Critical Analysis of the "Bench Dragons" Spiral Spinning

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Abstract. The "Bench Dragon" kinematic state requires an isometric solenoidal tangent model as an effective support. However, in the existing theories, there are only expressions for the displacement-velocity relationship obtained by derivation with the tangent starting point as the object of study. In this paper, while selecting the starting point as the first research object, according to the isometric spiral circling law of all the reference points of the kinematic relationship of the recursion, and finally completed the simulation of the motion process of the "Bench Dragon". The theory of coordinate system transformation and the application of numerical rooting method mentioned in the paper are also enriching and supplementing the previous theories. At the end of the article, the results of the basic parameters such as displacement and velocity at different moments from the starting point to the end point of the tairon are analyzed and calculated, which can be used as a support for the theoretical part of this paper.

Keywords: Isometric Solenoidal Tangent Insertion Model, Coordinate System Conversion, Numerical root finding, Law of Sines.

1. Introduction

The traditional Lantern Festival folklore activity "Bench Dragon" popular in Zhejiang and Fujian is one of the important intangible cultural heritages in China, and the wide dissemination of this folklore activity also reflects the people's worship and longing for the traditional national totem "Dragon" as well as their ideals and aspirations for a happy and beautiful life. and longing for a happy and beautiful life in previous theoretical studies, Liu Chongjun's analytical calculations on the principle of isometric solenoids and Hu Xing's theory on the combined application of the sine and cosine theorems have brought good inspiration to the study in this paper [1-3]. However, there is no study that combines the above two theories to form an overall kinematic model of the isospin. Therefore, in response to the overall kinematic model that has not been previously developed in relation to this section, this paper will refine and summarize the theoretical content of this section.

This paper takes the traditional folklore activity "Bench Dragon" as the actual research object, and conducts a more in-depth research on the mathematical-physical relationship embedded in the isometric spiral motion process. The "dragon head" is used as the initial reference point to solve the displacement-velocity relationship at different moments, and the displacement of the subsequent reference point (called the "dragon body reference point" in the actual problem) is gradually recursively deduced from the knowledge of the sine theorem in the trigonometric function relationship. and velocity relationship, and finally form a complete isometric solenoidal tangent model. Meanwhile, the coordinate system transformation and numerical root finding method adopted in the article also provide important help and support for the model simplification [4].

In order to better reflect the full picture of the article, this paper provides a detailed description of the isometric solenoidal tangent model in the following five sections [5]. The innovative nature of the model is described in the introductory section. The second part describes the relevant expertise required in the theoretical analysis process. And the third section describes the modeling process and the overall design flow. The corresponding theoretical results are obtained and some of the calculated values and travel routes are shown. In the last part this paper also give an outlook on the practical value and application prospects of the model in this paper.

2. Relevant theories

2.1. Cartesian Coordinate System Conversion

It is first necessary to transform the base Cartesian coordinate system in the problem into a form of expression in polar coordinates with respect to the relationship between polar diameters and polar angles. The basic transformation relationship applied to plane Cartesian coordinates and polar coordinates is shown in equation (1) below [6]:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \tag{1}$$

2.2. Basic theory of the solenoidal equation

Using the established isometric solenoid path and combining it with the polar coordinate transformation process one can write the basic expression for this isometric solenoid in a polar coordinate system as [7]:

$$r = a + b\theta \tag{2}$$

Where a is a constant and b is a coefficient on the angle, and their corresponding values are:

$$b = \frac{d_1}{2\pi} = \frac{0.55\text{m}}{2\pi} \tag{3}$$

3. Experiment

It is necessary to complete the modeling and solving of the “Bench Dragon” model and the following two basic tasks [8]:

(1) Analyzing and calculating the parameters such as traveling time, coordinate position and instantaneous speed;

(2) Construction of an overall isometric solenoidal kinematic model.3.2 Analysis of experimental results.

The study of the solenoid can be carried out by selecting the origin *O* as the reference point, thus setting the value of the constant *a* to zero, and writing the corresponding simplified solenoidal expression in the following form [9]:

$$r = b\theta = \frac{d_1\theta}{2\pi} \tag{4}$$

Based on the information that “the centers of the connecting handles are located on the threads of the screws”, the condition can be practically represented and the triangular relationship can be introduced by means of the top view of the bench, which is shown in Figure 1 below.

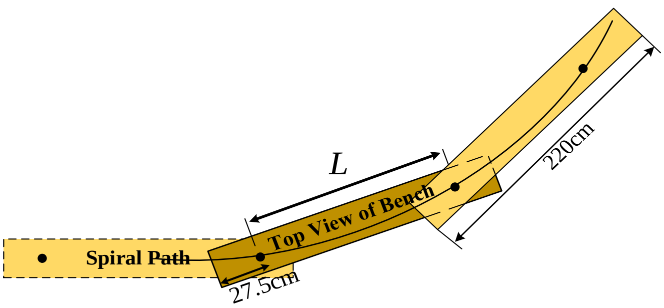

Figure 1. Schematic top view of bench (part)

Using the distance L between the centers of the two handles at the end of each bench given in the figure and the radial distance between the handles at the ends of that bench and the origin *O* of the solenoid r_1 and r_2 construct a triangular relationship between the three as shown in Figure 2 below.

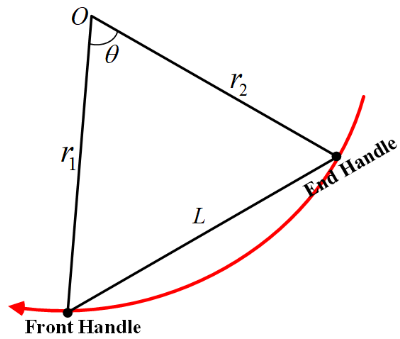

Figure 2. Triangulation between diameter length and centerline

Based on the pitch of the threads and the initial condition that "the initial position of the dragon's head is at point *A* of the 16th revolution", the polar coordinates of the disk entry point *A* can be determined as follows $(8.8, 32\pi)$. Set the position point corresponding to the front handle of the "dragon's head" as the moving point A_0. The corresponding polar angle is θ_0. Further by recursion the moving points of the subsequent handles can be expressed as A_i ($i = 0,1,2,...,223$). The corresponding angle is θ_i. Taking a certain traveling state as an example, a display of the correspondence between the arc length and angle of the disk-in passing in that case can be obtained, as shown in Figure 3.

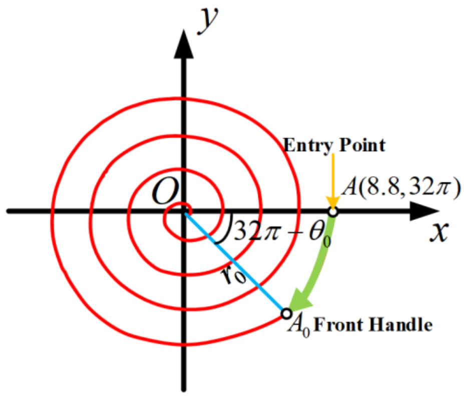

Figure 3. Correspondence between arc length and angle in a traveling state

As can be learned from the display diagram, the portion marked with a green arrow is the portion of the arc length through which the front handle of the dragon's head travels. Using the given conditions of "The A_0 point linear velocity is $v_0 = 1\text{m/s}$, and the value remains constant". A correspondence between the arc length *s* and the polar angle θ_0 can be established:

$$s = v_0 t = \int_{\theta_0}^{32\pi} \sqrt{r_0^2 + (\frac{\mathrm{d}r_0}{\mathrm{d}\theta})^2} \cdot \mathrm{d}\theta \tag{5}$$

A further simplification of the above equation shows that the corresponding expression between the cut-in time and the handle pole angle in front of the dragon's head is:

$$t = \frac{d_1}{2\pi} \cdot [\zeta\sqrt{4\zeta^2 + 1} + \frac{1}{2}\ln(2\zeta + \sqrt{4\zeta^2 + 1}) - \frac{\theta_0}{2}\sqrt{\theta_0^2 + 1} - \frac{1}{2}\ln(\theta_0 + \sqrt{\theta_0^2 + 1})] \tag{6}$$

Which ζ is 16π.

Using the relationship between the time t and the polar angle of the dragon's head front handle given above, the corresponding polar angle of the dragon's head per second can be determined by substituting the values of different moments. According to the polar angle of the dragon's head obtained from the solution, expression (5) and expression (3) can be used to finally solve the coordinates of the front handle of the dragon's head at each moment x_0 and y_0.

According to the process of "determining the angular position of the handle in front of the dragon's head at any time" given in the above content, it is possible to obtain the polar angle of the dragon's head at any time from the initial moment to the final time of 300s. Therefore, it is possible to obtain the coordinates of the position of each handle at different moments by establishing the relationship between the extreme angle of the dragon's head and the extreme angle of the handles in front of the bench in section 1, and then obtaining the corresponding extreme angle of each handle accordingly.

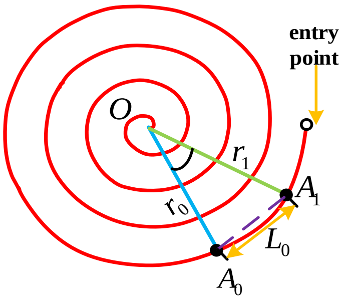

Figure 4. Establishment of the triangular relationship

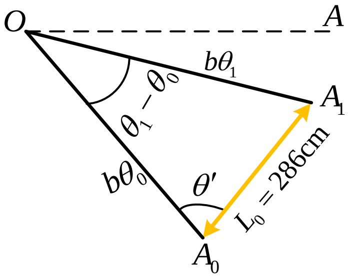

Figure 5. Enlarged Demonstration

According to the process of "determining the angular position of the handle in front of the dragon's head at any time" given in the above content, it is possible to obtain the polar angle of the dragon's head at any time from the initial moment to the final time of 300s. Therefore, it is possible to obtain the coordinates of the position of each handle at different moments by establishing the relationship between the extreme angle of the dragon's head and the extreme angle of the handles in front of the bench in section 1, and then obtaining the corresponding extreme angle of each handle accordingly. Combining the trigonometric relationship mentioned above, the trigonometric relationship between the front handle of the dragon's head located on the screw thread and the front handle of the bench in section 1 can be further constructed and the trigonometric recursive relationship equation can be obtained. The relationship between the angle of the front and rear handles and the length of the corresponding side as they move on the screw thread is shown in Figures 4 and 5 above.

The relationship between the polar angle of the front handle of the bench θ_1 and the angle of the front handle of the dragon's head θ_0 in Section 1 can be obtained by using the sine-cosine theorem in the triangle given in Figure 5. According to the cosine theorem, the two polar angles and the handle spacing L_0 can be expressed as a double covariate for the angles θ' labeled in Figure 5, i.e., there are:

$$\cos\theta' = \frac{(b\theta_1)^2-(b\theta_0)^2-L_0^2}{2b\theta_0 L_0} \tag{7}$$

The correspondence between the polar angle of the front handle of the bench in Section 1 and the front handle of the dragon's head can next be expressed by the sine theorem [10]:

$$\frac{L_0}{\sin(\theta_1-\theta_0)} = \frac{b\theta_1}{\sin\theta'} = \frac{b\theta_1}{\sqrt{1-\cos^2\theta'}} = \frac{b\theta_1}{\sqrt{1-[\frac{(b\theta_1)^2-(b\theta_0)^2-L_0^2}{2b\theta_0 L_0}]^2}} \tag{8}$$

Which b is $\frac{d_1}{2\pi}$.

For this purpose, after using the manual point of the front bench handle in section 1 as the advancement point and the manual point of the front bench handle in section 2 as the concurrent advancement point, the same modeling approach as described above was used to obtain the values of the angles at different moments of the polar angle of the front bench handle in section 2 and to obtain the recursive relationship, which is given by the following expression:

$$\frac{L_i}{\sin(\theta_{i+1}-\theta_i)}=\frac{b\theta_{i+1}}{\sin\theta_i'}=\frac{b\theta_{i+1}}{\sqrt{1-\cos^2\theta_i'}}=\frac{b\theta_{i+1}}{\sqrt{1-[\frac{(b\theta_{i+1})^2-(b\theta_i)^2-L_i^2}{2b\theta_i L_i}]^2}} \tag{9}$$

Using the above recursive relation expression, the corresponding coordinate values of any handle at different moments can be obtained.

In order to study the traveling linear velocity of each bench handle center at different moments , a tiny time microelement Δt must be intercepted for analysis (where Δt is a time interval much smaller than 1s). Taking the modeling of the linear velocity of the front handle of the first bench as an example, the study moment t_1 is selected, and the corresponding initial angle θ_{01} can be obtained by substituting this moment into the “Relation between time t and the polar angle of the front handle of the dragon’s head”, and according to the triangular recurrence relationship given by the above expression (9), the polar angle of the front handle of the first bench can be determined at this moment. θ_{11}, which is set to θ_{11}' after a very small time interval Δt According to the definition of arc length, this paper can establish the relationship between the linear velocity and the angular position of the handle in the state of time microelement:

$$\Delta s_1=v_1(t_1+\Delta t)-v_1 t_1=\int_{\theta_{11}}^{\theta_{11}'}\sqrt{r_1^2+(\frac{\mathrm{d}r_1}{\mathrm{d}\theta})^2}\cdot\mathrm{d}\theta \tag{10}$$

Simplification of the above equation yields the linear velocity of the first bench front handle travel at moment t_1:

$$v_1=\frac{\int_{\theta_{11}}^{\theta_{11}'}\sqrt{r_1^2+(\frac{\mathrm{d}r_1}{\mathrm{d}\theta})^2}\cdot\mathrm{d}\theta}{\Delta t} \tag{11}$$

Combining the above modeling process with recursion can obtain the traveling linear velocity of any handle center at any moment condition, expressed as follows:

$$v_i(t)=\frac{\int_{\theta_{it}}^{\theta_{it}'}\sqrt{r_i^2+(\frac{\mathrm{d}r_i}{\mathrm{d}\theta})^2}\cdot\mathrm{d}\theta}{\Delta t}=\frac{\int_{\theta_{it}}^{\theta_{it}'}\sqrt{(\frac{d_1\theta_{it}}{2\pi})^2+\frac{d_1^2}{4\pi^2}\cdot(\frac{\mathrm{d}\theta_{it}}{\mathrm{d}\theta})^2}\cdot\mathrm{d}\theta}{\Delta t} \tag{12}$$

Based on the relationship between the time t and the handle angle in front of the dragon’s head and the subsequent recurrence of the handle angle, the value of θ_i can be obtained for each handle at different moments in time.

4. Results

Through the law of transformation of the coordinate system, the position of each handle corresponding to the polar angle can be solved in right-angle coordinates. Then, by substituting theta i into the expression (12), the recursive relation for the traveling linear velocity can be deduced, and at the same time, the value of the linear velocity at the corresponding moments can also be solved, and some of the results of the positions and linear velocities are shown in Table 1 and 2 below. At the same time, this paper also show the distribution of positions at 0s and 300s as shown in Figures 6 and 7.

Table 1. Coordinate positions of some of the handles at a given moment in time

	0s	60s	120s	180s	240s	300s
Dragon's Head x(m)	8.800000	5.799212	-4.084890	-2.963608	2.594495	4.420273
Dragon's Head y(m)	0.000001	-5.771090	-6.304477	6.094781	-5.356742	2.320430
Body 1 x(m)	8.363823	7.456760	-1.445475	-5.237118	4.821222	2.459485
Body 1 y(m)	2.826547	-3.440395	-7.405882	4.359627	-3.561948	4.402478
Body 51 x(m)	-9.518734	-8.686313	-5.543147	2.890455	5.980009	-6.301346
Body 51 y(m)	1.341125	2.540121	6.377948	7.249289	-3.827761	0.465829
Body 101 x(m)	2.914003	5.687111	5.361930	1.898806	-4.917351	-6.237715
Body 101 y(m)	-9.918305	-8.001387	-7.557645	-8.471612	-6.379891	3.936019
Body 151 x(m)	10.861720	6.682329	2.388750	1.005151	2.965336	7.040752
Body 151 y(m)	1.828792	8.134529	9.727413	9.424751	8.399736	4.392994
Body 201 x(m)	4.555100	-6.619682	-10.627213	-9.287714	-7.457119	-7.458670
Body 201 y(m)	10.725119	9.025557	1.359827	-4.246686	-6.180765	-5.263373
Dragon's Tail x(m)	-5.305446	7.364583	10.974347	7.383893	3.241001	1.785042
Dragon's Tail y(m)	-10.676583	-8.797970	0.843485	7.492373	9.469354	9.301162

Table 2. Values of linear velocities of some handles at a specific moment in time

	0s	60s	120s	180s	240s	300s
Dragon's Head (m/s)	1.000012	1.000028	0.999971	0.999992	1.000002	1.000013
Body 1 (m/s)	0.999988	1.000004	0.999936	0.999917	0.999862	0.999739
Body 51 (m/s)	0.999913	0.999568	0.999497	0.999180	0.998966	0.998144
Body 101 (m/s)	0.999695	0.999327	0.999290	0.998906	0.998626	0.997293
Body 151 (m/s)	0.999755	0.999049	0.998939	0.998663	0.998368	0.996743
Body 201 (m/s)	0.999444	0.999206	0.998810	0.998637	0.998192	0.996486
Dragon's Tail (m/s)	0.999278	0.999305	0.998764	0.998509	0.998112	0.996427

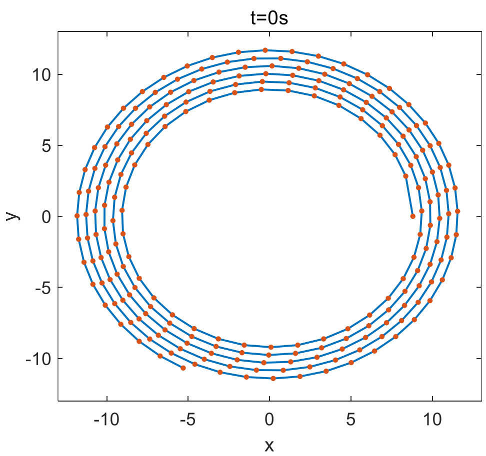

Figure 6. Initial positions of each point

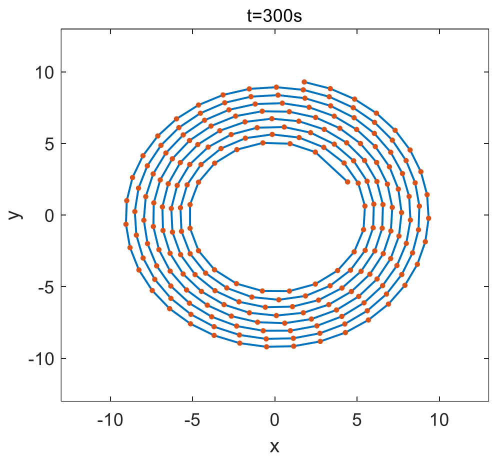

Figure 7. Distribution of point locations at 300s

5. Conclusion

In this paper, each handle node of the "Bench Dragon" is selected as a reference point, and through the construction of the triangular relationship between the lines connecting different reference points and the kinematics theory embedded in the equidistant solenoidal running model, the displacement and velocity information corresponding to each reference point at different moments is obtained by recursion, and then simulate the "Bench Dragon". The whole process of "Bench Dragon" traveling is simulated. By solving the triangular relationship between the reference points of the handles and the origin of the polar coordinates at the ends of each part and obtaining the recursive relationship, the corresponding polar angles and coordinate positions of all handles at different moments can be obtained. Finally, the instantaneous velocities are investigated in a very small time step of the microelement, and the instantaneous velocities of the handles at different moments are solved by using the equation of the definition of the arc length of the solenoid.

The “Bench Dragon” model studied in this paper is a simulated “dragon” shaped structure formed by a long bench and 223 short benches connected head to tail. The long bench with a board length of 341cm serves as the “dragon head” part, while the rest of the short benches with a board length of 220 cm serve as the subsequent dragon body and dragon tail parts. There are circular openings at the two fixed ends of each bench to insert connecting handles, so that the interconnection between the benches can be carried out by means of the handles, and the interconnection is completed to form a complete structure. In the circling process, the “dragon head” bench is required as the starting point of the movement to further drive the movement of the overall “dragon” shaped structure, and finally form the circling traveling “Bench Dragon” model.

The “Bench Dragon” model can not only understand the position and running speed of each node in the actual traveling process, but more importantly, it can get the overall improvement of flexibility and executability in the actual operation process.

In the course of future research, the model can also be applied to the analysis and simulation of the solenoidal equations, the analysis of the kinematics of solenoidal magnetic fields, and the recursion of the relationship between the states of motion of multiple objects, among many other fields.

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