Research on the Kinematic Model of "Benching Dragon" Based on Archimedes Spiral

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Abstract. Based on the Archimedes spiral, this paper establishes the kinematics model of "Bench Dragon" in order to optimize its motion trajectory and improve the scientific and artistic performance. By using the methods of plane geometry, physical kinematics and recursive calculation, this paper focuses on the analysis of the trajectory and position equations of the whole movement of the dragon team. Especially in the adjustment of pitch and motion path, detailed mathematical derivation and approximate analysis are carried out. In this paper, the size and connection mode of the dragon team are analyzed as a whole, and then the kinematics model of the dragon head is established as the benchmark of the movement of the dragon body and tail. The motion model of the dragon body and tail is extended by recursive method, and finally the position of each handle point per second are obtained. This research not only provides a scientific basis for optimizing the motion trajectory of the 'Bench Dragon' performance, enhancing its safety and visual appeal, but also serves as a model for integrating traditional culture with modern technology, offering new approaches for preserving and promoting folk customs.

Keywords: Kinematic model, Archimedes spiral, geometric approximation, recurrence method.

1. Introduction

The Archimedes spiral is widely applied in various fields due to its unique geometrical and mechanical properties. For instance, Liu Fang, Guo Zhongquan, and their colleagues designed an Archimedes spiral curved double-tooth disk soil covering device, addressing issues of uneven soil coverage and seed placement in precision agriculture equipment [1]. Similarly, Chen Hongyue and other researchers enhanced the stability and efficiency of compressors by optimizing coil spring designs [2]. While these studies have leveraged the Archimedes spiral to solve problems in macroscopic fluid mechanics and mechanical systems, there has been limited exploration of its application in fine motion trajectory analysis, especially in fields involving complex kinematics.

This paper aims to fill this gap by applying the Archimedes spiral to model the kinematics of the traditional folk performance known as the "Bench Dragon." This performance, which involves intricate synchronized movements of participants holding bench-like structures to form a dragon shape, presents a unique challenge in motion coordination and trajectory optimization. The primary objective of this study is to establish a kinematic model that optimizes the motion trajectory of the "Bench Dragon," ensuring synchronization of the dragon’s head, body, and tail while enhancing the artistic expression and safety of the performance.

The significance of this research lies in its innovative approach to merging traditional cultural practices with modern scientific techniques. By applying mathematical modeling, plane geometry, and recursive calculations, the study provides performers with accurate motion data and contributes to the preservation and development of traditional folk culture through technological means. This integration of modern technology with traditional art forms can inspire future interdisciplinary research aimed at promoting cultural heritage in a scientifically rigorous manner.

Moreover, the key innovation of this research is the application of the Archimedes spiral to optimize both the pitch and the motion path of the dragon, which has not been previously explored in fine motion trajectory studies. By establishing precise kinematic equations and using recursive methods, the study obtains the position of each handle point per second, enabling highly accurate control of the performance. This research not only advances the scientific understanding of the "Bench Dragon" but also serves as a model for other folk performances, offering new possibilities for integrating cultural expression with advanced kinematics.

2. The overall analysis of the dragon team

In order to ensure the accuracy of the subsequent model establishment, the overall size specification and connection mode of the dragon team were analyzed first. The front view and top view of the dragon team were shown in Figure 1.

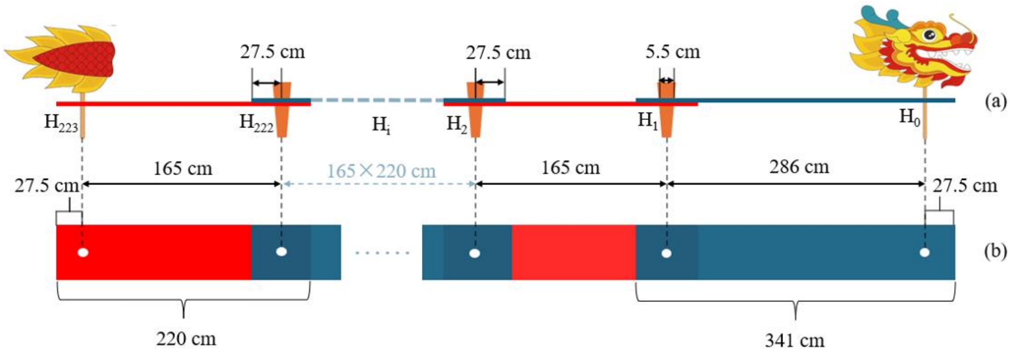

Figure 1. Front view and top view of the dragon team

In this paper, the dragon team is composed of 223 benches closely connected, where H0, H222 and H223 represent the handle points in front of the dragon head, the handle points in front of the dragon tail and the handle points in back of the dragon tail respectively. The first section is the dragon head, the length of the dragon head is 3.41m, which is different from the dragon body and tail. The following 221 sections constitute the dragon body, and the last section is the dragon tail, the length of the dragon body and tail is 2.20m. Therefore, the following modeling can handle the dragon body and tail together.

2.1. Kinematic model of the dragon head

In the process of building the whole "bench dragon" kinematics model, the kinematics model of the front handle of the dragon is analyzed first, and then the kinematics model of the body and tail [4] of the dragon is expanded and established.

Looking up data [5] found that the Archimedes spiral equation is more convenient to express in polar coordinates. However, to ultimately obtain the position of each "bench dragon" in Cartesian coordinate system. Therefore, it is necessary to establish the coordinate system as shown in Figure 3, and make the mathematical model can be converted between two coordinate systems: first, establish Cartesian coordinate system with the center point of the spiral as the origin O, OA as the x axis, perpendicular to OA as the y axis, establish Cartesian coordinate system; Then, a polar coordinate system with O as the origin and OA as the polar axis is established. The position coordinates of the front handle of the dragon head at the t-th second are defined as (r_{t0}, θ_{t0}) and (x_{t0}, y_{t0}).

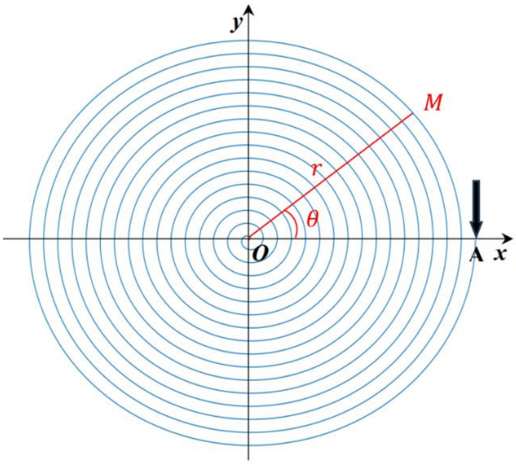

Figure 2. Establishment of Cartesian coordinate system and polar coordinate system

Based on the standard Archimedean spiral, the trajectory equation in polar coordinate system is obtained as follows:

$$r(t) = \frac{b}{2\pi}\theta(t) \tag{1}$$

For the time when the front handle of the dragon head is in, when it enters the clockwise disk along the isometric spiral with a pitch of 55 cm, adjust equation (1):

$$r_{t0} = \frac{b}{2\pi}\theta_{t0} \tag{2}$$

Where is the pitch of the isometric spiral, the dragon head is located at point A of the 16th circle of the spiral at the initial time, then $b = 0.55m$, $\theta_{t0} = 16 \times 2\pi = 32\pi$, θ_{t0} represents the pole Angle of the initial position, and r_{t0} represents the radius of the spiral.

According to the trajectory equation of the polar coordinate system, the coordinate equation in the Cartesian coordinate system:

$$\begin{cases} x = r\cos\theta = \left(\frac{b}{2\pi}\theta\right)\cos\theta \\ y = r\sin\theta = \left(\frac{b}{2\pi}\theta\right)\sin\theta \end{cases} \tag{3}$$

Standard Archimedean spirals define constant angular rate and variable line rate, which obviously does not apply here [6-7]. In this study, the spigot wire rate is defined as constant while the angular rate is variable [8]. The following is a recursion formula for dragon head position to obtain the position coordinates of the handle in front of the dragon head at different times.

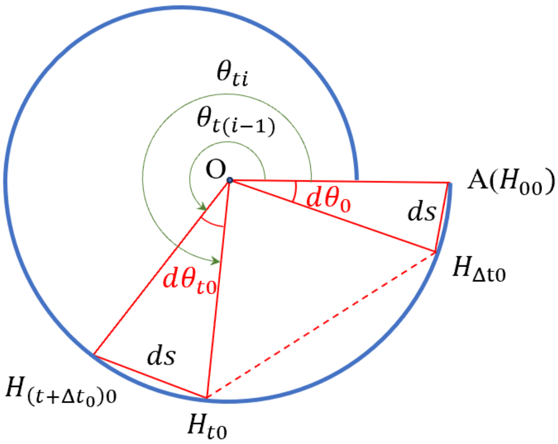

Figure 3. Schematic diagram of the position of the handle in front of the dragon head at different times

[1] As can be seen from Figure 3, Point H_{to} represents the position point of the front handle of the dragon head at time t_0, where $d\theta_{t0}$ is the angle that the front handle of the dragon head travels along the helix in d_t time.

[2] Suppose that the dragon moves a small angular displacement $d\theta_{t0}$ to reach the next point, and the distance from the helix center at that point is equal to the distance from the helix center at the previous point. Therefore, the helix $H_{(t+dt)0}$ arc length from point H_{t0} to point $H_{(t+dt)0}$ can be approximately approximated as a circular arc on a circle with H_{t0} as its center.

[3] The polar coordinate position of $H_{(t+dt)0}$ can be recursively derived from the geometric relationship in Figure 4:

$$\begin{cases} \theta_{(t+dt)0} = \theta_{t0} - d\theta_{t0} \\ r_{(t+dt)0} = \frac{b}{2\pi}\theta(t+dt) \end{cases} \tag{4}$$

Among them, since the front handle of the faucet rotates clockwise along the spiral, equation (4) can be substituted into equation (3) to obtain the Cartesian coordinate position.

In summary, the kinematic model of the dragon head is as follows:

$$\begin{cases} \theta_{(t+dt)0} = \theta_{t0} - d\theta_{t0} \\ r_{(t+dt)0} = \frac{b}{2\pi}\theta(t+dt) \\ x_{(t+dt)0} = \left[\frac{b}{2\pi}\theta(t+dt)\right] \cdot \cos(\theta_{t0} - d\theta_{t0}) \\ y_{(t+dt)0} = \left[\frac{b}{2\pi}\theta(t+dt)\right] \cdot \sin(\theta_{t0} - d\theta_{t0}) \\ (r_{00}, \theta_{00}) = (8.8m, 32\pi) \\ v \cdot dt = \frac{b}{2\pi}\theta(t) \cdot d\theta_{t0} \\ v = const \end{cases} \tag{5}$$

The known speed of the dragon head is substituted into equation (10), and the position of the dragon head per second during the process from the initial moment to 300s is obtained by using Matlab code $v = 1m/s$ iteration and recursion.

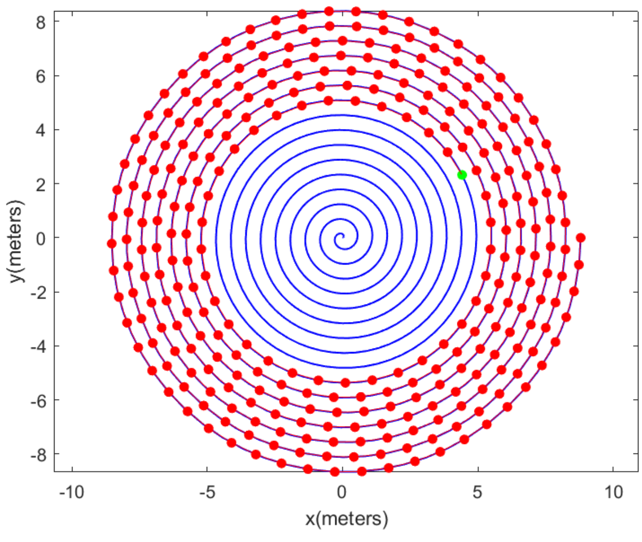

Figure 4. Schematic of the dragon head handle position from 0 to 300 seconds

According to Figure 4, the position of the front handle of the dragon head from the initial moment to 300s per second is analyzed as follows:

Firstly, during the process of inserting the dragon head along the spiral, the radius r of the spiral gradually decreases over time. Define the angle α between two adjacent position points.

$$\alpha = \frac{s}{r} \propto \frac{1}{r} \tag{6}$$

Where, $s = v \times \Delta t = 1\ m$. As the depth of the dragon head into the spiral increases, the radius of the spiral gradually decreases, and the Angle between the adjacent position points will increase α. Secondly, the dragon head is located at (8.800000, 0) meters at the 0 second, and it just starts to disk into the spiral and is in the outer ring. At the 300th second, the position of the dragon head becomes (4.418687, 2.323583) meters, which is located inside the spiral. Finally, since the speed of the front handle of the dragon head remains unchanged, the speed of the front handle of the dragon head from the initial moment to 300s is 1m/s.

2.2. Kinematic model of dragon body and tail

Since the head of the dragon is in front and the body and tail are circling together, Figure 5 shows the position of the handle in front of the dragon and the position of the other handles at t second. The recursive formula of the position coordinates of the front handle of the i segment dragon body at a certain time can be obtained through the geometric relationship, and the position coordinates of the front handle of the i segment dragon body at any time can be obtained by traversing all time points. The specific research is as follows.

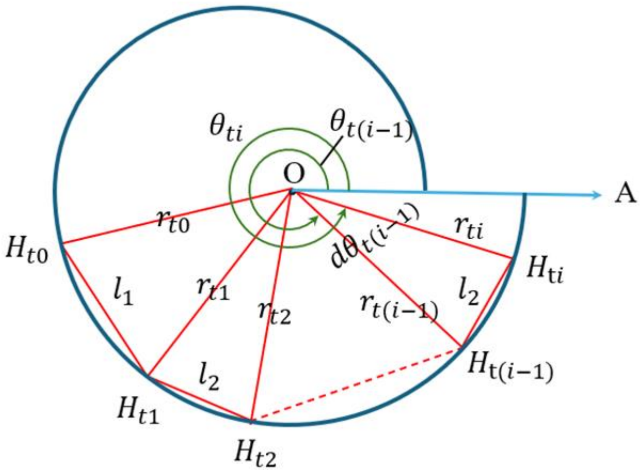

Figure 5. Schematic diagram of the position of each dragon body and tail at t second

The kinematic model (5) of the dragon head mentioned earlier can obtain the coordinates of H_{t0}, which is the position coordinates of the front handle of the dragon head at the t-th second (r_{t0}, θ_{t0}). Meanwhile, based on the overall analysis of the dragon team, it is known that the distance length l_1 between the front handle of the dragon head and the front handle of the first section of the dragon body is $2.86m$. The distance between the two handles of the dragon body and tail is $l_2 = 1.65m$.

For a specific time t-second, the front bench position (r_{ti}, θ_{ti}) can be used to derive the back bench position $\left(r_{t(i+1)}, \theta_{t(i+1)}\right)$, in order to determine the positions of the dragon body and tail.

By approximating the board length as the polar Angle difference between the adjacent benches corresponding to the arc length, other recursive formulas [9-10] for handle position can be obtained. Since the board length of the dragon head and the dragon body are different, the recursive formula has the following two cases:

a. Derive the position of the first dragon body front handle from the position of the dragon head front handle.

b. Derive the position of the i-th dragon body from the position of the (i-1) th dragon body recursively.

In summary, the position equation of the dragon body and tail is as follows:

$$\text{The first section}\begin{cases}\theta_{t1}=\theta_{t0}+d\theta_{t0}\\ r_{t1}=\dfrac{b}{2\pi}\theta_{t1}\\ l_1=2.86m\approx r_{t0}\cdot d\theta_{t0}\end{cases}$$

$$\text{The i}-\text{th segment of the dragon body}\begin{cases}\theta_{ti}=\theta_{t(i-1)}+d\theta_{t(i-1)}\\ r_{ti}=\frac{b}{2\pi}\theta_{ti}\\ l_2=1.65m\approx r_{t(i-1)}\cdot d\theta_{t(i-1)}\end{cases} \tag{7}$$

At this point, the polar coordinates of each bench handle point can be calculated for all time intervals, and then the polar coordinates are converted to Cartesian coordinates:

$$\begin{aligned} x_{ti}&=\frac{b}{2\pi}\theta_{ti}\cdot\cos(\theta_{ti})\\ y_{ti}&=\frac{b}{2\pi}\theta_{ti}\cdot\sin(\theta_{ti})\end{aligned} \tag{8}$$

According to the kinematics equations of the dragon body and tail, the complete position distribution of the dragon dance team at the end of t=300s can be drawn, and the results are presented in the form of Figure 6. Where, the green dot represents the position of the dragon head, while the red dot represents the position of the dragon body and tail, and the blue dot is the initial position point A of the dragon head.

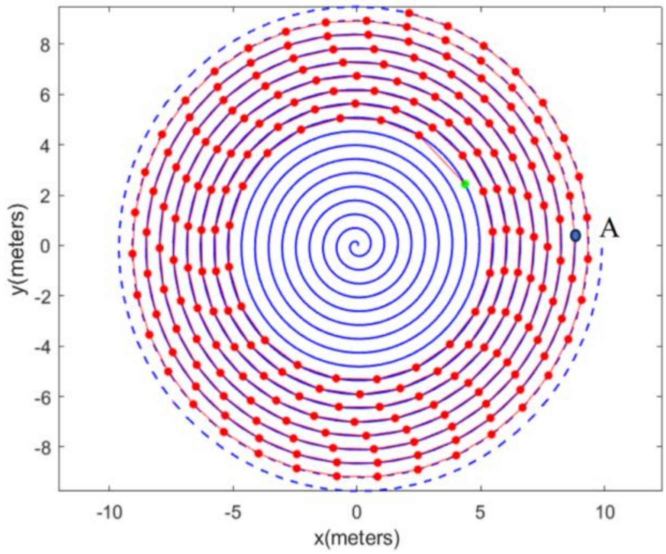

Figure 6. Schematic of the dragon dance team's position over 300 seconds

On further observation, it is found that at t=300s, the dragon dance team has entered the ninth circle of the spiral. By comparing the position of the dragon head with Figure 4, it is found that the position of the dragon head at 300s is consistent, which further verifies the accuracy and reliability of the model.

2.3. Result analysis: the position and speed of a specific node at a specific time

The kinematics model was solved by Matlab to obtain the position of each node per second from the initial moment to t=300s. At the same time, the positions of the front handle of the dragon head, the front handle of the 1st, 51st, 101, 151, 201 sections of the dragon body and the rear handle of the dragon tail at 0s, 60s, 120s, 180s, 240s and 300s are shown in FIG. 7.

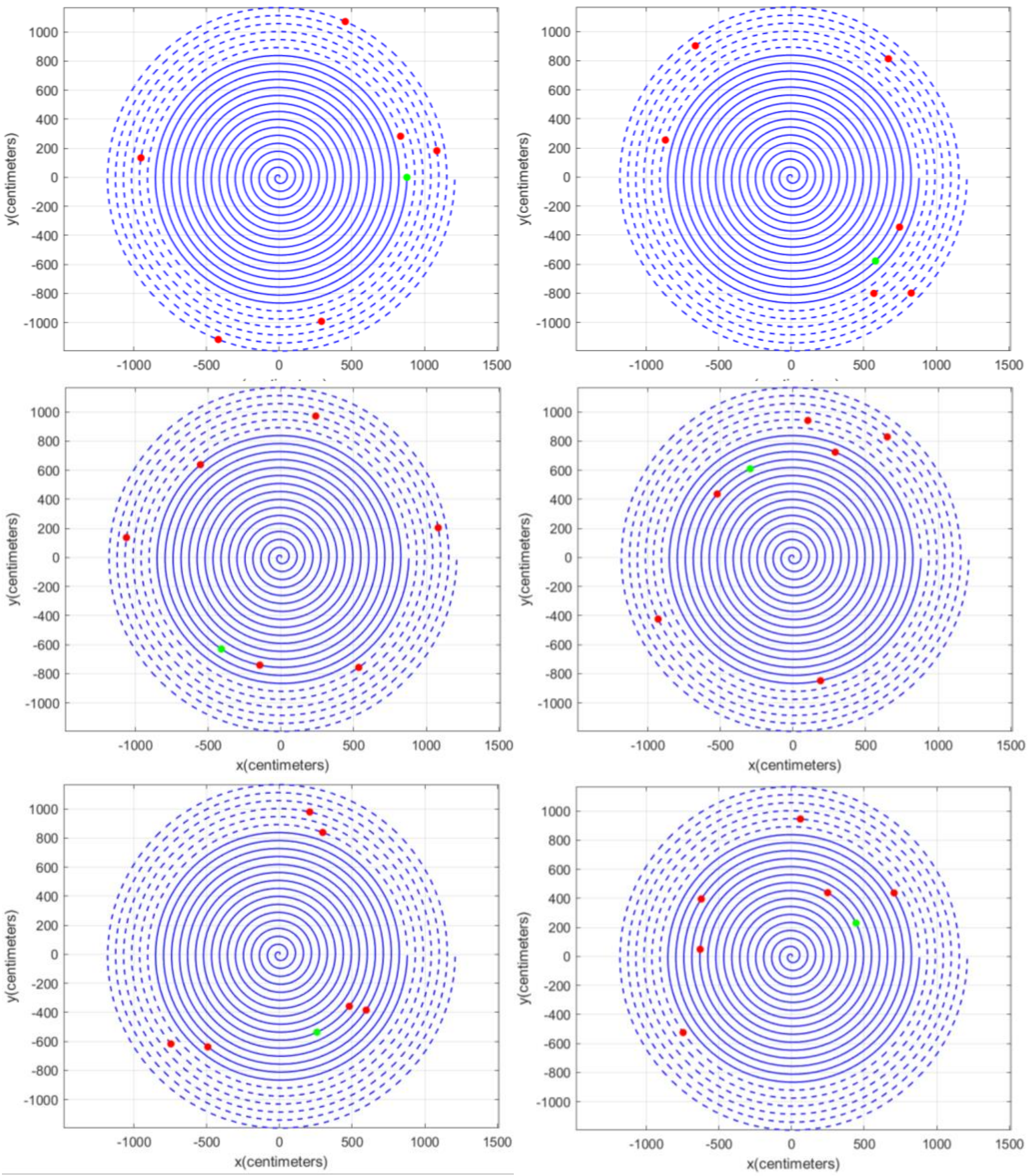

Figure 7. Position diagram of the dragon body at specific nodes

3. Conclusion

This research innovatively applies the Archimedes spiral to model the kinematics of the traditional "Bench Dragon" performance, creating a detailed and accurate motion model that enhances both the scientific and artistic aspects of the performance. Through rigorous mathematical derivation and recursive calculations, the study optimizes the motion trajectories of the dragon's head, body, and tail, ensuring synchronization and improving safety and visual appeal. The study combines modern science with traditional culture to show how technology can help protect and improve cultural heritage. The motion model created gives exact movement information to performers and also sets a base for future studies on similar systems. Verified through computer simulations, the model demonstrates its accuracy and reliability, offering valuable insights for both cultural preservation and technical application. This work deepens the understanding of the "Bench Dragon" and serves as a model for combining traditional culture with advanced kinematic analysis, opening up new avenues for interdisciplinary studies. Future research could apply this model to other traditional performances or enhance "Bench Dragon" shows with augmented reality.

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