragon dance teams in more complex environments is essential for providing comprehensive solutions for practical applications.

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